Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Thursday 14 January 2021

Morning (Time: 1 hour 30 minutes) | Paper Reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either two significant figures or three significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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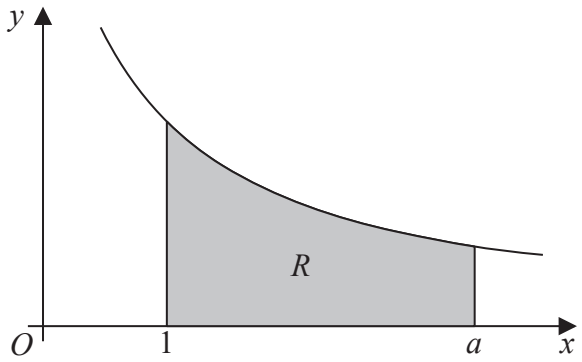


Figure 1

The region R, shown shaded in Figure 1, is bounded by the curve with equation $y = \frac{1}{x}$, the line with equation $x = 1$, the positive x-axis and the line with equation $x = a$ where $a > 1$

A uniform solid S is formed by rotating R through 2π radians about the x-axis.

(a) Show that the volume of S is

$$\pi\left(1 - \frac{1}{a}\right)$$

(3)

(b) Find the x coordinate of the centre of mass of S.

(5)

(a) $V = \pi\int_1^a y^2\,dx = \pi\int_1^a \frac{1}{x^2}\,dx$

$= \pi\left[-\frac{1}{x}\right]_1^a = \pi\left(-\frac{1}{a} + 1\right)$

$\therefore V = \pi\left(1 - \frac{1}{a}\right)$

(b) $\pi\int_1^a xy^2\,dx = \pi\int_1^a \frac{1}{x}\,dx = \pi[\ln x]_1^a = \pi \ln a$

$V\bar{x} = \pi\int_1^a xy^2\,dx \Rightarrow \pi\left(1 - \frac{1}{a}\right)\bar{x} = \pi \ln a$

$\bar{x} = \frac{\ln a}{1 - 1/a} = \frac{a \ln a}{a - 1}$

x coordinate of centre of mass: $\bar{x} = \frac{a \ln a}{a - 1}$

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Question 1 continued

Q1

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2. A particle P of mass m is at a distance x above the surface of the Earth. The Earth exerts a gravitational force on P. This force is directed towards the centre of the Earth. The magnitude of this force is inversely proportional to the square of the distance of P from the centre of the Earth.

At the surface of the Earth the acceleration due to gravity is g.

The Earth is modelled as a fixed sphere of radius R.

(a) Show that the magnitude of the gravitational force on P is $\dfrac{mgR^2}{(x+R)^2}$

(3)

A particle is released from rest from a point above the surface of the Earth. When the particle is at a distance R above the surface of the Earth, the particle has speed U.

Air resistance is modelled as being negligible.

(b) Find, in terms of U, g and R, the speed of the particle when it strikes the surface of the Earth.

(7)

(a) $F = \dfrac{k}{(x+R)^2}$ (inverse square of distance from centre)

At the surface, $x = 0$ and $F = mg$: $mg = \dfrac{k}{R^2} \Rightarrow k = mgR^2$

$\therefore F = \dfrac{mgR^2}{(x+R)^2}$

(b) Towards the Earth (x decreasing): $mv\dfrac{dv}{dx} = -\dfrac{mgR^2}{(x+R)^2}$

$\int v\,dv = -\int \dfrac{gR^2}{(x+R)^2}\,dx$

$\dfrac{1}{2}v^2 = \dfrac{gR^2}{x+R} + c$

$x = R,\ v = U$: $\dfrac{U^2}{2} = \dfrac{gR^2}{2R} + c \Rightarrow c = \dfrac{U^2 - gR}{2}$

$x = 0$: $\dfrac{1}{2}v^2 = \dfrac{gR^2}{R} + \dfrac{U^2 - gR}{2} = gR + \dfrac{U^2 - gR}{2}$

