Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 3 June 2025

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1: (a) Express $-2\sqrt{2} - \left(2\sqrt{6}\right)\mathrm{i}$ in the form $re^{i\theta}$ where $-\pi < \theta \leqslant \pi$

(3)

(b) Hence solve

$$z^5 = -2\sqrt{2} - \left(2\sqrt{6}\right)\mathrm{i}$$

Give your answers in the form $\sqrt{p}\,e^{i\theta}$ where $p \in \mathbb{Z}^+$ and $-\pi < \theta \leqslant \pi$

(3)

(a) $r = |z| = \sqrt{((2\sqrt{2})^2 + (2\sqrt{6})^2)} = \sqrt{(8 + 24)} = \sqrt{32} = 4\sqrt{2}$

z is in the third quadrant: $\theta = -\pi + \arctan\left(\frac{2\sqrt{6}}{2\sqrt{2}}\right) = -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$

$\therefore \quad -2\sqrt{2} - (2\sqrt{6})i = 4\sqrt{2}\, e^{-2\pi i/3}$

(b) $z^5 = 4\sqrt{2}\, e^{i(-2\pi/3 + 2k\pi)}$

$z = (4\sqrt{2})^{1/5}\, e^{i(-2\pi/3 + 2k\pi)/5} = (2^{5/2})^{1/5}\, e^{i(-2\pi + 6k\pi)/15} = \sqrt{2}\, e^{i(-2\pi + 6k\pi)/15}$

$k = 0$: $\theta = -\frac{2\pi}{15}$ $\quad k = 1$: $\theta = \frac{4\pi}{15}$ $\quad k = 2$: $\theta = \frac{10\pi}{15} = \frac{2\pi}{3}$

$k = -1$: $\theta = -\frac{8\pi}{15}$ $\quad k = -2$: $\theta = -\frac{14\pi}{15}$

$z = \sqrt{2}\, e^{-14\pi i/15},\ \sqrt{2}\, e^{-8\pi i/15},\ \sqrt{2}\, e^{-2\pi i/15},\ \sqrt{2}\, e^{4\pi i/15},\ \sqrt{2}\, e^{2\pi i/3}$

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Question 1 continued

(Total for Question 1 is 6 marks)

2: $$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y^2 = e^{x^2-9}$$

Given that $y = 2$ and $\frac{dy}{dx} = -1$ at $x = 3$, determine a Taylor series for y in ascending powers of $(x-3)$, up to and including the term in $(x-3)^3$

(5)

$\frac{d^2y}{dx^2} = 4\frac{dy}{dx} + 5y^2 + e^{x^2-9}$

Differentiating: $\frac{d^3y}{dx^3} = 4\frac{d^2y}{dx^2} + 10y\frac{dy}{dx} + 2xe^{x^2-9}$

At $x = 3$: $y = 2$, $\frac{dy}{dx} = -1$, $e^{x^2-9} = e^0 = 1$

$\frac{d^2y}{dx^2} = 4(-1) + 5(2)^2 + 1 = 17$

$\frac{d^3y}{dx^3} = 4(17) + 10(2)(-1) + 2(3)(1) = 68 - 20 + 6 = 54$

$y = y(3) + y'(3)(x-3) + \frac{y''(3)}{2!}(x-3)^2 + \frac{y'''(3)}{3!}(x-3)^3 + \ldots$

$y = 2 - (x-3) + \frac{17}{2}(x-3)^2 + \frac{54}{6}(x-3)^3 + \ldots$

$y = 2 - (x-3) + \frac{17}{2}(x-3)^2 + 9(x-3)^3 + \ldots$

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Question 2 continued

(Total for Question 2 is 5 marks)

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3: A complex number z is represented by the point P on an Argand diagram where $|z| = 1$

(a) Sketch the locus of P as z varies.

(1)

The transformation T from the z–plane, where $z = x + iy$, to the w–plane, where $w = u + iv$, is given by

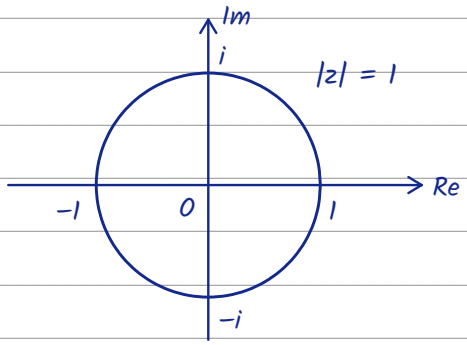
$$w = \frac{9iz - i}{z + 1} \qquad z \neq -1$$

Given that the image under T of the locus of P in the z–plane, where $z \neq -1$, is the line l in the w–plane,

