Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 3 June 2025

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM02/01A**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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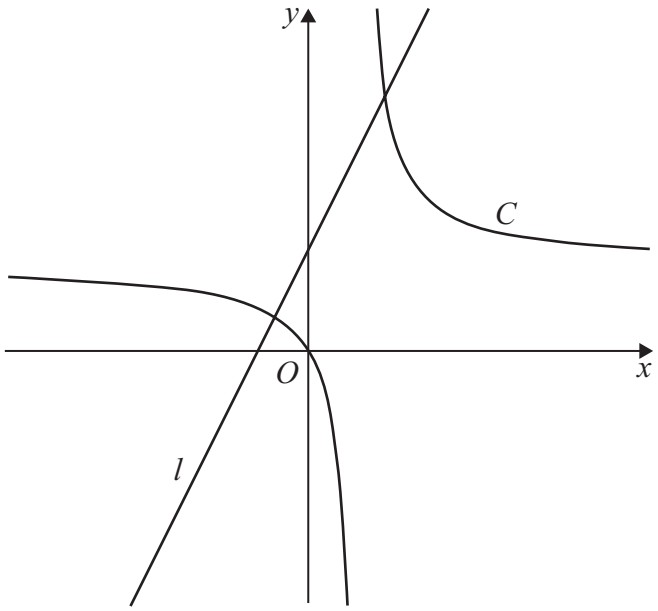


Figure 1

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{10x}{x-6} \qquad x \neq 6$$

and the line l with equation $y = 2x + 12$

(a) Use algebra to determine the x coordinates of the points of intersection of C and l

(2)

(b) Determine the range of values of x for which

(i) $2x + 12 > \dfrac{10x}{x-6}$

(ii) $|2x + 12| > \left|\dfrac{10x}{x-6}\right|$

(4)

(a) $\frac{10x}{x-6} = 2x + 12 \Rightarrow 10x = (2x + 12)(x - 6) = 2x^2 - 72$

$2x^2 - 10x - 72 = 0 \Rightarrow x^2 - 5x - 36 = 0$

$(x + 4)(x - 9) = 0 \Rightarrow x = -4, \; x = 9$

(b)(i) From the sketch, the line is above C for:

$-4 < x < 6, \quad x > 9$

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Question 1 continued

(b)(ii) Other critical values from $\frac{10x}{x-6} = -(2x+12)$

$10x = -(2x+12)(x-6) = -2x^2 + 72 \Rightarrow 2x^2 + 10x - 72 = 0$

$x^2 + 5x - 36 = 0 \Rightarrow (x+9)(x-4) = 0 \Rightarrow x = -9,\ x = 4$

Using the graphs reflected in the x-axis ($|y| \geqslant 0$), critical values

$-9, -4, 4, 9$ (and asymptote $x = 6$), so the regions are:

$x < -9, \quad -4 < x < 4, \quad x > 9$

(Total for Question 1 is 6 marks)

2. (a) Show that the differential equation

$$(x+2)\frac{dy}{dx} = 3x^2 + 6x - y \qquad x \neq -2$$

can be written in the form

$$\frac{dy}{dx} + f(x)y = kx$$

where f is a function to be determined and k is a constant to be found.

(2)

Given that $y = 18$ at $x = 4$

(b) use the answer to part (a) to determine, in simplest form, the particular solution of the differential equation.
Give the answer in the form $y = g(x)$

(6)

(a) $(x+2)\frac{dy}{dx} + y = 3x^2 + 6x = 3x(x+2)$

Divide by $(x+2)$: $\frac{dy}{dx} + \frac{y}{x+2} = \frac{3x(x+2)}{x+2}$

$\frac{dy}{dx} + \frac{1}{x+2}y = 3x$ $\quad f(x) = \frac{1}{x+2}$, $\quad k = 3$

