Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WST02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1 A local pottery makes cups. The number of faulty cups made by the pottery in a week follows a Poisson distribution with a mean of 6

In a randomly chosen week, the probability that there will be at least x faulty cups made is 0.1528

(a) Find the value of x

(3)

(b) Use a normal approximation to find the probability that in 6 randomly chosen weeks the total number of faulty cups made is fewer than 32

(4)

A week is called a "*poor week*" if at least x faulty cups are made, where x is the value found in part (a).

(c) Find the probability that in 50 randomly chosen weeks, more than 1 is a "*poor week*".

(4)

(a) $X \sim Po(6)$

$P(X \geqslant x) = 0.1528 \Rightarrow P(X \leqslant x - 1) = 1 - 0.1528 = 0.8472$

From tables, $P(X \leqslant 8) = 0.8472 \Rightarrow x - 1 = 8$

$x = 9$

(b) Y = number of faulty cups in 6 weeks, $Y \sim Po(36)$

Approximation: $Y \approx N(36, 36)$

$$P(Y < 32) \approx P(W < 31.5) = P\left(Z < \frac{31.5 - 36}{\sqrt{36}}\right) = P(Z < -0.75)$$

$= 1 - 0.7734 = 0.2266$

$P(\text{total fewer than } 32) = 0.227$

(c) W = number of poor weeks in 50, $W \sim B(50, 0.1528)$

$P(W > 1) = 1 - P(W \leqslant 1) = 1 - [P(W = 0) + P(W = 1)]$

$= 1 - (0.8472^{50} + 50 \times 0.1528 \times 0.8472^{49})$

$= 1 - (0.000250... + 0.002261...)$

$= 0.99748... = 0.997$

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Question 1 continued

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Question 1 continued

(Total 11 marks)

Q1

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2 The continuous random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < -k \\ \dfrac{x+k}{4k} & -k \leqslant x \leqslant 3k \\ 1 & x > 3k \end{cases}$$

where k is a positive constant.

(a) Specify fully, in terms of k, the probability density function of X

(2)

(b) Write down, in terms of k, the value of $E(X)$

(1)

(c) Show that $\text{Var}(X) = \dfrac{4}{3}k^2$

(2)

(d) Find, in terms of k, the value of $E(3X^2)$

(3)

(a) $f(x) = F'(x)$

$$f(x) = \begin{cases} \dfrac{1}{4k} & -k \leqslant x \leqslant 3k \\ 0 & \text{otherwise} \end{cases}$$

(b) $E(X) = k$

(c) X is uniform (rectangular) on $[-k, 3k]$

$$\text{Var}(X) = \frac{(\beta - \alpha)^2}{12} = \frac{(3k - (-k))^2}{12} = \frac{16k^2}{12}$$

$$\text{Var}(X) = \frac{4k^2}{3} \quad \text{as required}$$

(d) $E(X^2) = \text{Var}(X) + [E(X)]^2 = \dfrac{4k^2}{3} + k^2 = \dfrac{7k^2}{3}$

$$E(3X^2) = 3E(X^2) = 3 \times \frac{7k^2}{3} = 7k^2$$

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Question 2 continued

Q2

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3 A photocopier in a school is known to break down at random at a mean rate of 8 times per week.

(a) Give a reason why a Poisson distribution could be used to model the number of breakdowns.

(1)

The headteacher of the school replaces the photocopier with a refurbished one and wants to find out if the rate of breakdowns has increased or decreased.

(b) Write down suitable null and alternative hypotheses that the headteacher should use.

(1)

The refurbished photocopier was monitored for the first week after it was installed.

(c) Using a 5% level of significance, find the critical region to test whether the rate of breakdowns has now changed.

(3)

(d) Find the actual significance level of a test based on the critical region from part (c).

(2)

During the first week after it was installed there were 4 breakdowns.

(e) Comment on this finding in the light of the critical region found in part (c).

(2)

(a) *Breakdowns occur singly, independently and at random, at a constant (mean) rate.*

(b) $H_0: \lambda = 8$ $H_1: \lambda \neq 8$

(c) X = number of breakdowns in a week, $X \sim Po(8)$ under H_0

2.5% in each tail:

Lower tail: $P(X \leq 2) = 0.0138$, $P(X \leq 3) = 0.0424$

Upper tail: $P(X \geq 14) = 1 - 0.9658 = 0.0342$

$P(X \geq 15) = 1 - 0.9827 = 0.0173$

Critical region: $X \leq 2$ or $X \geq 15$

(d) Actual significance level = $P(X \leq 2) + P(X \geq 15)$

$= 0.0138 + 0.0173 = 0.0311$ (3.11%)

(e) 4 is not in the critical region, so do not reject H_0

There is insufficient evidence that the rate of breakdowns has changed with the refurbished photocopier.

