Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WME02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

P69289A

©2022 Pearson Education Ltd.

L:1/1/1/

P 6 9 2 8 9 A 0 1 2 8

Pearson

Leave blank

1. A particle of mass 0.5 kg is moving with velocity $(2\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse of $(-4\mathbf{i} + 6\mathbf{j})\,\text{N s}$.

(a) Find the speed of the particle immediately after it receives the impulse.

(5)

(b) Find the size of the angle between the direction of motion of the particle immediately before it receives the impulse and the direction of motion of the particle immediately after it receives the impulse.

(3)

(a) $I = mv - mu$

$-4i + 6j = 0.5(v - (2i + 4j))$

$v - (2i + 4j) = -8i + 12j$

$v = (-6i + 16j)\ m\ s^{-1}$

speed $= |v| = \sqrt{(-6)^2 + 16^2} = \sqrt{292}$

speed $= 2\sqrt{73} = 17.088\ldots = 17.1\ m\ s^{-1}$

(b) angle between $u = 2i + 4j$ and $v = -6i + 16j$:

$$\cos\theta = \frac{u \cdot v}{|u|\,|v|} = \frac{(2)(-6) + (4)(16)}{\sqrt{20}\ \sqrt{292}} = \frac{52}{\sqrt{5840}} = 0.68045\ldots$$

$\theta = 47.121\ldots°$

angle $= 47.1°$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 1 continued

Q1

(Total 8 marks)

Leave blank

2. A car of mass 600 kg tows a trailer of mass 200 kg up a hill along a straight road that is inclined at angle θ to the horizontal, where $\sin\theta = \frac{1}{20}$. The trailer is attached to the car by a light inextensible towbar. The resistance to the motion of the car from non-gravitational forces is modelled as a constant force of magnitude 150 N. The resistance to the motion of the trailer from non-gravitational forces is modelled as a constant force of magnitude 300 N.

When the engine of the car is working at a constant rate of P kW the car and the trailer have a constant speed of $15\,\text{m s}^{-1}$

(a) Find the value of P.

(5)

Later, at the instant when the car and the trailer are travelling up the hill with a speed of $20\,\text{m s}^{-1}$, the towbar breaks. When the towbar breaks the trailer is at the point X. The trailer continues to travel up the hill before coming to instantaneous rest at the point Y. The resistance to the motion of the trailer from non-gravitational forces is again modelled as a constant force of magnitude 300 N.

(b) Use the work-energy principle to find the distance XY.

(4)

(a) $\sin\theta = \frac{1}{20}$. Constant speed $\Rightarrow a = 0$. Car and trailer together:

$F - 150 - 300 - 800g \times \frac{1}{20} = 0$

$F = 450 + 392 = 842$ N

$P = Fv = 842 \times 15 = 12630$ W

$P = 12.6$ (kW)

(b) Trailer from X to Y, XY = d m. Work-energy principle:

KE lost = gain in GPE + work done against resistance

$\frac{1}{2} \times 200 \times 20^2 = 200g \times \frac{d}{20} + 300d$

$40000 = 98d + 300d = 398d$

$d = \frac{40000}{398} = 100.502...$

$XY = 101$ m

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 2 continued

Leave blank

Question 2 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 2 continued

Q2

(Total 9 marks)

Leave blank

3. A particle P of mass 0.25 kg is moving on a smooth horizontal surface under the action of a single force, **F** newtons.

At time t seconds $(t \geqslant 0)$, the velocity $\mathbf{v}\,\text{m s}^{-1}$ of P is given by

$$\mathbf{v} = (6\sin 3t)\mathbf{i} + (1 + 2\cos t)\mathbf{j}$$

(a) Find **F** in terms of t.

(3)

At time $t = 0$, the position vector of P relative to a fixed point O is $(4\mathbf{i} - \sqrt{3}\mathbf{j})\,\text{m}$.

(b) Find the position vector of P relative to O when P is first moving parallel to the vector **i**.

