Please check the examination details below before entering your candidate information

Candidate surname	Other names
StemUp Model Solution	

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. The curve C has equation

$$y = \frac{1}{2}\operatorname{arcosh}(2x) \qquad \frac{7}{2} \leqslant x \leqslant 13$$

Using calculus, determine the exact length of the curve C.

Give your answer in the form $p\sqrt{q}$, where p and q are constants to be found.

(6)

$\frac{dy}{dx} = \frac{1}{2} \times \frac{2}{\sqrt{((2x)^2 - 1)}} = \frac{1}{\sqrt{(4x^2 - 1)}}$

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{1}{4x^2 - 1} = \frac{4x^2}{4x^2 - 1}$

$s = \int_{7/2}^{13} \sqrt{\left(\frac{4x^2}{4x^2 - 1}\right)}\, dx = \int_{7/2}^{13} \frac{2x}{\sqrt{(4x^2 - 1)}}\, dx$

$= \left[\frac{2(4x^2 - 1)^{1/2}}{8 \times \frac{1}{2}}\right]_{7/2}^{13} = \left[(4x^2 - 1)^{1/2}\right]_{7/2}^{13}$

$= \frac{1}{2}\left(\sqrt{(4 \times 169 - 1)} - \sqrt{\left(4 \times \frac{49}{4} - 1\right)}\right)$

$= \frac{1}{2}(\sqrt{675} - \sqrt{48}) = \frac{1}{2}(15\sqrt{3} - 4\sqrt{3})$

$s = \frac{11}{2}\sqrt{3} \qquad (p = \frac{11}{2},\ q = 3)$

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Question 1 continued

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Question 1 continued

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Q1

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2. Given that

$$\cosh y = x \quad \text{and} \quad y < 0$$

use the definition of $\cosh y$ in terms of exponential functions to prove that

$$y = \ln\left(x - \sqrt{x^2 - 1}\right)$$

(6)

$\cosh y = x \Rightarrow x = \frac{e^y + e^{-y}}{2}$

$\Rightarrow 2x\,e^y = e^{2y} + 1$

$\Rightarrow e^{2y} - 2x\,e^y + 1 = 0$

$\Rightarrow e^y = \frac{2x \pm \sqrt{((2x)^2 - 4 \times 1 \times 1)}}{2} = \frac{2x \pm \sqrt{(4x^2 - 4)}}{2}$

$\Rightarrow e^y = x \pm \sqrt{(x^2 - 1)}$

Since $y < 0$, $e^y < 1$. $x = \cosh y > 1$, so $x + \sqrt{(x^2 - 1)} > 1$,
so we need the negative root: $e^y = x - \sqrt{(x^2 - 1)}$
$\therefore\ y = \ln(x - \sqrt{(x^2 - 1)})$ as required

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Question 2 continued

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Q2

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3. The ellipse E has equation

$$\frac{x^2}{64} + \frac{y^2}{36} = 1$$

The line l is the normal to E at the point $P(8\cos\theta, 6\sin\theta)$.

(a) Using calculus, show that an equation for l is

$$4x\sin\theta - 3y\cos\theta = 14\sin\theta\cos\theta$$

(4)

The line l meets the x-axis at the point A and meets the y-axis at the point B.

The point M is the midpoint of AB.

(b) Determine a Cartesian equation for the locus of M as θ varies, giving your answer in the form $ax^2 + by^2 = c$ where a, b and c are integers.

(5)

(a) $x = 8\cos\theta$, $y = 6\sin\theta$: $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{6\cos\theta}{-8\sin\theta} = -\frac{3\cos\theta}{4\sin\theta}$

Gradient of normal: $m_N = -\frac{1}{m_T} = \frac{4\sin\theta}{3\cos\theta}$

$y - 6\sin\theta = \frac{4\sin\theta}{3\cos\theta}(x - 8\cos\theta)$

$3y\cos\theta - 18\sin\theta\cos\theta = 4x\sin\theta - 32\sin\theta\cos\theta$

$\Rightarrow \quad 4x\sin\theta - 3y\cos\theta = 14\sin\theta\cos\theta$ as required