(b) $IF = e^{\int 1/(x+2)\,dx} = e^{\ln(x+2)} = x+2$

$\frac{d}{dx}[(x+2)y] = 3x(x+2) = 3x^2 + 6x$

$(x+2)y = \int(3x^2 + 6x)\,dx = x^3 + 3x^2 + c$

$x = 4,\ y = 18$: $\quad 6 \times 18 = 64 + 48 + c \quad \Rightarrow \quad 108 = 112 + c \quad \Rightarrow \quad c = -4$

$y = \frac{x^3 + 3x^2 - 4}{x+2} = \frac{(x+2)(x+2)(x-1)}{x+2}$

$y = (x+2)(x-1)$ $\quad$ i.e. $\quad y = x^2 + x - 2$

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Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 8 marks)

3. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Given that when $y = \arcsin 2x$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\left(1 - 4x^2\right)^{-\frac{1}{2}}$$

(a) show that

$$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \frac{Ax^2 + 8}{\left(1 - 4x^2\right)^{\frac{5}{2}}}$$

where A is a constant to be determined.

(3)

(b) Hence determine the Maclaurin series expansion for $\arcsin 2x$ in ascending powers of x up to and including the term in x^3

(2)

The Maclaurin series expansion for e^x is given by

$$e^x = 1 + x + \frac{x^2}{2} + \ldots + \frac{x^r}{r!} + \ldots$$

(c) Use the Maclaurin series expansion for e^{3x} and the answer to part (b) to show that, for small values of x

$$e^{3x}\arcsin 2x \approx Cx + Dx^2 + Ex^3$$

where C, D and E are constants to be determined.

(3)

(a) $\frac{dy}{dx} = 2(1 - 4x^2)^{-1/2}$

$\frac{d^2y}{dx^2} = 2 \times (-\frac{1}{2})(1 - 4x^2)^{-3/2} \times (-8x) = 8x(1 - 4x^2)^{-3/2}$

$\frac{d^3y}{dx^3} = 8(1 - 4x^2)^{-3/2} + 8x \times (-\frac{3}{2})(1 - 4x^2)^{-5/2} \times (-8x)$

$= 8(1 - 4x^2)^{-3/2} + 96x^2(1 - 4x^2)^{-5/2}$

$= \frac{8(1 - 4x^2) + 96x^2}{(1 - 4x^2)^{5/2}} = \frac{64x^2 + 8}{(1 - 4x^2)^{5/2}}$ $\quad A = 64$

(b) $y(0) = \arcsin 0 = 0$, $y'(0) = 2$, $y''(0) = 0$, $y'''(0) = 8$

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Question 3 continued

$\arcsin 2x = 0 + 2x + \frac{0}{2!}x^2 + \frac{8}{3!}x^3 + \ldots$

$\arcsin 2x \approx 2x + \frac{4}{3}x^3$

(c) $e^{3x} = 1 + 3x + \frac{(3x)^2}{2} + \ldots = 1 + 3x + \frac{9}{2}x^2 + \ldots$

$e^{3x}\arcsin 2x \approx \left(1 + 3x + \frac{9}{2}x^2\right)\left(2x + \frac{4}{3}x^3\right)$

$\approx 2x + 6x^2 + 9x^3 + \frac{4}{3}x^3$

$e^{3x}\arcsin 2x \approx 2x + 6x^2 + \frac{31}{3}x^3$

$C = 2, \quad D = 6, \quad E = \frac{31}{3}$

Question 3 continued

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Question 3 continued

(Total for Question 3 is 8 marks)

4.