(6)

(a) $a = \frac{dv}{dt} = (18\cos 3t)\,i - (2\sin t)\,j$

$F = ma = 0.25[(18\cos 3t)\,i - (2\sin t)\,j]$

$F = (4.5\cos 3t)\,i - (0.5\sin t)\,j$ N

(b) moving parallel to $i \Rightarrow j$ component of v is zero:

$1 + 2\cos t = 0 \Rightarrow \cos t = -\frac{1}{2} \Rightarrow t = \frac{2\pi}{3}$ (first time)

$r = \int v\,dt = (-2\cos 3t)\,i + (t + 2\sin t)\,j + C$

$t = 0$: $-2i + C = 4i - \sqrt{3}j \Rightarrow C = 6i - \sqrt{3}j$

$r = (6 - 2\cos 3t)\,i + (t + 2\sin t - \sqrt{3})\,j$

$t = \frac{2\pi}{3}$: i: $6 - 2\cos 2\pi = 6 - 2 = 4$

j: $\frac{2\pi}{3} + 2 \times \frac{\sqrt{3}}{2} - \sqrt{3} = \frac{2\pi}{3}$

$r = \left\langle 4i + \frac{2\pi}{3}j \right\rangle$ m

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 3 continued

Leave blank

Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 3 continued

Q3

(Total 9 marks)

Leave blank

4. Two small balls, A and B, are moving in opposite directions along the same straight line on smooth horizontal ground. The mass of A is $2m$ and the mass of B is $3m$. The balls collide directly. Immediately before the collision, the speed of A is $2u$ and the speed of B is u. The coefficient of restitution between A and B is e, where $e > 0$

By modelling the balls as particles,

(a) show that the speed of B immediately after the collision is $\frac{1}{5}u(1 + 6e)$.

(6)

After the collision with ball A, ball B hits a smooth fixed vertical wall which is perpendicular to the direction of motion of B.

The coefficient of restitution between B and the wall is $\frac{5}{7}$

Ball B rebounds from the wall and there is a second direct collision between A and B.

(b) Find the range of possible values of e.

(4)

(a) Take A's original direction (→) as positive.

After impact: A moves ← with speed v, B moves → with speed w.

CLM: $2m(2u) - 3m(u) = 3m(w) - 2m(v)$

$u = 3w - 2v$ ①

NEL: $w + v = e(2u + u) = 3eu$ ②

② × 2: $2v = 6eu - 2w$, substitute in ①:

$u = 3w - 6eu + 2w \Rightarrow 5w = u + 6eu$

$w = \frac{1}{5}u(1 + 6e)$ as required

(b) From ②:

$v = 3eu - w = 3eu - \frac{u(1 + 6e)}{5} = \frac{u}{5}(9e - 1)$

B rebounds from the wall with speed $x = \frac{5}{7}w = \frac{u}{7}(1 + 6e)$

For a second collision, B (moving ←) must catch A:

$\frac{u}{7}(1 + 6e) > \frac{u}{5}(9e - 1)$

$5 + 30e > 63e - 7 \Rightarrow 33e < 12$

$0 < e < \frac{4}{11}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 4 continued

Leave blank

Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 4 continued

Q4

(Total 10 marks)

Leave blank

5.

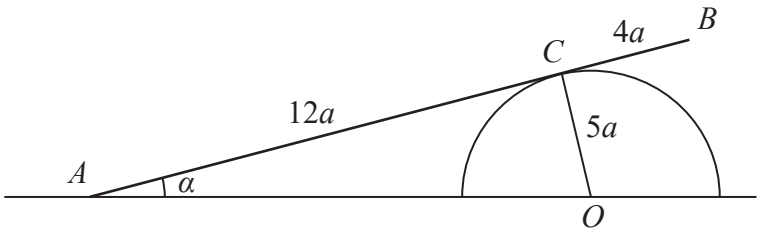


Figure 1

A smooth solid hemisphere is fixed with its flat surface in contact with rough horizontal ground. The hemisphere has centre O and radius $5a$.
A uniform rod AB, of length $16a$ and weight W, rests in equilibrium on the hemisphere with end A on the ground. The rod rests on the hemisphere at the point C, where $AC = 12a$ and angle $CAO = \alpha$, as shown in Figure 1.

Points A, C, B and O all lie in the same vertical plane.

(a) Explain why $AO = 13a$

(1)

The normal reaction on the rod at C has magnitude kW

(b) Show that $k = \frac{8}{13}$

(3)

The resultant force acting on the rod at A has magnitude R and acts upwards at $\theta°$ to the horizontal.