(b) $y = 0$: $4x\sin\theta = 14\sin\theta\cos\theta \Rightarrow x = \frac{7}{2}\cos\theta$, $A\left(\frac{7}{2}\cos\theta, 0\right)$

$x = 0$: $-3y\cos\theta = 14\sin\theta\cos\theta \Rightarrow y = -\frac{14}{3}\sin\theta$, $B\left(0, -\frac{14}{3}\sin\theta\right)$

$M = \left(\frac{(7/2)\cos\theta}{2}, \frac{-(14/3)\sin\theta}{2}\right) = \left(\frac{7}{4}\cos\theta, -\frac{7}{3}\sin\theta\right)$

$x = \frac{7}{4}\cos\theta \Rightarrow \cos\theta = \frac{4x}{7}$; $y = -\frac{7}{3}\sin\theta \Rightarrow \sin\theta = -\frac{3y}{7}$

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \left(-\frac{3y}{7}\right)^2 + \left(\frac{4x}{7}\right)^2 = 1$

$\frac{16x^2}{49} + \frac{9y^2}{49} = 1$

$16x^2 + 9y^2 = 49$

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4. The matrix **M** is given by

$$\begin{pmatrix} 2 & 0 & -1 \\ k & 3 & 2 \\ -2 & 1 & k \end{pmatrix}$$

(a) Show that $\det \mathbf{M} = 5k - 10$

(2)

Given that $k \neq 2$

(b) find $\mathbf{M}^{-1}$ in terms of k.

(4)

The points $O(0, 0, 0)$, $A(4, -8, 3)$, $B(-2, 5, -4)$ and $C(4, -6, 8)$ are the vertices of a tetrahedron T.

The transformation represented by matrix **M** transforms T to a tetrahedron with volume 50

(c) Determine the possible values of k.

(5)

(a) $\det M = 2\begin{vmatrix} 3 & 2 \\ 1 & k \end{vmatrix} - 0\begin{vmatrix} k & 2 \\ -2 & k \end{vmatrix} + (-1)\begin{vmatrix} k & 3 \\ -2 & 1 \end{vmatrix}$

$= 2(3k - 2) - 0 - (k + 6)$

$= 6k - 4 - k - 6 = 5k - 10$ as required

(b) Minors: $\begin{pmatrix} 3k-2 & k^2+4 & k+6 \\ 1 & 2k-2 & 2 \\ 3 & 4+k & 6 \end{pmatrix}$ Cofactors:

$\begin{pmatrix} 3k-2 & -k^2-4 & k+6 \\ -1 & 2k-2 & -2 \\ 3 & -4-k & 6 \end{pmatrix}$ $\text{Adj } M = \begin{pmatrix} 3k-2 & -1 & 3 \\ -k^2-4 & 2k-2 & -4-k \\ k+6 & -2 & 6 \end{pmatrix}$

$M^{-1} = \frac{1}{5k-10}\begin{pmatrix} 3k-2 & -1 & 3 \\ -k^2-4 & 2k-2 & -4-k \\ k+6 & -2 & 6 \end{pmatrix}$

(c) $a = OA = 4i - 8j + 3k$, $b = OB = -2i + 5j - 4k$, $c = OC = 4i - 6j + 8k$

$b \times c = (40 - 24)i - (-16 + 16)j + (12 - 20)k = 16i + 0j - 8k$

$a.(b \times c) = 4(16) + (-8)(0) + 3(-8) = 64 - 24 = 40$

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Question 4 continued

Volume of $T = \frac{1}{6}|a.(b \times c)| = \frac{40}{6} = \frac{20}{3}$

Volume of image $= |\det M| \times \frac{20}{3}$: $\frac{20}{3}|5k - 10| = 50 \Rightarrow |5k - 10| = \frac{15}{2}$

$5k - 10 = \frac{15}{2} \Rightarrow k = \frac{7}{2}$; $5k - 10 = -\frac{15}{2} \Rightarrow k = \frac{1}{2}$

$k = \frac{1}{2}$ or $k = \frac{7}{2}$

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Question 4 continued

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Question 4 continued

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5. The skew lines l_1 and l_2 have equations

$$l_1 : \mathbf{r} = (\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}) + \lambda(5\mathbf{i} + \mathbf{j})$$

and

$$l_2 : \mathbf{r} = (2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}) + \mu(8\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Determine a vector that is perpendicular to both l_1 and l_2

(2)

(b) Determine an equation of the plane parallel to l_1 that contains l_2

(i) in the form $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

(1)

(ii) in the form $\mathbf{r}.\mathbf{n} = p$

(2)

(c) Determine the shortest distance between l_1 and l_2

Give your answer in simplest form.