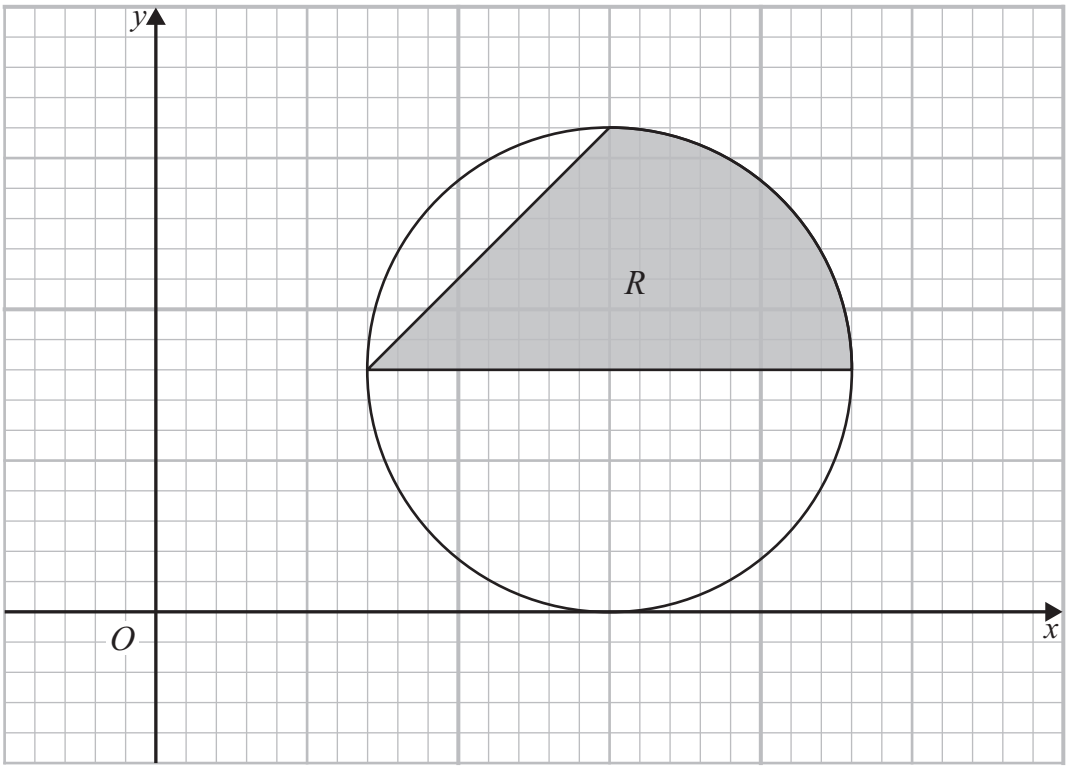


Figure 2

Figure 2 shows an Argand diagram for complex numbers of the form $z = x + \mathrm{i}y$.

The diagram is drawn accurately, although the scale is not shown on the axes.

Complex numbers that lie in the region R, shown shaded in Figure 2, satisfy all three of the inequalities

$$|z - 15 - 8\mathrm{i}| \leqslant a$$

$$0 \leqslant \arg(z + 1) \leqslant b\pi$$

$$|z + 2\mathrm{i}| \geqslant |z + c\mathrm{i}|$$

where a, b and c are real numbers.

(a) Determine the value of a, the value of b and the value of c

(3)

Given that the complex number w lies in the region R,

(b) determine the exact range of possible values of $|w|$

(3)

(c) determine the minimum value of arg w, giving the answer in radians to 3 significant figures.

(2)

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Question 4 continued

(a) Circle centre $15 + 8i$, radius 8 (touches the real axis): $a = 8$

Half-line from -1 through $7 + 8i$: gradient $\frac{8}{8} = 1$, angle $\frac{\pi}{4}$: $b = \frac{1}{4}$

Perpendicular bisector of $-2i$ and $-ci$ is the line $y = 8$:

$\frac{-2 + (-c)}{2} = 8 \Rightarrow -2 - c = 16 \Rightarrow c = -18$

(b) Nearest point of R to O is $7 + 8i$: $|w|_{min} = \sqrt{(7^2 + 8^2)} = \sqrt{113}$

Furthest point is on the line through O and the centre:

$|w|_{max} = \sqrt{(15^2 + 8^2)} + 8 = 17 + 8 = 25$

$\sqrt{113} \leqslant |w| \leqslant 25$

(c) Minimum arg w at the point $23 + 8i$ (right end of the diameter)

$\arg w_{min} = \arctan \frac{8}{23} = 0.33479... = 0.335$ (3 s.f.)