(c) Find

(i) an expression for R in terms of W

(ii) the value of θ

(8)

(a) AB is a tangent to the hemisphere at C, so angle ACO = 90°.

$AO = \sqrt{(12a)^2 + (5a)^2} = \sqrt{169a^2} = 13a$

(b) $\cos\alpha = \frac{12}{13}$, $\sin\alpha = \frac{5}{13}$

M(A): $W \times 8a \cos\alpha = kW \times 12a$

$W \times 8a \times \frac{12}{13} = 12akW$

$k = \frac{8}{13}$ as required

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 5 continued

(c) Normal reaction at C is perpendicular to the rod.

Let R have components X (horizontal) and Y (vertical up) at A.

$(\rightarrow)\quad X = kW\sin\alpha = \frac{8W}{13} \times \frac{5}{13} = \frac{40W}{169}$

$(\uparrow)\quad Y + kW\cos\alpha = W \Rightarrow Y = W - \frac{8W}{13} \times \frac{12}{13} = \frac{73W}{169}$

(i) $R = \sqrt{X^2 + Y^2} = \frac{W}{169}\sqrt{40^2 + 73^2} = \frac{\sqrt{6929}}{169}W$

$R = \frac{\sqrt{41}}{13}W \quad (= 0.493W)$

(ii) $\tan\theta° = \frac{Y}{X} = \frac{73}{40} = 1.825$

$\theta = 61.279\ldots \Rightarrow \theta = 61.3$

Leave blank

Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 5 continued

(Total 12 marks)

Q5

Leave blank

6. [*The centre of mass of a semicircular arc of radius r is* $\frac{2r}{\pi}$ *from the centre.*]

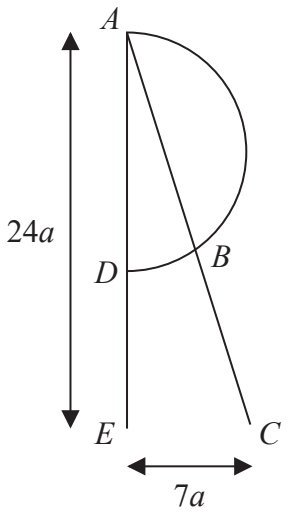


Figure 2

Uniform wire is used to form the framework shown in Figure 2.

In the framework,

- ABC is straight and has length $25a$
- ADE is straight and has length $24a$
- ABD is a semicircular arc of radius $7a$
- $EC = 7a$
- angle $AEC = 90°$
- the points A, B, C, D and E all lie in the same plane

The distance of the centre of mass of the framework from AE is d.

(a) Show that $d = \frac{53}{2(7+\pi)}a$

(4)

The framework is freely suspended from A and hangs in equilibrium with AC at angle $\alpha°$ to the downward vertical.

(b) Find the value of α.

(7)

(a) Lengths (∝ mass): AC = 25a, AE = 24a, arc ABD = 7πa

mass ratio 24a : 25a : 7πa : total 7a(7 + π)

distances from AE: 0, 3.5a, 14a/π

Moments about AE:

$25a \times \frac{7}{2}a + 7\pi a \times \frac{14a}{\pi} = 7a(7+\pi)\,d$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 6 continued

$\frac{37\pi}{2} a^2 = 7a(7 + \pi)\, d$

$d = \frac{37\pi a}{14(7 + \pi)} = \frac{53}{2(7 + \pi)} a$ as required

(b) Centre of semicircular arc is $7a$ from A along AE.

Moments about the axis through A perpendicular to AE (distance y of G from A, measured parallel to AE):

$24a \times 12a + 25a \times 12a + 7\pi a \times 7a = 7a(7 + \pi)\, y$

$49a^2(12 + \pi) = 7a(7 + \pi)\, y$

$y = \frac{7(12 + \pi)}{7 + \pi} a \quad (= 10.6858...a)$

Hanging from A, AG is vertical. Let φ = angle GAE:

$\tan \varphi = \frac{d}{y} = \frac{53}{14(12 + \pi)} = 0.25002... \Rightarrow \varphi = 14.037...°$

angle $CAE = \tan^{-1}\left(\frac{7}{24}\right) = 16.260...°$

$\alpha = 16.260... - 14.037... = 2.2228...$

$\alpha = 2.2$

Leave blank

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 6 continued

(Total 11 marks)

Q6

Leave blank

7. A particle P is projected from a fixed point O on horizontal ground. The particle is projected with speed u at an angle α above the horizontal. At the instant when the horizontal distance of P from O is x, the vertical distance of P above the ground is y.
The motion of P is modelled as that of a particle moving freely under gravity.