(5)

(a) $(5i + j) \times (8i - 2j + 3k) = \begin{vmatrix} i & j & k \\ 5 & 1 & 0 \\ 8 & -2 & 3 \end{vmatrix}$

$= (3 - 0)i - (15 - 0)j + (-10 - 8)k = 3i - 15j - 18k$

(b)(i) $r = (2i - 4j + 4k) + s(8i - 2j + 3k) + t(5i + j)$

(ii) $n = 3i - 15j - 18k$: $(2i - 4j + 4k).(3i - 15j - 18k) = 6 + 60 - 72 = -6$

$r.(3i - 15j - 18k) = -6$

(c) Point on l_1: $(1, 2, -5)$; point on l_2: $(2, -4, 4)$

$AB = (2i - 4j + 4k) - (i + 2j - 5k) = i - 6j + 9k$

$|n| = \sqrt{(3^2 + 15^2 + 18^2)} = \sqrt{558} = 3\sqrt{62}$

$d = \dfrac{|AB.n|}{|n|} = \dfrac{|(i - 6j + 9k).(3i - 15j - 18k)|}{\sqrt{558}} = \dfrac{|3 + 90 - 162|}{\sqrt{558}}$

$= \dfrac{69}{\sqrt{558}} = \dfrac{69}{3\sqrt{62}} = \dfrac{23}{\sqrt{62}} = \dfrac{23\sqrt{62}}{62}$

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Question 5 continued

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6.

$$I_n = \int_0^{\sqrt{\frac{\pi}{2}}} x^n \cos\left(x^2\right) \mathrm{d}x \qquad n \geqslant 1$$

(a) Prove that, for $n \geqslant 5$

$$I_n = \frac{1}{2}\left(\frac{\pi}{2}\right)^{\frac{n-1}{2}} - \frac{1}{4}(n-1)(n-3)I_{n-4}$$

(6)

(b) Hence, determine the exact value of I_5, giving your answer in its simplest form.

(3)

(a) $I_n = \int_0^{\sqrt{(\pi/2)}} x^{n-1} \cdot x\cos(x^2)\,dx$; $u = x^{n-1}$, $\frac{dv}{dx} = x\cos(x^2)$, $v = \frac{1}{2}\sin(x^2)$

$I_n = \left[\frac{1}{2}x^{n-1}\sin(x^2)\right]_0^{\sqrt{(\pi/2)}} - \int_0^{\sqrt{(\pi/2)}} (n-1)x^{n-2} \cdot \frac{1}{2}\sin(x^2)\,dx$

$= \frac{1}{2}\left(\sqrt{(\pi/2)}\right)^{n-1}\sin\frac{\pi}{2} - 0 - \frac{1}{2}(n-1)\int_0^{\sqrt{(\pi/2)}} x^{n-3} \cdot x\sin(x^2)\,dx$

Now $\int_0^{\sqrt{(\pi/2)}} x^{n-3} \cdot x\sin(x^2)\,dx$ $(u = x^{n-3},\ v = -\frac{1}{2}\cos(x^2))$

$= \left[-\frac{1}{2}x^{n-3}\cos(x^2)\right]_0^{\sqrt{(\pi/2)}} + \int_0^{\sqrt{(\pi/2)}} \frac{1}{2}(n-3)x^{n-4}\cos(x^2)\,dx$

$= -\frac{1}{2}\left(\sqrt{(\pi/2)}\right)^{n-3}\cos\frac{\pi}{2} + 0 + \frac{1}{2}(n-3)I_{n-4} = 0 + \frac{1}{2}(n-3)I_{n-4}$

$I_n = \frac{1}{2}\left(\frac{\pi}{2}\right)^{(n-1)/2} - \frac{1}{2}(n-1) \times \frac{1}{2}(n-3)I_{n-4}$