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Question 3 continued

Q3

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4 The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}k(x-1) & 1 \leqslant x \leqslant 3 \\ k & 3 < x \leqslant 6 \\ \frac{1}{4}k(10-x) & 6 < x \leqslant 10 \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

(a) Sketch $f(x)$ for all values of x

(2)

(b) Show that $k = \frac{1}{6}$

(2)

(c) Specify fully the cumulative distribution function $F(x)$ of X

(7)

Given that $E(X) = \frac{61}{12}$

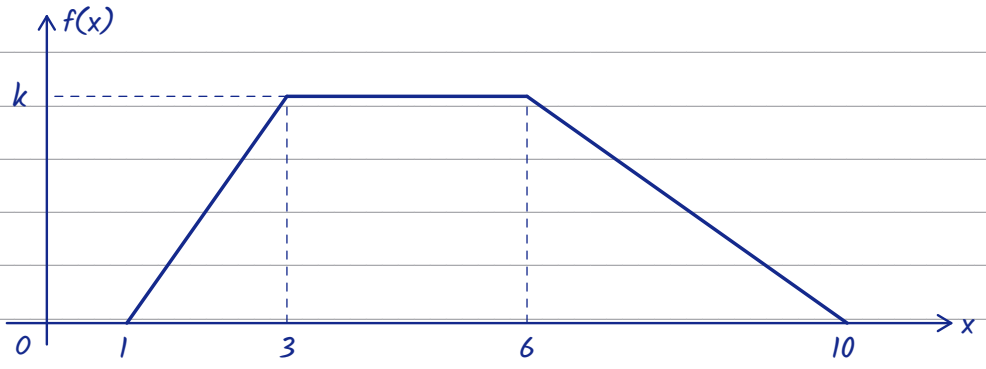
(d) find $P(X > E(X))$

(2)

(e) Describe the skewness of the distribution, giving a reason for your answer.

(2)

(a)



(b) Area of trapezium $= \frac{1}{2}(3 + 9) \times k = 1$

$6k = 1$

$k = \frac{1}{6}$ as required

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Question 4 continued

(c) For $1 \leqslant x \leqslant 3$: $F(x) = \int_1^x \frac{1}{12}(t-1)\,dt = \left[\frac{(t-1)^2}{24}\right]_1^x = \frac{(x-1)^2}{24}$

$F(3) = \frac{4}{24} = \frac{1}{6}$

For $3 < x \leqslant 6$: $F(x) = \frac{1}{6} + \int_3^x \frac{1}{6}\,dt = \frac{1}{6} + \frac{x-3}{6} = \frac{x-2}{6}$

$F(6) = \frac{4}{6} = \frac{2}{3}$

For $6 < x \leqslant 10$: $F(x) = \frac{2}{3} + \int_6^x \frac{10-t}{24}\,dt = \frac{2}{3} + \left[\frac{10t}{24} - \frac{t^2}{48}\right]_6^x$

$= \frac{2}{3} + \frac{10x}{24} - \frac{x^2}{48} - \frac{60}{24} + \frac{36}{48} = \frac{20x - x^2 - 52}{48}$

Check: $F(10) = \frac{200 - 100 - 52}{48} = 1$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{(x-1)^2}{24} & 1 \leqslant x \leqslant 3 \\ \frac{x-2}{6} & 3 < x \leqslant 6 \\ \frac{20x - x^2 - 52}{48} & 6 < x \leqslant 10 \\ 1 & x > 10 \end{cases}$$

(d) $P(X > E(X)) = 1 - F\left(\frac{61}{12}\right) = 1 - \frac{61/12 - 2}{6} = 1 - \frac{37}{72}$

$= \frac{35}{72} = 0.48611\ldots = 0.486$

(e) $P(X > \text{mean}) = 0.486 < 0.5$, so the median is less than the mean (median < mean), therefore the distribution has positive skew.

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Question 4 continued

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Question 4 continued

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Q4

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5 Applicants for a pilot training programme with a passenger airline are screened for colour blindness. Past records show that the proportion of applicants identified as colour blind is 0.045

(a) Write down a suitable model for the distribution of the number of applicants identified as colour blind from a total of n applicants.

(1)

(b) State one assumption necessary for this distribution to be a suitable model of this situation.

(1)

(c) Using a suitable approximation, find the probability that exactly 5 out of 120 applicants are identified as colour blind.

(3)

(d) Explain why the approximation that you used in part (c) is appropriate.

(2)

Jaymini claims that 75% of all applicants for this training programme go on to become pilots.

From a random sample of 96 applicants for this training programme 67 go on to become pilots.

(e) Using a suitable approximation, test Jaymini's claim at the 5% level of significance. State your hypotheses clearly.

(7)

(a) $X \sim B(n, 0.045)$

(b) Applicants are independent of each other (whether one applicant is colour blind does not affect another).