(b) determine, in simplest form, a Cartesian equation for l

(5)

(a)



(b) $w(z + 1) = 9iz - i \Rightarrow wz - 9iz = -w - i \Rightarrow z(w - 9i) = -w - i$

$z = \frac{-w - i}{w - 9i} = \frac{w + i}{9i - w}$

$|z| = 1 \Rightarrow \left|\frac{w + i}{w - 9i}\right| = 1 \Rightarrow |w + i| = |w - 9i|$

$w = u + iv$: $u^2 + (v + 1)^2 = u^2 + (v - 9)^2$

$v^2 + 2v + 1 = v^2 - 18v + 81$

$20v = 80 \Rightarrow v = 4$

l : $v = 4$

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 6 marks)

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4: Given that $y = \lambda x e^{3x}$ is a particular integral of the differential equation

$$4\frac{d^2y}{dx^2} - 11\frac{dy}{dx} - 3y = 78e^{3x}$$

(a) determine the value of the constant λ

(3)

(b) Hence determine the general solution of the differential equation.

(3)

Given also that $y = \frac{9}{2}$ and $\frac{dy}{dx} = 0$ at $x = 0$

(c) determine the particular solution of the differential equation.

(4)

(a) $y = \lambda x e^{3x}$

$\frac{dy}{dx} = \lambda e^{3x} + 3\lambda x e^{3x}$

$\frac{d^2y}{dx^2} = 3\lambda e^{3x} + 3\lambda e^{3x} + 9\lambda x e^{3x} = 6\lambda e^{3x} + 9\lambda x e^{3x}$

$4(6\lambda e^{3x} + 9\lambda x e^{3x}) - 11(\lambda e^{3x} + 3\lambda x e^{3x}) - 3\lambda x e^{3x} = 78e^{3x}$

$24\lambda e^{3x} + 36\lambda x e^{3x} - 11\lambda e^{3x} - 33\lambda x e^{3x} - 3\lambda x e^{3x} = 78e^{3x}$

$13\lambda e^{3x} = 78e^{3x} \Rightarrow \lambda = 6$

(b) $4m^2 - 11m - 3 = 0 \Rightarrow (4m + 1)(m - 3) = 0 \Rightarrow m = -\frac{1}{4}, 3$

CF: $y = Ae^{-x/4} + Be^{3x}$

$y = Ae^{-x/4} + Be^{3x} + 6xe^{3x}$

(c) $\frac{dy}{dx} = -\frac{1}{4}Ae^{-x/4} + 3Be^{3x} + 6e^{3x} + 18xe^{3x}$

$x = 0, y = \frac{9}{2}$: $\frac{9}{2} = A + B$

$x = 0, \frac{dy}{dx} = 0$: $0 = -\frac{1}{4}A + 3B + 6 \Rightarrow A - 12B = 24$

Subtracting: $13B = \frac{9}{2} - 24 = -\frac{39}{2} \Rightarrow B = -\frac{3}{2}$, $A = \frac{9}{2} + \frac{3}{2} = 6$

$y = 6e^{-x/4} - \frac{3}{2}e^{3x} + 6xe^{3x}$

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 10 marks)



5: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Express $\dfrac{2}{r(r+1)(r+2)}$ in partial fractions.

(2)

(b) Use the answer to part (a) and the method of differences to show that

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{n(n+a)}{2(n+b)(n+c)}$$

where a, b and c are integers to be determined.

(5)

(c) Hence form and solve a quadratic inequality to determine the smallest value of n for which

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} > \frac{7}{15}$$

(3)

(a) $\dfrac{2}{r(r+1)(r+2)} \equiv \dfrac{A}{r} + \dfrac{B}{r+1} + \dfrac{C}{r+2}$

$2 \equiv A(r+1)(r+2) + Br(r+2) + Cr(r+1)$

$r = 0$: $2 = 2A \Rightarrow A = 1$ $\quad r = -1$: $2 = -B \Rightarrow B = -2$ $\quad r = -2$: $2 = 2C \Rightarrow C = 1$

$\dfrac{2}{r(r+1)(r+2)} \equiv \dfrac{1}{r} - \dfrac{2}{r+1} + \dfrac{1}{r+2}$

(b) $r = 1$: $\dfrac{1}{1} - \dfrac{2}{2} + \dfrac{1}{3}$

$r = 2$: $\dfrac{1}{2} - \dfrac{2}{3} + \dfrac{1}{4}$

$r = 3$: $\dfrac{1}{3} - \dfrac{2}{4} + \dfrac{1}{5}$

$\vdots \qquad \vdots$

$r = n-1$: $\dfrac{1}{n-1} - \dfrac{2}{n} + \dfrac{1}{n+1}$

$r = n$: $\dfrac{1}{n} - \dfrac{2}{n+1} + \dfrac{1}{n+2}$

$\sum_{r=1}^{n} \dfrac{2}{r(r+1)(r+2)} = 1 - 1 + \dfrac{1}{2} + \dfrac{1}{n+1} - \dfrac{2}{n+1} + \dfrac{1}{n+2} = \dfrac{1}{2} - \dfrac{1}{n+1} + \dfrac{1}{n+2}$