(a) Show that $y = x\tan\alpha - \dfrac{gx^2}{2u^2}\left(1 + \tan^2\alpha\right)$

(6)

A small ball is projected from the fixed point O on horizontal ground. The ball is projected with speed $20\,\text{m s}^{-1}$ at angle $\theta°$ above the horizontal. A vertical pole AB, of height 2 m, stands on the ground with $OA = 10\,\text{m}$, as shown in Figure 3.

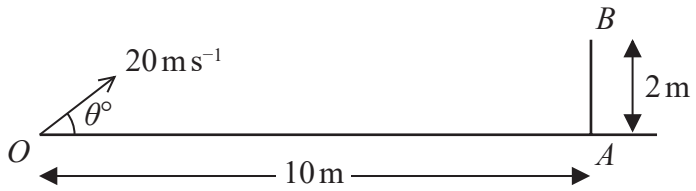


Figure 3

The ball is modelled as a particle moving freely under gravity and the pole is modelled as a rod.
The path of the ball lies in the vertical plane containing O, A and B.

Using the model,

(b) find the range of values of θ for which the ball will pass over the pole.

(3)

Given that $\theta = 40$ and that the ball first hits the ground at the point C

(c) find the speed of the ball at the instant it passes over the pole,

(5)

(d) find the distance OC.

(2)

(a) $(\rightarrow)\quad x = u\cos\alpha\, t \quad\Rightarrow\quad t = x / (u\cos\alpha)$

$(\uparrow)\quad y = u\sin\alpha\, t - \frac{1}{2}gt^2$

$y = u\sin\alpha \times \dfrac{x}{u\cos\alpha} - \dfrac{g}{2}\left(\dfrac{x}{u\cos\alpha}\right)^2$

$y = x\tan\alpha - \dfrac{gx^2}{2u^2\cos^2\alpha} = x\tan\alpha - \dfrac{gx^2}{2u^2}\sec^2\alpha$

since $\sec^2\alpha = 1 + \tan^2\alpha$: $\quad y = x\tan\alpha - \dfrac{gx^2}{2u^2}(1 + \tan^2\alpha)$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 7 continued

(b) $u = 20$, $x = 10$, need $y > 2$:

$$10 \tan\theta - \frac{9.8 \times 100}{2 \times 400}(1 + \tan^2\theta) > 2$$

$$10 \tan\theta - 1.225 - 1.225\tan^2\theta > 2$$

$$1.225\tan^2\theta - 10 \tan\theta + 3.225 < 0$$

$$\tan\theta = \frac{10 \pm \sqrt{84.1975}}{2.45} = 7.8269\ldots \text{ or } 0.33636\ldots$$

critical values $\theta = 82.719\ldots^\circ$ or $18.590\ldots^\circ$

$18.6 < \theta < 82.7$

(c) $\theta = 40$: at $x = 10$,

$$y = 10 \tan 40^\circ - \frac{9.8 \times 100}{2 \times 400}(1 + \tan^2 40^\circ) = 6.3034\ldots \text{ m}$$

Conservation of energy from O to the top of the pole:

$\frac{1}{2}mv^2 = \frac{1}{2}m(20)^2 - mg(6.3034\ldots)$

$v^2 = 400 - 19.6 \times 6.3034\ldots = 276.45\ldots$

$v = 16.626\ldots = 16.6 \text{ m s}^{-1}$

(d) At C, $y = 0$:

$$0 = x \tan 40^\circ - \frac{9.8x^2}{800}(1 + \tan^2 40^\circ)$$

$$x = \frac{800 \tan 40^\circ}{9.8(1 + \tan^2 40^\circ)} = 40.196\ldots$$

OC = 40.2 m

Leave blank

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 16 marks)

TOTAL FOR PAPER IS 75 MARKS

END

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA