Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Friday 15 May 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. $$\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} = 2\cos x$$

(a) Express $\frac{d^3y}{dx^3}$ in terms of x, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

(3)

At $x = 0, y = 2$ and $\frac{dy}{dx} = 5$

(b) Determine the value of $\frac{d^3y}{dx^3}$ at $x = 0$

(1)

(c) Express y as a series in ascending powers of x, up to and including the term in x^3

(3)

(a) Differentiate with respect to x:

$\frac{d^3y}{dx^3} + 3\frac{dy}{dx} + 3x\frac{d^2y}{dx^2} = -2\sin x$

$\frac{d^3y}{dx^3} = -2\sin x - 3\frac{dy}{dx} - 3x\frac{d^2y}{dx^2}$

(b) At x = 0: $\frac{d^3y}{dx^3} = -2\sin 0 - 3(5) - 3(0)\frac{d^2y}{dx^2} = -15$

(c) At x = 0: $\frac{d^2y}{dx^2} = 2\cos 0 - 3(0)(5) = 2$

$y = y_0 + x\left(\frac{dy}{dx}\right)_0 + \frac{x^2}{2!}\left(\frac{d^2y}{dx^2}\right)_0 + \frac{x^3}{3!}\left(\frac{d^3y}{dx^3}\right)_0 + \ldots$

$y = 2 + 5x + \frac{2}{2}x^2 + \frac{-15}{6}x^3 + \ldots$

$y = 2 + 5x + x^2 - \frac{5}{2}x^3 + \ldots$

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Question 1 continued

Q1

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2. (a) Write $\dfrac{3r+1}{r(r-1)(r+1)}$ in partial fractions.

(2)

(b) Hence find

$$\sum_{r=2}^{n} \frac{3r+1}{r(r-1)(r+1)} \qquad n \geqslant 2$$

giving your answer in the form

$$\frac{an^2+bn+c}{2n(n+1)}$$

where a, b and c are integers to be determined.

(5)

(c) Hence determine the exact value of

$$\sum_{r=15}^{20} \frac{3r+1}{r(r-1)(r+1)}$$

(2)

(a) $\dfrac{3r+1}{r(r-1)(r+1)} \equiv \dfrac{A}{r} + \dfrac{B}{r-1} + \dfrac{C}{r+1}$

$3r+1 \equiv A(r-1)(r+1) + Br(r+1) + Cr(r-1)$

$r = 0$: $1 = -A \Rightarrow A = -1$; $r = 1$: $4 = 2B \Rightarrow B = 2$; $r = -1$: $-2 = 2C \Rightarrow C = -1$

$\dfrac{3r+1}{r(r-1)(r+1)} = -\dfrac{1}{r} + \dfrac{2}{r-1} - \dfrac{1}{r+1}$

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Question 2 continued

(b) $r = 2$: $\frac{2}{1} - \frac{1}{2} - \frac{1}{3}$

$r = 3$: $\frac{2}{2} - \frac{1}{3} - \frac{1}{4}$

$r = 4$: $\frac{2}{3} - \frac{1}{4} - \frac{1}{5}$

$r = 5$: $\frac{2}{4} - \frac{1}{5} - \frac{1}{6}$

$\vdots$

$r = n - 1$: $\frac{2}{n-2} - \frac{1}{n-1} - \frac{1}{n}$

$r = n$: $\frac{2}{n-1} - \frac{1}{n} - \frac{1}{n+1}$

$$\text{Sum} = 2 + \frac{2}{2} - \frac{1}{2} - \frac{1}{n} - \frac{1}{n} - \frac{1}{n+1}$$

$$= \frac{5}{2} - \frac{2}{n} - \frac{1}{n+1}$$

$$= \frac{5n(n+1) - 4(n+1) - 2n}{2n(n+1)}$$

$$= \frac{5n^2 + 5n - 4n - 4 - 2n}{2n(n+1)}$$

$$\sum_{r=2}^{n} \frac{3r+1}{r(r-1)(r+1)} = \frac{5n^2 - n - 4}{2n(n+1)} \qquad a = 5,\ b = -1,\ c = -4$$