Question 4 continued

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Question 4 continued

(Total for Question 4 is 8 marks)

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Express $1 + \dfrac{2}{2r+5}$ as a single fraction in simplest form.

(1)

(b) Hence use the method of differences to determine an expression for

$$\sum_{r=1}^{n} \log_3\left(1 + \frac{2}{2r+5}\right)$$

giving the answer in the form $\log_3(f(n))$ where f is a function to be found.

(3)

(c) Hence determine the value of n for which

$$\sum_{r=n+2}^{10n} \log_3\left(1 + \frac{2}{2r+5}\right) = 2$$

(4)

(a) $1 + \frac{2}{2r+5} = \frac{2r+5+2}{2r+5} = \frac{2r+7}{2r+5}$

(b) $\sum_{r=1}^{n} \log_3\left(\frac{2r+7}{2r+5}\right) = \sum_{r=1}^{n} [\log_3(2r+7) - \log_3(2r+5)]$

$r = 1$: $\log_3 9 - \log_3 7$

$r = 2$: $\log_3 11 - \log_3 9$

$r = 3$: $\log_3 13 - \log_3 11$

⋮ ⋮

$r = n$: $\log_3(2n+7) - \log_3(2n+5)$

All terms cancel except $\log_3(2n+7)$ and $-\log_3 7$

$\sum_{r=1}^{n} \log_3\left(1 + \frac{2}{2r+5}\right) = \log_3(2n+7) - \log_3 7 = \log_3\left(\frac{2n+7}{7}\right)$

(c) Sum from $r = n+2$ to $10n$ = $S(10n) - S(n+1)$

$= \log_3\left(\frac{20n+7}{7}\right) - \log_3\left(\frac{2(n+1)+7}{7}\right) = \log_3\left(\frac{20n+7}{2n+9}\right)$

$\log_3\left(\frac{20n+7}{2n+9}\right) = 2 \Rightarrow \frac{20n+7}{2n+9} = 3^2 = 9$

$20n + 7 = 18n + 81 \Rightarrow 2n = 74 \Rightarrow n = 37$

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 8 marks)

6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C_1 has equation

$$r = \sqrt{3} + \tan\theta \qquad 0 < \theta < \frac{\pi}{2}$$

The tangent to C_1 is perpendicular to the initial line at the point P

(a) Use calculus to determine, in simplest form, the exact polar coordinates of P

(4)

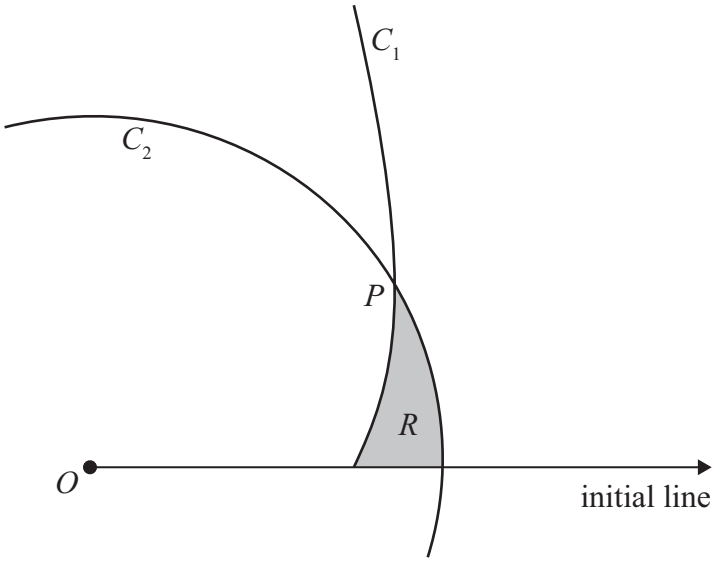


Figure 3

Figure 3 shows a sketch of part of the curve C_1 and part of the curve C_2

The curve C_2 is a circle with centre at the pole O.