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$= \frac{(n+1)(n+2) - 2(n+2) + 2(n+1)}{2(n+1)(n+2)} = \frac{n^2 + 3n + 2 - 2n - 4 + 2n + 2}{2(n+1)(n+2)}$

$= \frac{n^2 + 3n}{2(n+1)(n+2)} = \frac{n(n+3)}{2(n+1)(n+2)}$

$a = 3, \quad b = 1, \quad c = 2$

(c) $\frac{n(n+3)}{2(n+1)(n+2)} > \frac{7}{15}$

$15n(n+3) > 14(n+1)(n+2)$ (denominators positive)

$15n^2 + 45n > 14n^2 + 42n + 28$

$n^2 + 3n - 28 > 0 \Rightarrow (n+7)(n-4) > 0 \Rightarrow n > 4 \quad (n > 0)$

Smallest value: $n = 5$

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 10 marks)

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6: (a) Show that the substitution $z = \frac{1}{y^2}$ transforms the differential equation

$$2\frac{dy}{dx} + (\cot x)y + (\tan x \sec x)y^3 = 0 \qquad 0 < x < \frac{\pi}{2} \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - (\cot x)z = \tan x \sec x \qquad 0 < x < \frac{\pi}{2} \qquad \text{(II)}$$

(3)

(b) Hence determine the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$

(5)

Given that $y^2 = \frac{4\sqrt{3}}{3}$ when $x = \frac{\pi}{6}$

(c) determine the exact values of y when $x = \frac{\pi}{3}$

(3)

(a) $z = y^{-2} \Rightarrow \frac{dz}{dx} = -2y^{-3}\frac{dy}{dx} \Rightarrow 2\frac{dy}{dx} = -y^3\frac{dz}{dx}$

Substituting into (I): $-y^3\frac{dz}{dx} + (\cot x)y + (\tan x \sec x)y^3 = 0$

Dividing by $-y^3$: $\frac{dz}{dx} - \frac{\cot x}{y^2} - \tan x \sec x = 0$

$\therefore \frac{dz}{dx} - (\cot x)z = \tan x \sec x \qquad$ (II)

(b) I.F. $= e^{-\int \cot x\, dx} = e^{-\ln \sin x} = \frac{1}{\sin x}$

$\frac{d}{dx}\left(\frac{z}{\sin x}\right) = \frac{1}{\sin x} \times \frac{\sin x}{\cos x} \times \frac{1}{\cos x} = \sec^2 x$

$\frac{z}{\sin x} = \int \sec^2 x\, dx = \tan x + c$

$z = \sin x(\tan x + c)$

$y^2 = \frac{1}{\sin x(\tan x + c)}$

(c) $x = \frac{\pi}{6}$: $\frac{4\sqrt{3}}{3} = \frac{1}{\sin(\pi/6)(\tan(\pi/6) + c)} = \frac{1}{\frac{1}{2}(\sqrt{3}/3 + c)}$

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Question 6 continued

$\frac{\sqrt{3}}{3} + c = \frac{2 \times 3}{4\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{6}$

$x = \frac{\pi}{3}$: $y^2 = \frac{1}{(\sqrt{3}/2)(\sqrt{3} + \sqrt{3}/6)} = \frac{1}{(\sqrt{3}/2)(7\sqrt{3}/6)} = \frac{1}{7/4} = \frac{4}{7}$

$y = \pm\frac{2}{\sqrt{7}} = \pm\frac{2\sqrt{7}}{7}$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 11 marks)

7: **In this question you must show all stages of your working.**
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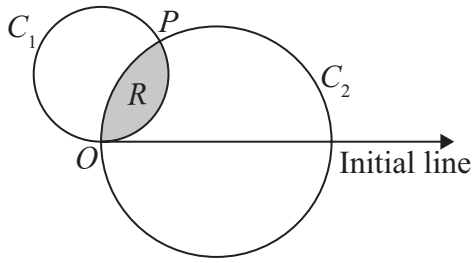


Figure 1

Figure 1 shows a sketch of the circles C_1 and C_2

The circle C_1 has polar equation

$$r = \sqrt{3}\sin\theta \qquad 0 \leqslant \theta \leqslant \pi$$

The circle C_2 has polar equation

$$r = 3\cos\theta \qquad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$$

Circles C_1 and C_2 intersect at the origin O and at the point P.