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Question 2 continued

(c) $\sum_{r=15}^{20} = \sum_{r=2}^{20} - \sum_{r=2}^{14}$

$= \frac{5(20)^2 - 20 - 4}{2(20)(21)} - \frac{5(14)^2 - 14 - 4}{2(14)(15)}$

$= \frac{1976}{840} - \frac{962}{420} = \frac{1976 - 1924}{840} = \frac{52}{840}$

$= \frac{13}{210}$

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Question 2 continued

Q2

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3. Use algebra to obtain the set of values of x for which

$$\left|\frac{x^2 + 3x + 10}{x + 2}\right| < 7 - x$$

(9)

$x^2 + 3x + 10 = (x + \frac{3}{2})^2 + \frac{31}{4} > 0$ for all x

Case $x > -2$: $\frac{x^2 + 3x + 10}{x + 2} = 7 - x$

$x^2 + 3x + 10 = (7 - x)(x + 2) = 14 + 5x - x^2$

$2x^2 - 2x - 4 = 0 \Rightarrow x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0 \Rightarrow x = 2 , \ x = -1$

Case $x < -2$: $\frac{-(x^2 + 3x + 10)}{x + 2} = 7 - x$

$-x^2 - 3x - 10 = 14 + 5x - x^2$

$8x = -24 \Rightarrow x = -3$

Critical values: $x = -3 , \ -2 , \ -1 , \ 2$

Test: $x = -4$: LHS $= 7 < 11$ ✓ $\quad x = -2.5$: LHS $= 17.5 > 9.5$ ✗

$x = 0$: LHS $= 5 < 7$ ✓ $\quad x = 3$: LHS $= 5.6 > 4$ ✗

$x < -3$ or $-1 < x < 2$

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Question 3 continued

Q3

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4. (a) Express the complex number $18\sqrt{3} - 18i$ in the form

$$r(\cos\theta + i\sin\theta) \quad -\pi < \theta \leqslant \pi$$

(3)

(b) Solve the equation

$$z^4 = 18\sqrt{3} - 18i$$

giving your answers in the form $re^{i\theta}$ where $-\pi < \theta \leqslant \pi$

(5)

(a) $r = |18\sqrt{3} - 18i| = 18\sqrt{(\sqrt{3})^2 + 1^2} = 18 \times 2 = 36$

$\tan\theta = \frac{-18}{18\sqrt{3}} = -\frac{1}{\sqrt{3}}$ (4th quadrant) $\Rightarrow \theta = -\frac{\pi}{6}$

$18\sqrt{3} - 18i = 36\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$

(b) $z^4 = 36\left(\cos\left(2k\pi - \frac{\pi}{6}\right) + i\sin\left(2k\pi - \frac{\pi}{6}\right)\right)$

$z = 36^{1/4}\left(\cos\left(\frac{12k\pi - \pi}{24}\right) + i\sin\left(\frac{12k\pi - \pi}{24}\right)\right)$, $36^{1/4} = \sqrt{6}$

$k = 0$: $z = \sqrt{6}\, e^{-\pi i/24}$

$k = 1$: $z = \sqrt{6}\, e^{11\pi i/24}$

$k = 2$: $z = \sqrt{6}\, e^{23\pi i/24}$

$k = -1$: $z = \sqrt{6}\, e^{-13\pi i/24}$

$z = \sqrt{6}\, e^{-13\pi i/24}$, $\sqrt{6}\, e^{-\pi i/24}$, $\sqrt{6}\, e^{11\pi i/24}$, $\sqrt{6}\, e^{23\pi i/24}$

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Question 4 continued

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Question 4 continued

Q4

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5. The transformation T from the z-plane to the w-plane is given by

$$w = \frac{z - 3i}{z + 2i} \qquad z \neq -2i$$

The circle with equation $|z| = 1$ in the z-plane is mapped by T onto the circle C in the w-plane.

Determine

(i) the centre of C,

(ii) the radius of C.