$v^2 = 2gR + U^2 - gR = U^2 + gR$

Speed when it strikes the surface: $v = \sqrt{U^2 + gR}$

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Question 2 continued

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Question 2 continued

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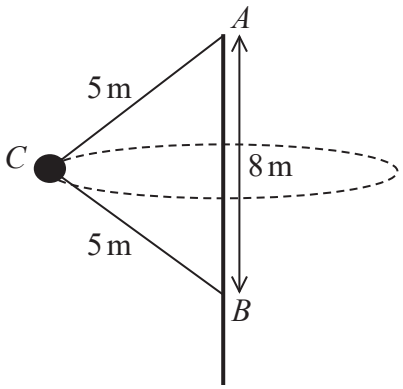


Figure 2

A fairground ride consists of a cabin C that travels in a horizontal circle with a constant angular speed about a fixed vertical central axis. The cabin is attached to one end of each of two rigid arms, each of length 5 m. The other end of the top arm is attached to the fixed point A at the top of the central axis of the ride. The other end of the lower arm is attached to the fixed point B on the central axis, where AB is 8 m, as shown in Figure 2.

Both arms are free to rotate about the central axis.

The arms are modelled as light inextensible rods.

The cabin, together with the people inside, is modelled as a particle.

The cabin completes one revolution every 2 seconds.

Given that the combined mass of the cabin and the people is 600 kg,

(a) find

(i) the tension in the upper arm of the ride,

(ii) the tension in the lower arm of the ride.

(9)

In a refined model, it is assumed that both arms stretch to a length of 5.1 m.

(b) State how this would affect the sum of the tensions in the two arms, justifying your answer.

(2)

(a) Radius of circle: $r = \sqrt{(5^2 - 4^2)} = 3$ m

Angle θ between each arm and the vertical: $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$

$\omega = \frac{2\pi}{2} = \pi$ rad s^{-1}

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Question 3 continued

$R(\uparrow)$: $T_A \cos\theta - T_B \cos\theta = 600g \Rightarrow \frac{4}{5}(T_A - T_B) = 600g$

$\Rightarrow T_A - T_B = 750g = 7350$ (1)

$R(\rightarrow)$: $T_A \sin\theta + T_B \sin\theta = 600 \times \omega^2 \times r = 600 \times \pi^2 \times 3$

$\frac{3}{5}(T_A + T_B) = 1800\pi^2 \Rightarrow T_A + T_B = 3000\pi^2$ (2)

(1) + (2): $2T_A = 3000\pi^2 + 750g$

$T_A = 1500\pi^2 + 375g = 18479.4...$

(i) Tension in upper arm = 18 500 N (3 s.f.)

$T_B = 1500\pi^2 - 375g = 11129.4...$

(ii) Tension in lower arm = 11 100 N (3 s.f.)

(b) If the arms stretch to 5.1 m, the radius of the circle increases ($r = \sqrt{(5.1^2 - 4^2)} = 3.16...$ m > 3 m).

$R(\rightarrow)$: $(T_A + T_B)\sin\theta = 600\omega^2(5.1\sin\theta) \Rightarrow T_A + T_B = 3060\pi^2$

So the sum of the tensions in the two arms would increase.

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Question 3 continued

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Question 3 continued

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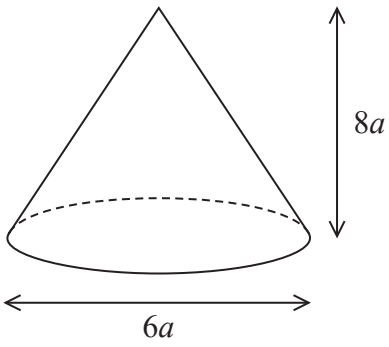


Figure 3

A uniform right solid cone C has diameter $6a$ and height $8a$, as shown in Figure 3.