(c) $X \sim B(120, 0.045)$, n large and p small, so $X \approx Po(5.4)$

$$P(X = 5) = \frac{e^{-5.4} \times 5.4^5}{5!} = 0.17282\ldots = 0.173$$

(d) The number of applicants $n = 120$ is large and the probability $p = 0.045$ is small ($np = 5.4 < 10$).

(e) $H_0: p = 0.75$ $H_1: p \neq 0.75$

Y = number of applicants who become pilots, $Y \sim B(96, 0.75)$

$np = 72$, $np(1 - p) = 18$, so $Y \approx N(72, 18)$

$$P(Y \leqslant 67) \approx P(W < 67.5) = P\left(Z < \frac{67.5 - 72}{\sqrt{18}}\right)$$

$$= P(Z < -1.0607) = P(Z < -1.06) = 1 - 0.8554 = 0.1446$$

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Question 5 continued

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Two-tailed test: 0.1446 > 0.025

Not significant: there is insufficient evidence to reject H_0

There is insufficient evidence against Jaymini's claim that 75% of applicants for this training programme go on to become pilots.

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Question 5 continued

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Question 5 continued

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Q5

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6 (a) Explain what you understand by the sampling distribution of a statistic.

(1)

At Sam's cafe a standard breakfast consists of 6 breakfast items. Customers can then choose to upgrade to a medium breakfast by adding 1 extra breakfast item or they can upgrade to a large breakfast by adding 2 extra breakfast items. Standard, medium and large breakfasts are sold in the ratio 6 : 3 : 2 respectively.

A random sample of 2 customers is taken from customers who have bought a breakfast from Sam's cafe on a particular day.

(b) Find the sampling distribution for the total number, T, of breakfast items bought by these 2 customers. Show your working clearly.

(7)

(c) Find E(T)

(2)

(a) The sampling distribution of a statistic is the probability distribution of the statistic: all the possible values of the statistic (from all possible random samples) and their probabilities.

(b) $P(6) = \frac{6}{11}$, $P(7) = \frac{3}{11}$, $P(8) = \frac{2}{11}$

Possible totals T: 12, 13, 14, 15, 16

Combinations: (6, 6), (6, 7), (7, 6), (6, 8), (8, 6), (7, 7), (7, 8), (8, 7), (8, 8)

$P(T = 12) = \left(\frac{6}{11}\right)^2 = \frac{36}{121}$

$P(T = 13) = 2 \times \frac{6}{11} \times \frac{3}{11} = \frac{36}{121}$

$P(T = 14) = 2 \times \frac{6}{11} \times \frac{2}{11} + \left(\frac{3}{11}\right)^2 = \frac{24}{121} + \frac{9}{121} = \frac{33}{121}$

$P(T = 15) = 2 \times \frac{3}{11} \times \frac{2}{11} = \frac{12}{121}$

$P(T = 16) = \left(\frac{2}{11}\right)^2 = \frac{4}{121}$

t	12	13	14	15	16
$P(T = t)$	$\frac{36}{121}$	$\frac{36}{121}$	$\frac{33}{121}$	$\frac{12}{121}$	$\frac{4}{121}$

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Question 6 continued

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(c) $E(T) = 12 \times \frac{36}{121} + 13 \times \frac{36}{121} + 14 \times \frac{33}{121} + 15 \times \frac{12}{121} + 16 \times \frac{4}{121}$

$E(T) = \frac{1606}{121} = \frac{146}{11} = 13.2727\ldots = 13.3$

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Question 6 continued

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Question 6 continued

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Q6

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7 The sides of a square are each of length L cm and its area is A cm^2

Given that A is uniformly distributed on the interval [10, 30]

(a) find P($L \geqslant 4.5$)

(2)

(b) find Var(L)

(6)

(a) $P(L \geq 4.5) = P(A \geq 4.5^2) = P(A \geq 20.25)$

$= (30 - 20.25) \times \frac{1}{20} = \frac{9.75}{20} = 0.4875$

(b) $Var(L) = E(L^2) - [E(L)]^2$

$E(L^2) = E(A) = \frac{10 + 30}{2} = 20$

$f(a) = \frac{1}{20}$ for $10 \leq a \leq 30$, $L = \sqrt{A}$

$E(L) = E(\sqrt{A}) = \frac{1}{20} \int_{10}^{30} a^{1/2}\, da = \frac{1}{20} \left[\frac{2}{3} a^{3/2} \right]_{10}^{30}$

$= \frac{1}{30} (30^{3/2} - 10^{3/2}) = \frac{1}{30} (164.3167... - 31.6227...)$

$E(L) = 4.42313...$

$Var(L) = 20 - (4.42313...)^2 = 20 - 19.56410...$

$Var(L) = 0.43589... = 0.436$

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Question 7 continued

Q7

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TOTAL FOR PAPER: 75 MARKS

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