$\therefore\ I_n = \frac{1}{2}\left(\frac{\pi}{2}\right)^{(n-1)/2} - \frac{1}{4}(n-1)(n-3)I_{n-4}$ as required

(b) $I_1 = \int_0^{\sqrt{(\pi/2)}} x\cos(x^2)\,dx = \left[\frac{1}{2}\sin(x^2)\right]_0^{\sqrt{(\pi/2)}} = \frac{1}{2}\sin\frac{\pi}{2} - 0 = \frac{1}{2}$

$I_5 = \frac{1}{2}\left(\frac{\pi}{2}\right)^2 - \frac{1}{4}(4)(2)I_1 = \frac{\pi^2}{8} - 2 \times \frac{1}{2}$

$I_5 = \frac{\pi^2}{8} - 1$

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Question 6 continued

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7. A hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{25} = 1$$

where a is a positive constant.

The eccentricity of H is e.

(a) Determine an expression for e^2 in terms of a.

(1)

The line l is the directrix of H for which $x > 0$

The points A and A' are the points of intersection of l with the asymptotes of H.

(b) Determine, in terms of e, the length of the line segment AA'.

(3)

The point F is the focus of H for which $x < 0$

Given that the area of triangle AFA' is $\frac{164}{3}$

(c) show that a is a solution of the equation

$$30a^3 - 164a^2 + 375a - 4100 = 0$$

(4)

(d) Hence, using algebra and making your reasoning clear, show that the only possible value of a is $\frac{20}{3}$

(3)

(a) $b^2 = a^2(e^2 - 1) \Rightarrow 25 = a^2(e^2 - 1) \Rightarrow e^2 = \frac{25}{a^2} + 1 = \frac{25 + a^2}{a^2}$

(b) Directrix l: $x = \frac{a}{e}$; asymptotes: $\frac{x}{a} = \pm\frac{y}{5}$, i.e. $y = \pm\frac{5x}{a}$

$x = \frac{a}{e}$: $y = \pm\frac{5}{a} \times \frac{a}{e} = \pm\frac{5}{e}$, so $A\left(\frac{a}{e}, \frac{5}{e}\right)$, $A'\left(\frac{a}{e}, -\frac{5}{e}\right)$

$AA' = \frac{5}{e} - \left(-\frac{5}{e}\right) = \frac{10}{e}$

(c) $F = (-ae, 0)$. Height of triangle $= \frac{a}{e} - (-ae) = ae + \frac{a}{e}$

Area $= \frac{1}{2} \times \frac{10}{e} \times \left(ae + \frac{a}{e}\right) = 5\left(a + \frac{a}{e^2}\right) = \frac{164}{3}$

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Question 7 continued

$\Rightarrow 15\left(a + \frac{a}{e^2}\right) = 164 \quad \Rightarrow \quad 15a\left(1 + \frac{a^2}{25 + a^2}\right) = 164$

$\Rightarrow 15a\left(\frac{25 + 2a^2}{25 + a^2}\right) = 164$

$\Rightarrow 375a + 30a^3 = 164(25 + a^2) = 4100 + 164a^2$

$\Rightarrow 30a^3 - 164a^2 + 375a - 4100 = 0$ as required

(d) $a = \frac{20}{3}$: $30\left(\frac{20}{3}\right)^3 - 164\left(\frac{20}{3}\right)^2 + 375\left(\frac{20}{3}\right) - 4100 = 0$

so $(3a - 20)$ is a factor:

$30a^3 - 164a^2 + 375a - 4100 = (3a - 20)(10a^2 + 12a + 205)$

$10a^2 + 12a + 205 = 0$: $b^2 - 4ac = 12^2 - 4(10)(205) = -8056 < 0$

so the quadratic has no real roots, and the only real root of the cubic is from $3a - 20 = 0$. Hence $a = \frac{20}{3}$ is the only possible value.

