Please check the examination details below before entering your candidate information

Candidate surname	StemUp Model Solution	Other names	

Pearson Edexcel
International
Advanced Level

Centre Number						Candidate Number				

Tuesday 12 January 2021

Afternoon (Time: 1 hour 30 minutes)	Paper Reference **WME02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M2

You must have: Mathematical Formulae and Statistical Tables (Blue), calculator	Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. A particle P of mass 1.5 kg is moving with velocity $(4\mathbf{i} + 6\mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse of magnitude 15 Ns. Immediately after P receives the impulse, the velocity of P is $v\mathbf{i}\,\text{m s}^{-1}$.

Find the two possible values of v.

(7)

$I = mv - mu$

$I = 1.5\{vi - (4i + 6j)\} = 1.5(v - 4)i - 9j$

$|I| = 15$:

$15^2 = 1.5^2(v - 4)^2 + 9^2$

$225 = 2.25(v - 4)^2 + 81$

$(v - 4)^2 = 64$

$v^2 - 8v - 48 = 0$

$(v - 12)(v + 4) = 0$

$v = 12 \quad \text{or} \quad v = -4$

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Question 1 continued

Q1

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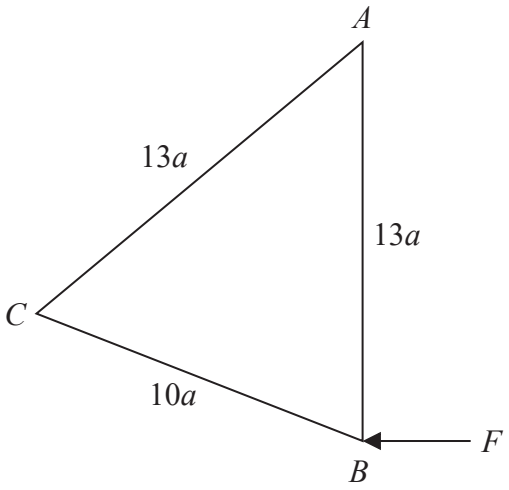


Figure 1

The uniform lamina ABC has sides $AB = AC = 13a$ and $BC = 10a$. The lamina is freely suspended from A. A horizontal force of magnitude F is applied to the lamina at B, as shown in Figure 1. The line of action of the force lies in the vertical plane containing the lamina. The lamina is in equilibrium with AB vertical. The weight of the lamina is W.

Find F in terms of W.

(5)

M = midpoint of BC: $AM = \sqrt{(13^2 - 5^2)}\ a = 12a$

G on AM with $AG = \frac{2}{3} \times 12a = 8a$

Angle $BAM = \theta$: $\sin\theta = \frac{5}{13}$

Horizontal distance of G from $AB = 8a \sin\theta = \frac{40a}{13}$

$M(A)$: $F \times 13a = W \times 8a \times \frac{5}{13}$

$13aF = \frac{40aW}{13}$

$F = \frac{40W}{169}$

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Question 2 continued

Q2

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3. A car of mass 600 kg travels along a straight horizontal road with the engine of the car working at a constant rate of P watts. The resistance to the motion of the car is modelled as a constant force of magnitude R newtons. At the instant when the speed of the car is $15\,\text{m s}^{-1}$, the magnitude of the acceleration of the car is $0.2\,\text{m s}^{-2}$.

Later the same car travels up a straight road inclined at angle θ to the horizontal, where $\sin\theta = \frac{1}{20}$. The resistance to the motion of the car from non-gravitational forces is modelled as a constant force of magnitude R newtons. When the engine of the car is working at a constant rate of P watts, the car has a constant speed of $10\,\text{m s}^{-1}$.

Find the value of P.

