Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WMA13/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P3

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

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**1.** Find, using calculus, the $x$ coordinate of the stationary point on the curve with equation

$$y = (2x + 5)e^{3x}$$

**(4)**

$y = (2x + 5)e^{3x}$

$\frac{dy}{dx} = 2e^{3x} + 3(2x + 5)e^{3x}$

$= e^{3x}(2 + 6x + 15) = e^{3x}(6x + 17)$

Stationary point: $\frac{dy}{dx} = 0 \Rightarrow e^{3x}(6x + 17) = 0$

$e^{3x} \neq 0$ so $6x + 17 = 0$

$x = -\frac{17}{6}$

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**Question 1 continued**

**Q1**

**(Total 4 marks)**

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**2.** (a) Show that the equation

$$8\cos\theta = 3\operatorname{cosec}\theta$$

can be written in the form

$$\sin 2\theta = k$$

where $k$ is a constant to be found.

**(3)**

(b) Hence find the smallest positive solution of the equation

$$8\cos\theta = 3\operatorname{cosec}\theta$$

giving your answer, in degrees, to one decimal place.

**(2)**

(a) $8\cos\theta = 3\operatorname{cosec}\theta = \dfrac{3}{\sin\theta}$ since $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\Rightarrow 8\sin\theta\cos\theta = 3$

$\Rightarrow 4(2\sin\theta\cos\theta) = 3$ using $\sin 2\theta = 2\sin\theta\cos\theta$

$\Rightarrow 4\sin 2\theta = 3$

$\Rightarrow \sin 2\theta = \dfrac{3}{4}$ $k = \dfrac{3}{4}$

(b) $2\theta = \arcsin\left(\dfrac{3}{4}\right) = 48.590...°$

$\theta = \dfrac{48.590...}{2} = 24.295...°$

$\theta = 24.3°$ (1 d.p.)

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**Question 2 continued**

**Q2**

**(Total 5 marks)**

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**3.** (i) Find, in simplest form,

$$\int (2x - 5)^7 \, dx$$

**(2)**

(ii) Show, by algebraic integration, that

$$\int_0^{\frac{\pi}{3}} \frac{4\sin x}{1 + 2\cos x} dx = \ln a$$

where $a$ is a rational constant to be found.

**(4)**

(i) $\int (2x - 5)^7 \, dx = \frac{(2x - 5)^8}{8 \times 2} + c$

$= \frac{(2x - 5)^8}{16} + c$

(ii) $\frac{d}{dx}[\ln(1 + 2\cos x)] = \frac{-2\sin x}{1 + 2\cos x}$

so $\int \frac{4\sin x}{1 + 2\cos x} dx = -2\ln(1 + 2\cos x) + c$

$\int_0^{\pi/3} \frac{4\sin x}{1 + 2\cos x} dx = \left[-2\ln(1 + 2\cos x)\right]_0^{\pi/3}$

$= -2\ln\left(1 + 2\cos\frac{\pi}{3}\right) - (-2\ln(1 + 2\cos 0))$

$= -2\ln(1 + 1) + 2\ln(1 + 2)$

$= -2\ln 2 + 2\ln 3$

$= \ln 9 - \ln 4 = \ln\frac{9}{4}$ $\quad a = \frac{9}{4}$

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**Question 3 continued**

**Q3**

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**4.** The growth of a weed on the surface of a pond is being studied.

The surface area of the pond covered by the weed, $A\,\text{m}^2$, is modelled by the equation

$$A = \frac{80pe^{0.15t}}{pe^{0.15t} + 4}$$

where $p$ is a positive constant and $t$ is the number of days after the start of the study.

Given that

- $30\,\text{m}^2$ of the surface of the pond was covered by the weed at the start of the study
- $50\,\text{m}^2$ of the surface of the pond was covered by the weed $T$ days after the start of the study

(a) show that $p = 2.4$

**(2)**

(b) find the value of $T$, giving your answer to one decimal place.

(*Solutions relying entirely on graphical or numerical methods are not acceptable.*)

**(4)**

The weed grows until it covers the surface of the pond.

(c) Find, according to the model, the maximum possible surface area of the pond.