The curves C_1 and C_2 intersect at P.

The region R, shown shaded in Figure 3, is bounded by C_1, C_2 and the initial line.

(b) Use algebraic integration to determine the area of R, giving the answer in the form

$$a\pi + \frac{\sqrt{3}}{2}(\ln b + c)$$

where a, b and c are rational numbers.

(8)

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Question 6 continued

(a) $x = r\cos\theta = (\sqrt{3} + \tan\theta)\cos\theta = \sqrt{3}\cos\theta + \sin\theta$

Tangent perpendicular to initial line: $\frac{dx}{d\theta} = 0$

$\frac{dx}{d\theta} = -\sqrt{3}\sin\theta + \cos\theta = 0 \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$

$r = \sqrt{3} + \tan\frac{\pi}{6} = \sqrt{3} + \frac{\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$

$P\left(\frac{4\sqrt{3}}{3}, \frac{\pi}{6}\right)$

(b) Area R = sector of C_2 $(0 \leq \theta \leq \pi/6)$ − area under C_1 $(0 \leq \theta \leq \pi/6)$

Sector $= \frac{1}{2}\left(\frac{4\sqrt{3}}{3}\right)^2 \times \frac{\pi}{6} = \frac{1}{2} \times \frac{16}{3} \times \frac{\pi}{6} = \frac{4\pi}{9}$

$\frac{1}{2}\int_0^{\pi/6} (\sqrt{3} + \tan\theta)^2\, d\theta = \frac{1}{2}\int_0^{\pi/6} (3 + 2\sqrt{3}\tan\theta + \tan^2\theta)\, d\theta$

$\tan^2\theta = \sec^2\theta - 1$: $= \frac{1}{2}\int_0^{\pi/6} (2 + 2\sqrt{3}\tan\theta + \sec^2\theta)\, d\theta$

$= \frac{1}{2}\left[2\theta - 2\sqrt{3}\ln\cos\theta + \tan\theta\right]_0^{\pi/6}$

$= \frac{1}{2}\left[\left(\frac{\pi}{3} - 2\sqrt{3}\ln\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{3}\right) - (0 - 0 + 0)\right]$

$= \frac{\pi}{6} - \sqrt{3}\ln\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{6}$

Area $R = \frac{4\pi}{9} - \left(\frac{\pi}{6} - \sqrt{3}\ln\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{6}\right) = \frac{5\pi}{18} + \sqrt{3}\ln\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6}$

$\sqrt{3}\ln\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}\ln\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{\sqrt{3}}{2}\ln\frac{3}{4}$

Area $R = \frac{5\pi}{18} + \frac{\sqrt{3}}{2}\left(\ln\frac{3}{4} - \frac{1}{3}\right)$

$a = \frac{5}{18}$, $b = \frac{3}{4}$, $c = -\frac{1}{3}$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 12 marks)

7. (a) Show that the substitution $x = e^u$, where u is a function of x, transforms the differential equation

$$2x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - y = 27x^2 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$2\frac{d^2y}{du^2} + \frac{dy}{du} - y = 27e^{2u} \qquad \text{(II)}$$

(4)

(b) By solving differential equation (II), determine the general solution of differential equation (I).

Give the answer in the form $y = f(x)$ where f is a fully simplified function.

(4)

Given that when $x = \frac{1}{4}, y = \frac{11}{16}$ and $\frac{dy}{dx} = 1$

(c) determine the value of y when $x = \frac{1}{8}$, giving the answer in the form $\frac{1}{64}\left(p\sqrt{2} + q\right)$ where p and q are integers.