(8)

Horizontal road: driving force $F_1 = P \div 15$

N2L: $F_1 - R = 600 \times 0.2$

$\frac{P}{15} - R = 120$ (1)

Up the slope: driving force $F_2 = P \div 10$, $a = 0$

$F_2 - R - 600g \sin\theta = 0$

$\frac{P}{10} - R - 600 \times 9.8 \times \frac{1}{20} = 0$

$\frac{P}{10} - R - 30g = 0 \Rightarrow \frac{P}{10} - R = 294$ (2)

(2) − (1):

$\frac{P}{10} - \frac{P}{15} = 294 - 120 = 174$

$\frac{P}{30} = 174$

$P = 5220$ (W)

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Question 3 continued

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Question 3 continued

Q3

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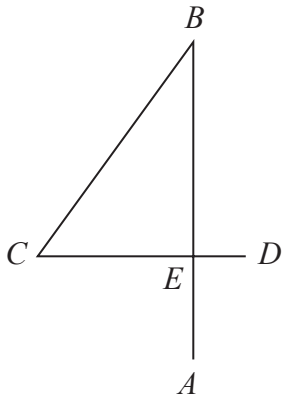


Figure 2

The number "4", shown in Figure 2, is a rigid framework made from three uniform rods, AB, BC and CD, where

$$AB = 6a,\ BC = 5a \text{ and } CD = 4a$$

The point E is on AB and CD, where $BE = 4a$, $CE = 3a$ and angle $CEB = 90°$

The three rods are all made from the same material and they all lie in the same plane.

The framework is suspended from B and hangs in equilibrium with BA at an angle θ to the downward vertical.

Find θ to the nearest degree.

(9)

Masses proportional to lengths: AB 6m, BC 5m, CD 4m, total 15m

ED = 4a − 3a = a, EA = 6a − 4a = 2a

Centres: AB: 3a below B, on AB

BC: 2a below B, 1.5a left of AB

CD: 4a below B, 3a − 2a = a left of AB

Moments about a horizontal axis through B (distances below B):

$6m \times 3a + 5m \times 2a + 4m \times 4a = 15m \times y$

$44ma = 15my$

$y = \frac{44a}{15}$ below B

Moments about AB (distances to the left of AB):

$5m \times \frac{3a}{2} + 4m \times a = 15m \times x$

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Question 4 continued

$\frac{23ma}{2} = 15mx$

$x = \frac{23a}{30}$ from AB

G is vertically below B when hanging, so

$\tan\theta = \frac{x}{y} = \frac{23a}{30} \div \frac{44a}{15} = \frac{23}{88} = 0.2613...$

$\theta = 14.64...°$

$\theta = 15°$ (nearest degree)

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Question 4 continued

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Question 4 continued

Q4

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5. At time t seconds, $t \geqslant 0$, a particle P has velocity $\mathbf{v}\,\text{m s}^{-1}$, where

$$\mathbf{v} = (5t^2 - 12t + 15)\mathbf{i} + (t^2 + 8t - 10)\mathbf{j}$$

When $t = 0$, P is at the origin O.

At time T seconds, P is moving in the direction of $(\mathbf{i} + \mathbf{j})$.

(a) Find the value of T.

(3)

When $t = 3$, P is at the point A.

(b) Find the magnitude of the acceleration of P as it passes through A.

(4)

(c) Find the position vector of A.

(4)

(a) Parallel to $i + j$ $\Rightarrow$ i and j components equal:

$5T^2 - 12T + 15 = T^2 + 8T - 10$

$4T^2 - 20T + 25 = 0$

$(2T - 5)^2 = 0$

$T = \frac{5}{2} = 2.5$

(b) $a = \frac{dv}{dt} = (10t - 12)i + (2t + 8)j$

$t = 3$: $a = 18i + 14j$

$|a| = \sqrt{(18^2 + 14^2)} = \sqrt{520} = 22.803...$

$|a| = 22.8\ m\ s^{-2}$

(c) $r = \int v\,dt = \left(\frac{5}{3}t^3 - 6t^2 + 15t\right)i + \left(\frac{1}{3}t^3 + 4t^2 - 10t\right)j + c$