**(1)**

(a) $t = 0$, $A = 30$, $e^0 = 1$: $30 = \dfrac{80p \times 1}{p \times 1 + 4} = \dfrac{80p}{p + 4}$

$30(p + 4) = 80p \Rightarrow 30p + 120 = 80p$

$50p = 120 \Rightarrow p = \dfrac{120}{50} = 2.4$ as required

(b) $50 = \dfrac{80 \times 2.4e^{0.15T}}{2.4e^{0.15T} + 4} = \dfrac{192e^{0.15T}}{2.4e^{0.15T} + 4}$

$120e^{0.15T} + 200 = 192e^{0.15T}$

$72e^{0.15T} = 200 \Rightarrow e^{0.15T} = \dfrac{200}{72}$

$0.15T = \ln\left(\dfrac{200}{72}\right) \Rightarrow T = \dfrac{1}{0.15}\ln\left(\dfrac{200}{72}\right)$

$T = 6.8110\ldots = 6.8$ (1 d.p.)

(c) As $t \to \infty$, $A = \dfrac{80p}{p + 4e^{-0.15t}} \to \dfrac{80p}{p} = 80$

Maximum surface area = $80\ \text{m}^2$

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**Question 4 continued**

**Q4**

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**5.**

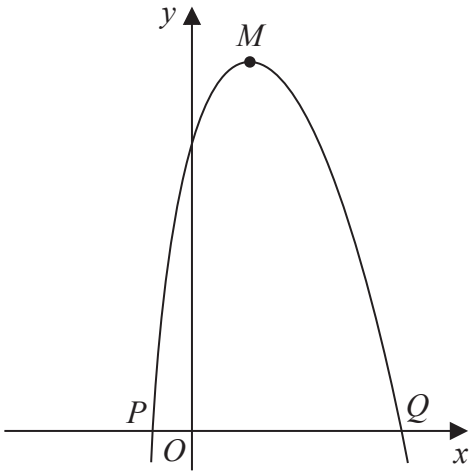



**Figure 1**

Figure 1 shows a sketch of part of the curve with equation

$$y = 6\ln(2x + 3) - \frac{1}{2}x^2 + 4 \qquad x > -\frac{3}{2}$$

The curve cuts the negative $x$-axis at the point $P$, as shown in Figure 1.

(a) Show that the $x$ coordinate of $P$ lies in the interval $[-1.25, -1.2]$

**(2)**

The curve cuts the positive $x$-axis at the point $Q$, also shown in Figure 1.

Using the iterative formula

$$x_{n+1} = \sqrt{12\ln(2x_n + 3) + 8} \quad \text{with} \quad x_1 = 6$$

(b) (i) find, to 4 decimal places, the value of $x_2$

(ii) find, by continued iteration, the $x$ coordinate of $Q$. Give your answer to 4 decimal places.

**(3)**

The curve has a maximum turning point at $M$, as shown in Figure 1.

(c) Using calculus and showing each stage of your working, find the $x$ coordinate of $M$.

**(4)**

(a) $y(-1.25) = 6\ln(0.5) - 0.5(-1.25)^2 + 4 = -0.940...$

$y(-1.2) = 6\ln(0.6) - 0.5(-1.2)^2 + 4 = 0.215...$

Sign change and y is continuous on $[-1.25, -1.2]$,

so the x coordinate of P lies in $[-1.25, -1.2]$

(b)(i) $x_2 = \sqrt{12\ln(2 \times 6 + 3) + 8} = \sqrt{12\ln 15 + 8}$

$x_2 = 6.36369... = 6.3637$ (4 d.p.)

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**Question 5 continued**

(ii) $x_3 = 6.40818...$, $x_4 = 6.41346...$, $x_5 = 6.41409...$

$x_6 = 6.414164...$, $x_7 = 6.414173...$, $x_8 = 6.414174...$

x coordinate of Q = 6.4142 (4 d.p.)

(c) $\frac{dy}{dx} = 6 \times \frac{2}{2x+3} - x = \frac{12}{2x+3} - x$

At M, $\frac{dy}{dx} = 0 \Rightarrow \frac{12}{2x+3} = x$

$12 = x(2x+3) \Rightarrow 2x^2 + 3x - 12 = 0$

$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-12)}}{2(2)} = \frac{-3 \pm \sqrt{105}}{4}$

$x > -\frac{3}{2}$ so reject $x = \frac{-3 - \sqrt{105}}{4} = -3.31...$

x coordinate of M $= \frac{-3 + \sqrt{105}}{4} = 1.81$ (3 s.f.)

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**Question 5 continued**

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**Question 5 continued**

**Q5**

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**6.** The function f is defined by

$$f(x) = \frac{5x - 3}{x - 4} \qquad x > 4$$

(a) Show, by using calculus, that f is a decreasing function.