(5)

(a) $x = e^u \Rightarrow \frac{dx}{du} = e^u = x \Rightarrow \frac{du}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{x}\frac{dy}{du} \Rightarrow x\frac{dy}{dx} = \frac{dy}{du}$

$\frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x}\frac{d^2y}{du^2}\frac{du}{dx} = \frac{1}{x^2}\frac{d^2y}{du^2} - \frac{1}{x^2}\frac{dy}{du}$

$\Rightarrow x^2\frac{d^2y}{dx^2} = \frac{d^2y}{du^2} - \frac{dy}{du}$

Substitute into (I): $2\left(\frac{d^2y}{du^2} - \frac{dy}{du}\right) + 3\frac{dy}{du} - y = 27(e^u)^2$

$2\frac{d^2y}{du^2} + \frac{dy}{du} - y = 27e^{2u}$ as required (II)

(b) AE: $2m^2 + m - 1 = 0 \Rightarrow (2m - 1)(m + 1) = 0 \Rightarrow m = \frac{1}{2}, -1$

CF: $y = Ae^{u/2} + Be^{-u}$

PI: $y = \lambda e^{2u}$, $y' = 2\lambda e^{2u}$, $y'' = 4\lambda e^{2u}$

$8\lambda + 2\lambda - \lambda = 27 \Rightarrow 9\lambda = 27 \Rightarrow \lambda = 3$

$y = Ae^{u/2} + Be^{-u} + 3e^{2u}$ and $e^u = x$:

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Question 7 continued

$y = Ax^{1/2} + Bx^{-1} + 3x^2$

(c) $\frac{dy}{dx} = \frac{1}{2}Ax^{-1/2} - Bx^{-2} + 6x$

$x = \frac{1}{4}, y = \frac{11}{16}$: $\frac{11}{16} = \frac{1}{2}A + 4B + \frac{3}{16} \Rightarrow \frac{1}{2}A + 4B = \frac{1}{2}$

$x = \frac{1}{4}, \frac{dy}{dx} = 1$: $1 = A - 16B + \frac{3}{2} \Rightarrow A - 16B = -\frac{1}{2}$

Solving: $A + 8B = 1$, $A - 16B = -\frac{1}{2} \Rightarrow 24B = \frac{3}{2} \Rightarrow B = \frac{1}{16}$, $A = \frac{1}{2}$

$y = \frac{1}{2}\sqrt{x} + \frac{1}{16x} + 3x^2$

$x = \frac{1}{8}$: $y = \frac{1}{2}\left(\frac{1}{8}\right)^{1/2} + \frac{1}{16}\left(\frac{1}{8}\right)^{-1} + 3\left(\frac{1}{8}\right)^2 = \frac{1}{4\sqrt{2}} + \frac{1}{2} + \frac{3}{64}$

$= \frac{\sqrt{2}}{8} + \frac{35}{64} = \frac{1}{64}(8\sqrt{2} + 35)$

Question 7 continued

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Question 7 continued

(Total for Question 7 is 13 marks)

8. Given that $z = \cos\theta + \mathrm{i}\sin\theta$

(a) show that, for $n \in \mathbb{Z}$

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

(2)

(b) Hence show that

$$\cos^4\theta = \frac{1}{8}(\cos 4\theta + a\cos 2\theta + b)$$

where a and b are integers to be determined.

(4)