$t = 0, r = 0 \Rightarrow c = 0$

$t = 3$: $r = (45 - 54 + 45)i + (9 + 36 - 30)j$

$r = 36i + 15j\ m$

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Question 5 continued

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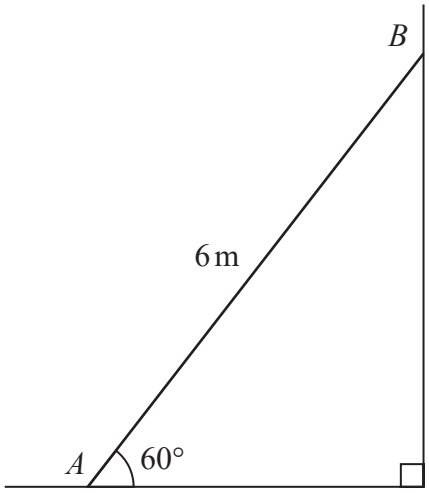


Figure 3

A ladder AB has length 6 m and mass 30 kg. The ladder rests in equilibrium at 60° to the horizontal with the end A on rough horizontal ground and the end B against a smooth vertical wall, as shown in Figure 3.

A man of mass 70 kg stands on the ladder at the point C, where $AC = 2$ m, and the ladder remains in equilibrium. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall. The man is modelled as a particle.

(a) Find the magnitude of the force exerted on the ladder by the ground.

(6)

The man climbs further up the ladder. When he is at the point D on the ladder, the ladder is about to slip.

Given that the coefficient of friction between the ladder and the ground is 0.4

(b) find the distance AD.

(4)

(c) State how you have used the modelling assumption that the ladder is a rod.

(1)

(a) Forces: R up and F (towards wall) at A, N from wall at B,
30g at the midpoint, 70g at C

M(A): $30g \times 3\cos 60° + 70g \times 2\cos 60° = N \times 6 \sin 60°$

$45g + 70g = 3\sqrt{3}\,N$

$N = \frac{115g}{3\sqrt{3}} = 216.89...$

(↑): $R = 30g + 70g = 100g = 980$

(→): $F = N = 216.89...$

Force from ground $= \sqrt{(R^2 + F^2)} = \sqrt{(980^2 + 216.89...^2)}$

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Question 6 continued

$= 1003.7... = 1000$ N (3 s.f.)

(b) $R = 100g$ still, limiting friction: $F = \mu R = 0.4 \times 100g = 392$

$(\rightarrow)$: $N = F = 392 = 40g$

Let AD = x.

M(A): $N \times 6 \sin 60° = 30g \times 3 \cos 60° + 70g \times x \cos 60°$

$40g \times 3\sqrt{3} = 45g + 35gx$

$x = \dfrac{120\sqrt{3} - 45}{35} = 4.6527...$

AD = 4.65 m

(c) The ladder does not bend (it is straight and has no thickness), so the distances along the ladder are used in the moments.

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Question 6 continued

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Question 6 continued

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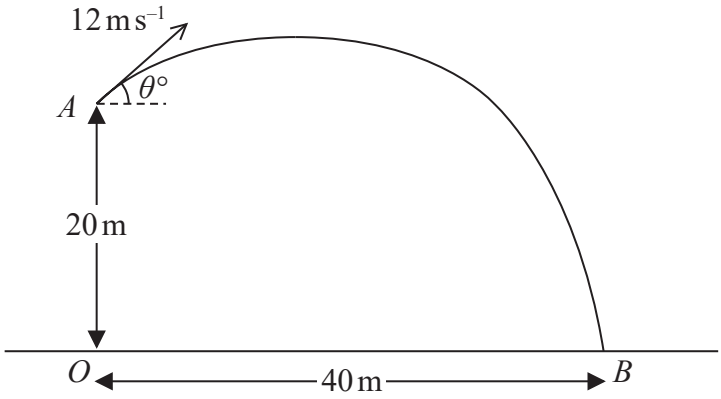


Figure 4

The fixed point A is 20 m vertically above the point O which is on horizontal ground. At time $t = 0$, a particle P is projected from A with speed 12 m s^{-1} at an angle $\theta°$ above the horizontal. The particle moves freely under gravity. At time $t = 5$ seconds, P strikes the ground at the point B, where $OB = 40$ m, as shown in Figure 4.