**(3)**

(b) Find $f^{-1}$

**(3)**

(c) (i) Show that $ff(x) = \frac{ax + b}{x + c}$ where $a$, $b$ and $c$ are constants to be found.

(ii) Deduce the range of ff.

**(5)**

(a) $f'(x) = \frac{5(x-4) - (5x-3)(1)}{(x-4)^2} = \frac{5x - 20 - 5x + 3}{(x-4)^2}$

$= \frac{-17}{(x-4)^2}$

$(x-4)^2 > 0$ so $f'(x) < 0$ for all $x > 4$

hence f is a decreasing function

(b) $y = \frac{5x-3}{x-4} \Rightarrow xy - 4y = 5x - 3$

$xy - 5x = 4y - 3 \Rightarrow x(y-5) = 4y - 3$

$x = \frac{4y-3}{y-5}$

$f^{-1}(x) = \frac{4x-3}{x-5}, \quad x > 5$

(c)(i) $ff(x) = f\left(\frac{5x-3}{x-4}\right) = (5 \times \frac{5x-3}{x-4} - 3) \div (\frac{5x-3}{x-4} - 4)$

$= \frac{5(5x-3) - 3(x-4)}{(5x-3) - 4(x-4)}$ (x (x − 4) top and bottom)

$= \frac{25x - 15 - 3x + 12}{5x - 3 - 4x + 16} = \frac{22x - 3}{x + 13}$

$a = 22, \ b = -3, \ c = 13$

(ii) $f(x) = 5 + \frac{17}{x-4} > 5$, so range of f is $f(x) > 5$

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**Question 6 continued**

$ff(x) = f(u)$ with $u > 5$: $f(5) = \frac{22}{1} = 22$, $f(u) \to 5$ as $u \to \infty$

$f$ decreasing, so range of $ff$: $5 < ff(x) < 22$

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**Question 6 continued**

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**Question 6 continued**

**Q6**

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**7.**

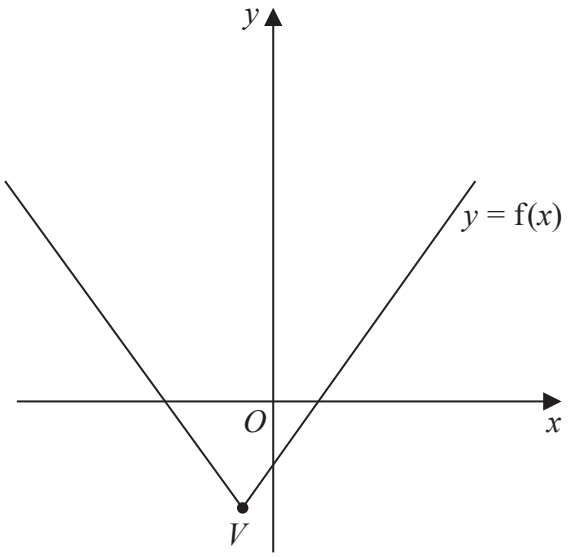



**Figure 2**

Figure 2 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{1}{2}|2x + 7| - 10$$

(a) State the coordinates of the vertex, $V$, of the graph.

**(2)**

(b) Solve, using algebra,

$$\frac{1}{2}|2x + 7| - 10 \geqslant \frac{1}{3}x + 1$$

**(4)**

(c) Sketch the graph with equation

$$y = |\mathrm{f}(x)|$$

stating the coordinates of the local maximum point and each local minimum point.

**(4)**

(a) vertex when $2x + 7 = 0 \Rightarrow x = -\frac{7}{2}$, $y = -10$

$V = \left(-\frac{7}{2}, -10\right)$

(b) critical values from $\frac{1}{2}|2x + 7| - 10 = \frac{1}{3}x + 1$

$2x + 7 \geq 0$: $\frac{1}{2}(2x + 7) - 10 = \frac{1}{3}x + 1$

$\times 6$: $6x + 21 - 60 = 2x + 6 \Rightarrow 4x = 45$

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**Question 7 continued**

$x = \frac{45}{4}$

$2x + 7 < 0$: $-\frac{1}{2}(2x + 7) - 10 = \frac{1}{3}x + 1$

$\times 6$: $-6x - 21 - 60 = 2x + 6 \Rightarrow -8x = 87$

$x = -\frac{87}{8}$

The graph of $y = f(x)$ is above the line outside the critical values:

$x \leq -\frac{87}{8}$ or $x \geq \frac{45}{4}$

(c) $|f(x)| = 0$ when $|2x + 7| = 20 \Rightarrow 2x + 7 = \pm 20$

$x = \frac{13}{2}$ or $x = -\frac{27}{2}$

local maximum $\left(-\frac{7}{2}, 10\right)$ (reflection of V in the x-axis)

local minima $\left(-\frac{27}{2}, 0\right)$ and $\left(\frac{13}{2}, 0\right)$

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**Question 7 continued**

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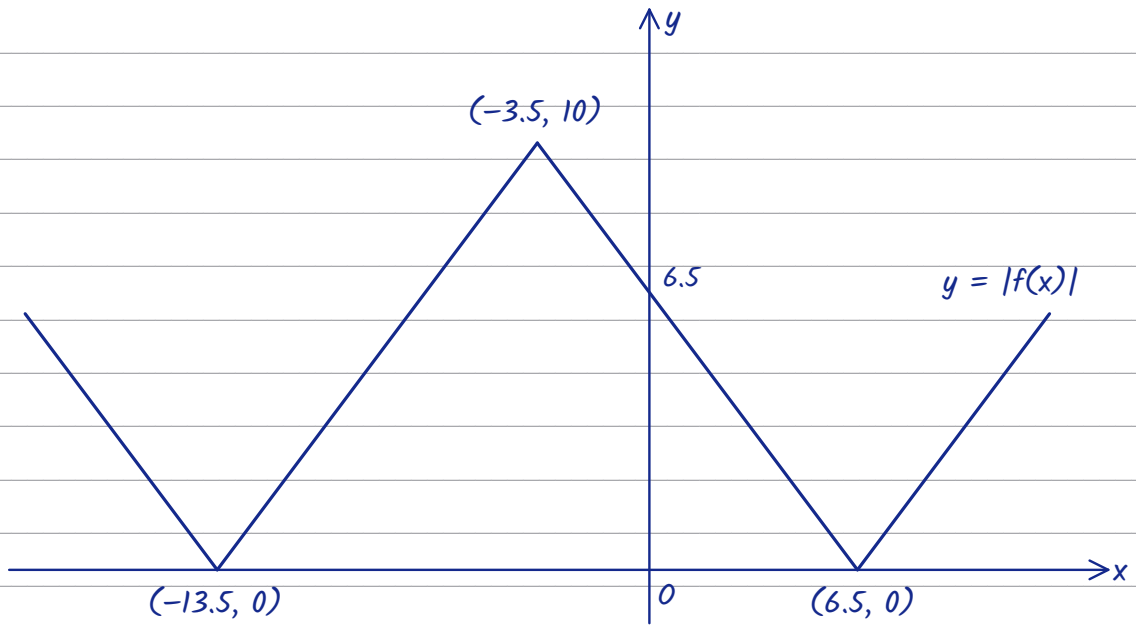
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**Question 7 continued**




Q7

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**8.** A dose of antibiotics is given to a patient.

The amount of the antibiotic, $x$ milligrams, in the patient's bloodstream $t$ hours after the dose was given, is found to satisfy the equation

$$\log_{10} x = 2.74 - 0.079t$$

(a) Show that this equation can be written in the form

$$x = pq^{-t}$$

where $p$ and $q$ are constants to be found. Give the value of $p$ to the nearest whole number and the value of $q$ to 2 significant figures.

**(4)**

(b) With reference to the equation in part (a), interpret the value of the constant $p$.

**(1)**

When a different dose of the antibiotic is given to another patient, the values of $x$ and $t$ satisfy the equation

$$x = 400 \times 1.4^{-t}$$

(c) Use calculus to find, to 2 significant figures, the value of $\frac{dx}{dt}$ when $t = 5$

**(3)**

(a) $\log_{10} x = 2.74 - 0.079t$

$x = 10^{2.74 - 0.079t} = 10^{2.74} \times 10^{-0.079t}$

$x = 10^{2.74} \times (10^{0.079})^{-t}$

so $x = pq^{-t}$ with $p = 10^{2.74}$, $q = 10^{0.079}$

$p = 10^{2.74} = 549.54\ldots = 550$ (nearest whole number)

$q = 10^{0.079} = 1.1995\ldots = 1.2$ (2 s.f.)

$x = 550 \times 1.2^{-t}$

(b) $p$ is the amount of antibiotic (550 mg) in the patient's bloodstream at the start, when $t = 0$ (just after the dose is given).