(7)

$w = \frac{z - 3i}{z + 2i} \Rightarrow wz + 2iw = z - 3i \Rightarrow z(1 - w) = 2iw + 3i$

$z = \frac{i(2w + 3)}{1 - w}$

$|z| = 1 \Rightarrow \frac{|i|\,|2w + 3|}{|1 - w|} = 1 \Rightarrow |2w + 3| = |1 - w|$

Let $w = u + iv$: $|(2u + 3) + 2iv| = |(1 - u) - iv|$

$(2u + 3)^2 + 4v^2 = (1 - u)^2 + v^2$

$4u^2 + 12u + 9 + 4v^2 = 1 - 2u + u^2 + v^2$

$3u^2 + 3v^2 + 14u + 8 = 0$

$u^2 + v^2 + \frac{14}{3}u + \frac{8}{3} = 0$

$\left(u + \frac{7}{3}\right)^2 + v^2 = \frac{49}{9} - \frac{8}{3} = \frac{25}{9}$

(i) Centre of C is $\left(-\frac{7}{3}, 0\right)$ (i.e. $w = -\frac{7}{3}$)

(ii) Radius of C is $\frac{5}{3}$

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Question 5 continued

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Question 5 continued

Q5

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6. Obtain the general solution of the equation

$$x^2 \frac{dy}{dx} + (x\cot x + 2)xy = 4\sin x \qquad 0 < x < \pi$$

Give your answer in the form $y = f(x)$

(8)

$\div x^2$: $\frac{dy}{dx} + \frac{x\cot x + 2}{x} y = \frac{4\sin x}{x^2}$

$IF = e^{\int (\cot x + 2/x)\,dx} = e^{\ln \sin x + 2\ln x} = e^{\ln \sin x} \times e^{2\ln x} = x^2 \sin x$

$\frac{d}{dx}(yx^2 \sin x) = x^2 \sin x \times \frac{4\sin x}{x^2} = 4\sin^2 x$

$yx^2 \sin x = \int 4\sin^2 x\,dx = \int 2(1 - \cos 2x)\,dx$

$yx^2 \sin x = 2x - \sin 2x + C$

$y = \frac{2x - \sin 2x + C}{x^2 \sin x}$

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Question 6 continued

Q6

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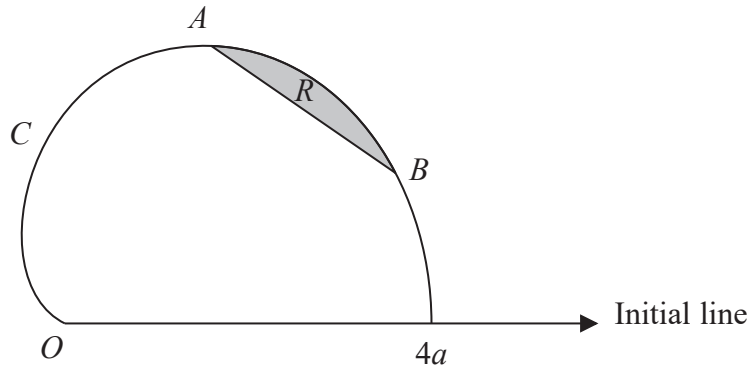


Figure 1

The curve C, shown in Figure 1, has polar equation

$$r = 2a(1 + \cos\theta) \qquad 0 \leqslant \theta \leqslant \pi$$

where a is a positive constant.

The tangent to C at the point A is parallel to the initial line.

(a) Determine the polar coordinates of A.

(6)

The point B on the curve has polar coordinates $\left(a\left(2+\sqrt{3}\right), \frac{\pi}{6}\right)$

The finite region R, shown shaded in Figure 1, is bounded by the curve C and the line AB.

(b) Use calculus to determine the exact area of the shaded region R.

Give your answer in the form

$$\frac{a^2}{4}\left(d\pi - e + f\sqrt{3}\right)$$

where d, e and f are integers.