The solid S is formed by removing a cone of height $4a$ from the top of C and then removing an identical, inverted cone. The vertex of the removed cone is at the point O in the centre of the base of C, as shown in Figure 4.

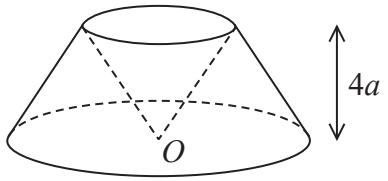


Figure 4

(a) Find the distance of the centre of mass of S from O.

(5)

The point A lies on the circumference of the base of S and the point B lies on the circumference of the top of S. The points O, A and B all lie in the same vertical plane, as shown in Figure 5.

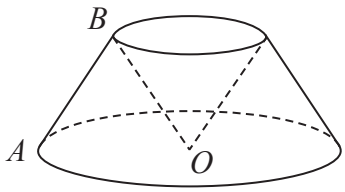


Figure 5

The solid S is freely suspended from the point B and hangs in equilibrium.

(b) Find the size of the angle that AB makes with the downward vertical.

(4)

(a) Volume of $C = \frac{1}{3}\pi(3a)^2(8a) = 24\pi a^3$; each small cone $= \frac{1}{3}\pi(1.5a)^2(4a) = 3\pi a^3$

Mass ratios: C : top cone : inside cone : S = 8 : 1 : 1 : 6

Distances of centres of mass above O: $2a$, $5a$, $3a$, $\bar{y}$

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Question 4 continued

Moments about O: $8(2a) - 1(5a) - 1(3a) = 6\bar{y}$

$6\bar{y} = 8a \Rightarrow \bar{y} = \frac{4}{3}a$

Distance of centre of mass of S from O $= \frac{4}{3}a$

(b) B hangs vertically above G, so BG is vertical.

Measure angles at B from the line through B parallel to the axis:

BA: $\tan\alpha = \frac{3a - 1.5a}{4a} = \frac{3}{8} \Rightarrow \alpha = 20.556...°$

BG: $\tan\beta = \frac{1.5a}{4a - 4a/3} = \frac{9}{16} \Rightarrow \beta = 29.357...°$

(BA and BG lie on opposite sides of this line)

Angle between AB and the downward vertical $= \alpha + \beta$

$= 20.556...° + 29.357...° = 49.913...° = 50°$ (2 s.f.)

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Question 4 continued

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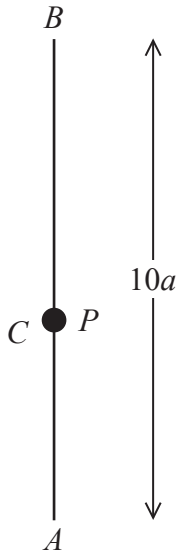


Figure 6

The fixed points, A and B, are a distance $10a$ apart, with B vertically above A.

One end of a light elastic string, of natural length $2a$ and modulus of elasticity $2mg$, is attached to a particle P of mass m and the other end is attached to A.

One end of another light elastic string, of natural length $4a$ and modulus of elasticity $6mg$, is attached to P and the other end is attached to B.

The particle P rests in equilibrium at the point C, as shown in Figure 6.

(a) Show that each string has an extension of $2a$.

(5)

The particle P is now pulled down vertically, so that it is a distance a below C and then released from rest.

(b) Show that in the subsequent motion, P performs simple harmonic motion.

(4)

(c) Find, in terms of a and g, the speed of P when it is a distance $\frac{7}{2}a$ above A.

(4)

(a) Let lower string PA have extension e_1, upper string PB extension e_2.

$e_1 + e_2 = 10a - 2a - 4a = 4a$ so $e_2 = 4a - e_1$

$T_{PA} = \frac{2mg\,e_1}{2a}$, $T_{PB} = \frac{6mg(4a - e_1)}{4a}$

R(↑): $T_{PB} = mg + T_{PA} \Rightarrow \frac{6mg(4a - e_1)}{4a} = mg + \frac{2mg\,e_1}{2a}$

$\times 4a / mg$: $24a - 6e_1 = 4a + 4e_1 \Rightarrow 10e_1 = 20a$

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Question 5 continued

$e_1 = 2a$ and $e_2 = 4a - 2a = 2a$

$\therefore$ each string has an extension of $2a$

(b) Let P be a distance x below C (downwards positive).

$$mg + \frac{2mg(2a - x)}{2a} - \frac{6mg(2a + x)}{4a} = m\ddot{x}$$

$$mg + 2mg - \frac{mgx}{a} - 3mg - \frac{3mgx}{2a} = m\ddot{x}$$

$$\ddot{x} = -\frac{5g}{2a}x$$

This is of the form $\ddot{x} = -\omega^2 x$, so P moves with simple harmonic motion (lower string stays taut, since amplitude $a < 2a$).

(c) $\omega^2 = \frac{5g}{2a}$, amplitude $= a$

C is $2a + 2a = 4a$ above A, so at $\frac{7}{2}a$ above A, $x = \frac{a}{2}$

$$v^2 = \omega^2(a^2 - x^2) = \frac{5g}{2a}\left(a^2 - \left(\frac{a}{2}\right)^2\right) = \frac{5g}{2a} \times \frac{3a^2}{4} = \frac{15ga}{8}$$

$$v = \sqrt{15ga/8} = \frac{\sqrt{(30ga)}}{4}$$

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Question 5 continued

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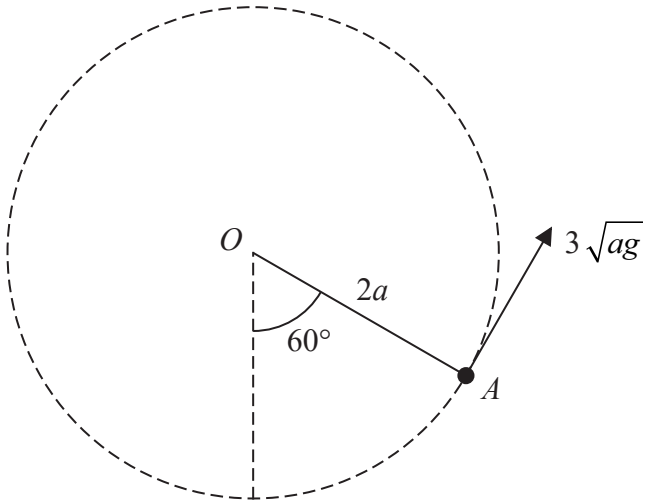


Figure 7

A particle of mass m is attached to one end of a light inextensible string of length $2a$. The other end of the string is attached to a fixed point O. The particle is initially held at the point A with the string taut and OA making an angle of 60° with the downward vertical.

The particle is then projected upwards with a speed of $3\sqrt{ag}$, perpendicular to OA, in the vertical plane containing OA, as shown in Figure 7.

In an initial model of the motion of the particle, it is assumed that the string does not break.

Using this model,

(a) show that the particle performs complete vertical circles.

(8)

In a refined model it is assumed that the string will break if the tension in it exceeds $7mg$.

Using this refined model,

(b) show that the particle still performs complete vertical circles.

(5)

(a) At A, P is $2a\cos 60° = a$ below O. Let v = speed at the top.

Energy: $\frac{1}{2}mv^2 + mg(2a) = \frac{1}{2}m(3\sqrt{ag})^2 - mg(2a\cos 60°)$

$\frac{1}{2}v^2 = \frac{9ag}{2} - ga - 2ag \Rightarrow v^2 = 3ag$

At the top, towards O: $T + mg = \frac{mv^2}{2a}$

$T = \frac{m(3ag)}{2a} - mg = \frac{mg}{2}$

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Question 6 continued

The tension is least at the top of the circle and $T = \frac{1}{2}mg > 0$ there, so the string remains taut and the particle performs complete vertical circles.

(b) Let V = speed at the bottom (2a below O). From A:

$$\frac{1}{2}mV^2 = \frac{1}{2}m(3\sqrt{ag})^2 + mg(2a - 2a\cos 60°)$$

$$\frac{1}{2}V^2 = \frac{9ag}{2} + ag \quad \Rightarrow \quad V^2 = 11ag$$

At the bottom, towards O: $T - mg = \frac{mV^2}{2a} = \frac{m(11ag)}{2a}$

$$T = \frac{11mg}{2} + mg = \frac{13mg}{2} = 6.5mg$$

The maximum tension (at the bottom) is $6.5mg < 7mg$, so the string does not break and the particle still performs complete vertical circles.

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Question 6 continued

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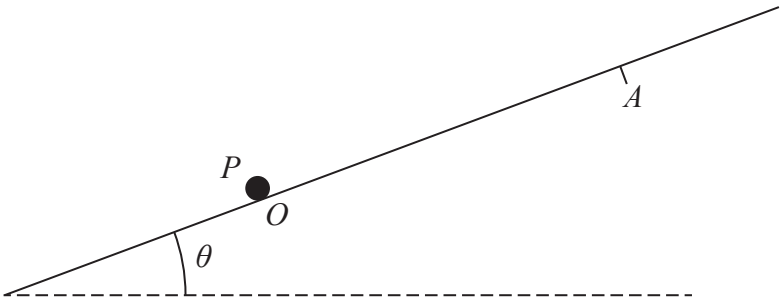


Figure 8

A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 2 m and modulus of elasticity 3 N. The other end of the string is attached to a fixed point O on a rough plane. The plane is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{2}{7}$

The coefficient of friction between P and the plane is $\frac{\sqrt{5}}{5}$

The particle P is initially at rest at the point O, as shown in Figure 8.

The particle P then receives an impulse of magnitude 4 Ns, directed up a line of greatest slope of the plane.

The particle P moves up the plane and comes to rest at the point A.

(a) Find the extension of the elastic string when P is at A.

(8)

(b) Show that the particle does not remain at rest at A.

(3)

(a) Impulse: $0.5u = 4 \Rightarrow u = 8\ m\ s^{-1}$

$\cos\theta = \frac{\sqrt{45}}{7} = \frac{3\sqrt{5}}{7}$; $R = 0.5g\cos\theta$

$F_{max} = \mu R = \frac{\sqrt{5}}{5} \times 0.5g \times \frac{3\sqrt{5}}{7} = \frac{3g}{14} = 2.1\ N$

Weight component down plane $= 0.5g \times \frac{2}{7} = 1.4\ N$

Let x m be the extension at A, so OA $= (x + 2)$ m.

Energy: $\frac{1}{2} \times 0.5 \times 8^2 = 0.5g(x + 2)\sin\theta + 2.1(x + 2) + \frac{3x^2}{2 \times 2}$

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Question 7 continued

$16 = 1.4(x + 2) + 2.1(x + 2) + \frac{3x^2}{4}$

$64 = 14(x + 2) + 3x^2$

$3x^2 + 14x - 36 = 0$

$x = \frac{-14 + \sqrt{(14^2 + 4 \times 3 \times 36)}}{6} = \frac{-14 + \sqrt{628}}{6} = 1.8433...$

Extension of the string at A = 1.84 m (3 s.f.)

(b) At A: $T = \frac{3 \times 1.8433...}{2} = 2.765...$ N

Forces down the plane: $T + 0.5g \sin\theta = 2.765... + 1.4 = 4.165...$ N

Maximum friction up the plane = 2.1 N

$4.165... > 2.1$, so there is a resultant force down the plane:

$0.5a = 4.165... - 2.1 \Rightarrow a = 4.13$ m s^{-2} down the slope

Hence the particle does not remain at rest at A.

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Question 7 continued

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Question 7 continued

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