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Question 7 continued

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8.

$$y = \arccos\left(2\sqrt{x}\right)$$

(a) Determine $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(3)

(b) Show that

$$\int y\,\mathrm{d}x = x\arccos\left(2\sqrt{x}\right) + \int \frac{\sqrt{x}}{\sqrt{1-4x}}\,\mathrm{d}x$$

(2)

(c) Use the substitution $\sqrt{x} = \dfrac{1}{2}\cos\theta$ to show that

$$\int_0^{\frac{1}{8}} \frac{\sqrt{x}}{\sqrt{1-4x}}\,\mathrm{d}x = \frac{1}{4}\int_a^b \cos^2\theta\,\mathrm{d}\theta$$

where a and b are limits to be determined.

(4)

(d) Hence, determine the exact value of

$$\int_0^{\frac{1}{8}} \arccos\left(2\sqrt{x}\right)\mathrm{d}x$$

(4)

(a) $\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{(1-(2\sqrt{x})^2)}} \times \dfrac{d}{dx}(2x^{1/2}) = \dfrac{-1}{\sqrt{(1-4x)}} \times x^{-1/2}$

$\dfrac{dy}{dx} = -\dfrac{1}{\sqrt{x}\sqrt{(1-4x)}}$

(b) $u = \arccos(2\sqrt{x})$, $\dfrac{du}{dx} = -\dfrac{1}{\sqrt{x}\sqrt{(1-4x)}}$; $\dfrac{dv}{dx} = 1$, $v = x$

$\int y\,dx = \int 1 \times \arccos(2\sqrt{x})\,dx = x\arccos(2\sqrt{x}) - \int x \times \dfrac{-1}{\sqrt{x}\sqrt{(1-4x)}}\,dx$

$= x\arccos(2\sqrt{x}) + \int \dfrac{\sqrt{x}}{\sqrt{(1-4x)}}\,dx$ as required

(c) $\sqrt{x} = \dfrac{1}{2}\cos\theta \Rightarrow x = \dfrac{1}{4}\cos^2\theta$, $\dfrac{dx}{d\theta} = -\dfrac{1}{2}\cos\theta\sin\theta$

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Question 8 continued

$\sqrt{(1-4x)} = \sqrt{(1-\cos^2\theta)} = \sin\theta$

$\int \frac{\sqrt{x}}{\sqrt{(1-4x)}}\,dx = \int \frac{(1/2)\cos\theta}{\sin\theta} \times \left(-\frac{1}{2}\cos\theta\sin\theta\right) d\theta = -\frac{1}{4}\int \cos^2\theta\,d\theta$

$x = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}; \quad x = \frac{1}{8} \Rightarrow \cos\theta = 2\sqrt{\frac{1}{8}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

$\int_0^{1/8} \frac{\sqrt{x}}{\sqrt{(1-4x)}}\,dx = -\frac{1}{4}\int_{\pi/2}^{\pi/4} \cos^2\theta\,d\theta = \frac{1}{4}\int_{\pi/4}^{\pi/2} \cos^2\theta\,d\theta$

$a = \frac{\pi}{4}, \quad b = \frac{\pi}{2}$

(d) $\frac{1}{4}\int_{\pi/4}^{\pi/2} \cos^2\theta\,d\theta = \frac{1}{4}\int_{\pi/4}^{\pi/2} \frac{1}{2}(1+\cos 2\theta)\,d\theta = \frac{1}{8}\left[\theta + \frac{1}{2}\sin 2\theta\right]_{\pi/4}^{\pi/2}$

$= \frac{1}{8}\left[\left(\frac{\pi}{2}+0\right) - \left(\frac{\pi}{4}+\frac{1}{2}\right)\right] = \frac{1}{8}\left(\frac{\pi}{4}-\frac{1}{2}\right) = \frac{\pi}{32} - \frac{1}{16}$

$\int_0^{1/8} \arccos(2\sqrt{x})\,dx = \left[x\arccos(2\sqrt{x})\right]_0^{1/8} + \frac{\pi}{32} - \frac{1}{16}$

$= \frac{1}{8}\arccos\left(\frac{1}{\sqrt{2}}\right) - 0 + \frac{\pi}{32} - \frac{1}{16} = \frac{1}{8}\times\frac{\pi}{4} + \frac{\pi}{32} - \frac{1}{16}$

$= \frac{\pi}{16} - \frac{1}{16}$

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Question 8 continued

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TOTAL FOR PAPER: 75 MARKS

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