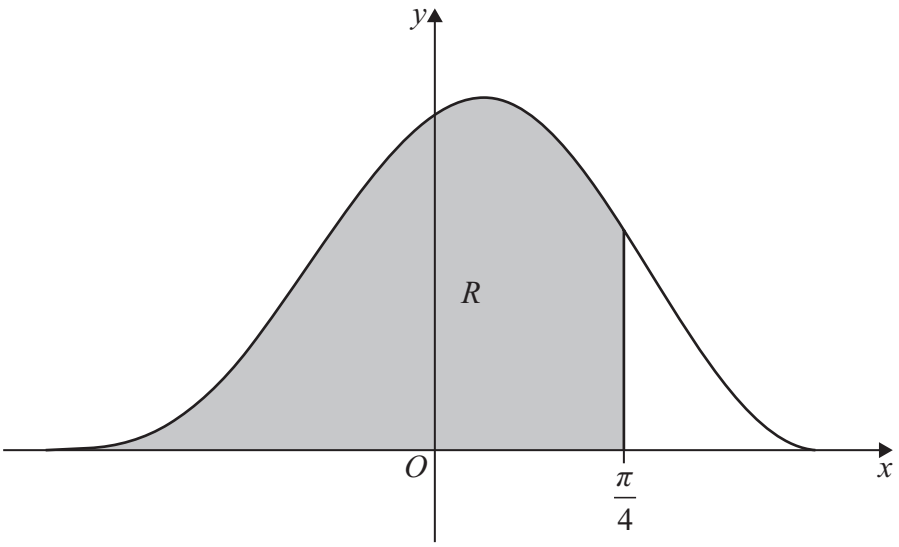


Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \cos^2 x\sqrt{1 + \sin x} \qquad -\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 4, is bounded by the curve, the x–axis and the line with equation $x = \frac{\pi}{4}$

The region R is rotated through 2π radians about the x–axis to form a solid of revolution.

(c) Use the answer to part (b) and algebraic integration to determine the exact volume of this solid.

Give the answer in the form $\frac{\pi}{160}\left(p + q\pi + r\sqrt{2}\right)$ where p, q and r are integers.

(6)

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Question 8 continued

(a) By de Moivre: $z^n = \cos n\theta + i \sin n\theta$

$\frac{1}{z^n} = z^{-n} = \cos(-n\theta) + i \sin(-n\theta) = \cos n\theta - i \sin n\theta$

$z^n + \frac{1}{z^n} = \cos n\theta + i \sin n\theta + \cos n\theta - i \sin n\theta = 2\cos n\theta$ as required

(b) $\left(z + \frac{1}{z}\right)^4 = z^4 + 4z^3\left(\frac{1}{z}\right) + 6z^2\left(\frac{1}{z}\right)^2 + 4z\left(\frac{1}{z}\right)^3 + \left(\frac{1}{z}\right)^4$

$= \left(z^4 + \frac{1}{z^4}\right) + 4\left(z^2 + \frac{1}{z^2}\right) + 6$

$(2\cos\theta)^4 = 2\cos 4\theta + 4(2\cos 2\theta) + 6$

$16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$

$\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$ $\quad a = 4,\ b = 3$

(c) $V = \pi\int_{-\pi/2}^{\pi/4} y^2\,dx = \pi\int_{-\pi/2}^{\pi/4} \cos^4 x(1 + \sin x)\,dx$

$= \pi\int_{-\pi/2}^{\pi/4} \left(\frac{1}{8}(\cos 4x + 4\cos 2x + 3) + \sin x \cos^4 x\right)dx$

$= \pi\left[\frac{1}{8}\left(\frac{1}{4}\sin 4x + 2\sin 2x + 3x\right) - \frac{1}{5}\cos^5 x\right]_{-\pi/2}^{\pi/4}$

At $\frac{\pi}{4}$: $\frac{1}{8}\left(0 + 2 + \frac{3\pi}{4}\right) - \frac{1}{5}\left(\frac{\sqrt{2}}{2}\right)^5 = \frac{1}{4} + \frac{3\pi}{32} - \frac{\sqrt{2}}{40}$

At $-\frac{\pi}{2}$: $\frac{1}{8}\left(0 + 0 - \frac{3\pi}{2}\right) - 0 = -\frac{3\pi}{16}$

$V = \pi\left(\frac{1}{4} + \frac{3\pi}{32} - \frac{\sqrt{2}}{40} + \frac{3\pi}{16}\right) = \pi\left(\frac{1}{4} + \frac{9\pi}{32} - \frac{\sqrt{2}}{40}\right)$

$V = \frac{\pi}{160}(40 + 45\pi - 4\sqrt{2})$

Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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