(c) $x = 400 \times 1.4^{-t} \Rightarrow \frac{dx}{dt} = -400 \ln 1.4 \times 1.4^{-t}$

$t = 5$: $\frac{dx}{dt} = -400 \ln 1.4 \times 1.4^{-5} = -25.024\ldots$

$\frac{dx}{dt} = -25$ (2 s.f.) mg per hour

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**Question 8 continued**

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**Question 8 continued**

**Q8**

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**9.** **In this question you must show detailed reasoning.**

**Solutions relying entirely on calculator technology are not acceptable.**

(i) Solve, for $0 < x \leqslant \pi$, the equation

$$2\sec^2 x - 3\tan x = 2$$

giving the answers, as appropriate, to 3 significant figures.

**(4)**

(ii) Prove that

$$\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} \equiv 2$$

**(4)**

(i) $2\sec^2 x - 3\tan x = 2$, using $\sec^2 x = 1 + \tan^2 x$

$2(1 + \tan^2 x) - 3\tan x = 2$

$2\tan^2 x - 3\tan x = 0$

$\tan x(2\tan x - 3) = 0$

$\tan x = 0$ or $\tan x = \frac{3}{2}$

$\tan x = \frac{3}{2} \Rightarrow x = \arctan 1.5 = 0.98279\ldots = 0.983$

$(\pi + 0.983 = 4.12$ is outside $0 < x \leqslant \pi)$

$\tan x = 0 \Rightarrow x = \pi$ ($x = 0$ is not in the range)

$x = 0.983,\ x = \pi$

(ii) $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} \equiv \frac{\sin 3\theta \cos\theta - \cos 3\theta \sin\theta}{\sin\theta \cos\theta}$

$\equiv \frac{\sin(3\theta - \theta)}{\sin\theta \cos\theta}$ (compound angle formula)

$\equiv \frac{\sin 2\theta}{\frac{1}{2}\sin 2\theta}$ since $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$

$\equiv 2$ as required

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**Question 9 continued**

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**Question 9 continued**

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**Question 9 continued**

**Q9**

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**10.**

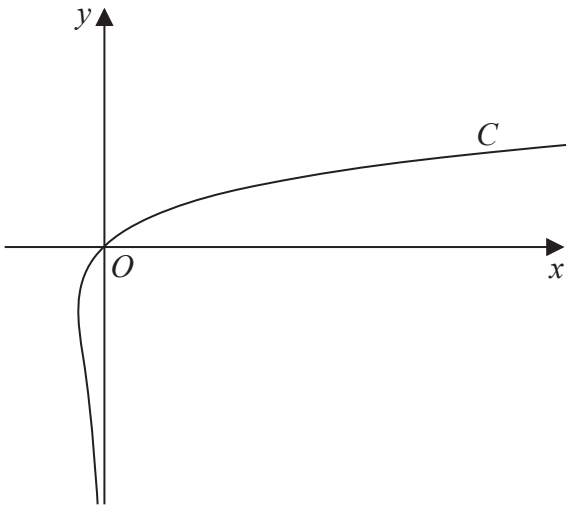



**Figure 3**

Figure 3 shows a sketch of the curve $C$ with equation

$$x = ye^{2y} \qquad y \in \mathbb{R}$$

(a) Show that

$$\frac{dy}{dx} = \frac{y}{x(1+2y)}$$

**(4)**

Given that the straight line with equation $x = k$, where $k$ is a constant, cuts $C$ at exactly two points,

(b) find the range of possible values for $k$.

**(3)**

(a) $x = ye^{2y} \Rightarrow \frac{dx}{dy} = e^{2y} + 2ye^{2y}$

$e^{2y} = \frac{x}{y}$ and $ye^{2y} = x$, so $\frac{dx}{dy} = \frac{x}{y} + 2x$

$= \frac{x + 2xy}{y}$

$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{y}{x + 2xy} = \frac{y}{x(1 + 2y)}$ as required

(b) The curve turns back (vertical tangent) where $\frac{dx}{dy} = 0$

$e^{2y}(1 + 2y) = 0 \Rightarrow 1 + 2y = 0 \Rightarrow y = -\frac{1}{2}$

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**Question 10 continued**

$x = -\frac{1}{2}e^{-1} = -\frac{1}{2e}$ $(= -0.1839...)$

For $x = k$ to cut $C$ at exactly two points:

$-\frac{1}{2e} < k < 0$

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**Question 10 continued**

**Q10**

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**TOTAL FOR PAPER IS 75 MARKS**

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