(a) By considering energy, find the speed of P as it hits the ground at B.

(4)

(b) Find the least speed of P as it moves from A to B.

(2)

(c) Find the length of time for which the speed of P is more than 10 m s^{-1}.

(6)

(a) Energy from A to B (loss of PE = gain in KE):

$\frac{1}{2} m \times 12^2 + m \times 9.8 \times 20 = \frac{1}{2} mv^2$

$72 + 196 = \frac{1}{2}v^2$

$v^2 = 536 \Rightarrow v = 23.151...$

$v = 23.2 \text{ m s}^{-1}$

(b) Horizontal: $12 \cos\theta \times 5 = 40 \Rightarrow 12 \cos\theta = 8$

Least speed is at the top, where speed = horizontal component

Least speed = 8 m s^{-1}

(c) Speed = 10 when vertical component = $\sqrt{(10^2 - 8^2)} = 6$ m s^{-1}

$\cos\theta = \frac{8}{12} = \frac{2}{3} \Rightarrow \sin\theta = \frac{\sqrt{5}}{3}$, $12 \sin\theta = 4\sqrt{5}$

Vertical velocity: $\pm 6 = 4\sqrt{5} - 9.8t$

$t_1 = \frac{4\sqrt{5} - 6}{9.8} = 0.3004...$, $t_2 = \frac{4\sqrt{5} + 6}{9.8} = 1.5249...$

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Question 7 continued

Speed ≤ 10 for 0.3004... ≤ t ≤ 1.5249...:

time interval = 1.5249... − 0.3004... = 1.2244... s

Required time = 5 − 1.2244... = 3.7755...

= 3.78 s

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Question 7 continued

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Question 7 continued

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8. Two particles, A and B, have masses $3m$ and $4m$ respectively. The particles are moving towards each other along the same straight line on a smooth horizontal surface. The particles collide directly. Immediately after the collision, A and B are moving in the same direction with speeds $\frac{u}{3}$ and u respectively. In the collision, A receives an impulse of magnitude $8mu$.

(a) Find the coefficient of restitution between A and B.

(6)

When A and B collide they are at a distance d from a smooth vertical wall, which is perpendicular to their direction of motion. After the collision with A, particle B collides directly with the wall and rebounds so that there is a second collision between A and B. This second collision takes place at distance x from the wall.

Given that the coefficient of restitution between B and the wall is $\frac{1}{4}$

(b) find x in terms of d.

(6)

(a) Take the direction of motion after impact as positive.

Before: A speed v (→), B speed w (←). After: A u/3 (→), B u (→)

Impulse on A: $8mu = 3mv - 3m \times \frac{u}{3}$

$8mu = 3mv - mu \Rightarrow v = 3u$

Impulse on B: $8mu = 4mu - 4m(-w) = 4mu + 4mw \Rightarrow w = u$

NEL: $u - \frac{u}{3} = e(3u + u)$

$\frac{2u}{3} = 4ue$

$e = \frac{1}{6}$

(b) B reaches wall after time $\frac{d}{u}$; A has moved $\frac{u}{3} \times \frac{d}{u} = \frac{d}{3}$

Gap between A and B when B hits wall $= d - \frac{d}{3} = \frac{2d}{3}$

Speed of B after rebound $= \frac{1}{4} u = \frac{u}{4}$ (away from wall)

Time to close gap $= \frac{2d}{3} \div \left(\frac{u}{3} + \frac{u}{4} \right) = \frac{2d}{3} \div \frac{7u}{12} = \frac{8d}{7u}$

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Question 8 continued

Distance from wall: $x = \frac{u}{4} \times \frac{8d}{7u}$

$x = \frac{2d}{7}$

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Question 8 continued

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Question 8 continued

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