(7)

(a) $y = r\sin\theta = 2a(1+\cos\theta)\sin\theta = 2a\sin\theta + a\sin 2\theta$

$\frac{dy}{d\theta} = 2a\cos\theta + 2a\cos 2\theta = 0$ (tangent parallel to initial line)

$\cos\theta + 2\cos^2\theta - 1 = 0 \Rightarrow 2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0 \Rightarrow \cos\theta = \frac{1}{2}$ ($\cos\theta = -1$ gives $\theta = \pi$, at O)

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Question 7 continued

$\theta = \frac{\pi}{3}$, $r = 2a\left(1 + \frac{1}{2}\right) = 3a$

A is $\left(3a, \frac{\pi}{3}\right)$

(b) Area under arc AB $= \frac{1}{2}\int_{\pi/6}^{\pi/3} r^2\, d\theta = \frac{1}{2}\int_{\pi/6}^{\pi/3} 4a^2(1 + \cos\theta)^2\, d\theta$

$= 2a^2\int_{\pi/6}^{\pi/3} (1 + 2\cos\theta + \cos^2\theta)\, d\theta$

$= 2a^2\int_{\pi/6}^{\pi/3} \left(1 + 2\cos\theta + \frac{1}{2}(1 + \cos 2\theta)\right) d\theta$

$= 2a^2\left[\frac{3\theta}{2} + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_{\pi/6}^{\pi/3}$

$= 2a^2\left[\left(\frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8}\right) - \left(\frac{\pi}{4} + 1 + \frac{\sqrt{3}}{8}\right)\right]$

$= 2a^2\left(\frac{\pi}{4} + \sqrt{3} - 1\right)$

Area of triangle OAB $= \frac{1}{2} \times 3a \times (2 + \sqrt{3})a \times \sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{3a^2(2 + \sqrt{3})}{4}$

$R = 2a^2\left(\frac{\pi}{4} + \sqrt{3} - 1\right) - \frac{3a^2(2 + \sqrt{3})}{4}$

$= \frac{a^2}{4}(2\pi + 8\sqrt{3} - 8 - 6 - 3\sqrt{3})$

Area of $R = \frac{a^2}{4}(2\pi - 14 + 5\sqrt{3})$

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Question 7 continued

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Question 7 continued

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Q7

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8. (a) Show that the transformation $x = e^u$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 8y = 4 \ln x \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{du^2} + 2\frac{dy}{du} - 8y = 4u \qquad \text{(II)}$$

(6)

(b) Determine the general solution of differential equation (II), expressing y as a function of u.

(7)

(c) Hence obtain the general solution of differential equation (I).

(1)

(a) $x = e^u \Rightarrow \frac{dx}{du} = e^u \Rightarrow \frac{du}{dx} = e^{-u}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^{-u}\frac{dy}{du}$

$\frac{d^2y}{dx^2} = -e^{-u}\frac{du}{dx}\frac{dy}{du} + e^{-u}\frac{d^2y}{du^2}\frac{du}{dx} = e^{-2u}\left(\frac{d^2y}{du^2} - \frac{dy}{du}\right)$

Substitute into (I):

$e^{2u} \times e^{-2u}\left(\frac{d^2y}{du^2} - \frac{dy}{du}\right) + 3e^u \times e^{-u}\frac{dy}{du} - 8y = 4\ln(e^u)$

$\frac{d^2y}{du^2} - \frac{dy}{du} + 3\frac{dy}{du} - 8y = 4u$

$\frac{d^2y}{du^2} + 2\frac{dy}{du} - 8y = 4u$ (II) as required

(b) Auxiliary equation: $m^2 + 2m - 8 = 0 \Rightarrow (m+4)(m-2) = 0$

$m = -4$, $m = 2$ CF: $y = Ae^{-4u} + Be^{2u}$

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Question 8 continued

PI: try $y = au + b \Rightarrow \frac{dy}{du} = a, \quad \frac{d^2y}{du^2} = 0$

$0 + 2a - 8(au + b) = 4u$

u: $-8a = 4 \Rightarrow a = -\frac{1}{2}$; constant: $2a - 8b = 0 \Rightarrow b = -\frac{1}{8}$

$y = Ae^{-4u} + Be^{2u} - \frac{1}{2}u - \frac{1}{8}$

(c) $u = \ln x$: $y = Ax^{-4} + Bx^2 - \frac{1}{2}\ln x - \frac{1}{8}$

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Question 8 continued

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Question 8 continued

Q8

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TOTAL FOR PAPER: 75 MARKS

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