(a) Determine the polar coordinates of P.

(2)

The finite region R is bounded by C_1 and C_2 and is shown shaded in Figure 1.

(b) Use algebraic integration to determine the exact area of R, giving your answer in the form $a\pi + b\sqrt{3}$ where a and b are simplified rational numbers.

(6)

(a) $\sqrt{3}\sin\theta = 3\cos\theta \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$, $r = 3\cos\frac{\pi}{3} = \frac{3}{2}$

$P\left(\frac{3}{2}, \frac{\pi}{3}\right)$

(b) $\int(\sqrt{3}\sin\theta)^2\,d\theta = \int\left(\frac{3}{2} - \frac{3}{2}\cos 2\theta\right)d\theta = \frac{3}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right)$

$\int(3\cos\theta)^2\,d\theta = \int\left(\frac{9}{2} + \frac{9}{2}\cos 2\theta\right)d\theta = \frac{9}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right)$

Area $R = \frac{1}{2}\int_0^{\pi/3} 3\sin^2\theta\,d\theta + \frac{1}{2}\int_{\pi/3}^{\pi/2} 9\cos^2\theta\,d\theta$

$= \frac{3}{4}\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi/3} + \frac{9}{4}\left[\theta + \frac{1}{2}\sin 2\theta\right]_{\pi/3}^{\pi/2}$

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Question 7 continued

$$= \frac{3}{4}\left[\frac{\pi}{3} - \frac{1}{2}\sin\frac{2\pi}{3} - 0\right] + \frac{9}{4}\left[\frac{\pi}{2} + 0 - \left(\frac{\pi}{3} + \frac{1}{2}\sin\frac{2\pi}{3}\right)\right]$$

$$= \frac{3}{4}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) + \frac{9}{4}\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

$$= \frac{\pi}{4} - \frac{3\sqrt{3}}{16} + \frac{3\pi}{8} - \frac{9\sqrt{3}}{16}$$

Area $R = \frac{5}{8}\pi - \frac{3}{4}\sqrt{3}$

Question 7 continued

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Question 7 continued

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8: **In this question you must show all stages of your working.**
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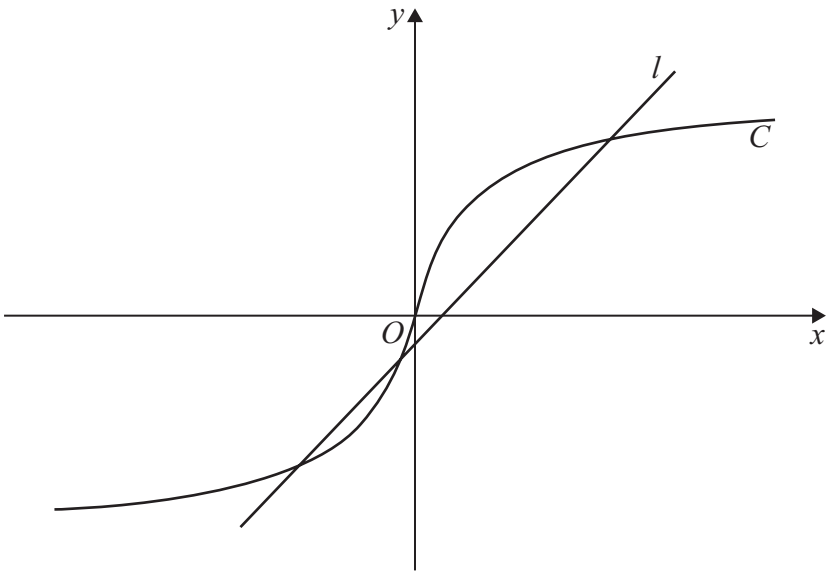


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \dfrac{15x}{|x| + 4}$ and the line l with equation $y = x - 2$

(a) Use algebra to determine the exact values of x for which

$$x - 2 > \frac{15x}{|x| + 4}$$

(6)

(b) Hence use algebra to determine the exact values of x for which

$$|x - 2| > \left|\frac{15x}{|x| + 4}\right|$$

(4)

(a) $x > 0$: $\dfrac{15x}{x + 4} = x - 2 \Rightarrow 15x = x^2 + 2x - 8 \Rightarrow x^2 - 13x - 8 = 0$

$x = \dfrac{13 \pm \sqrt{(169 + 32)}}{2} = \dfrac{13 \pm \sqrt{201}}{2} \Rightarrow x = \dfrac{13 + \sqrt{201}}{2}$ $(x > 0)$

$x < 0$: $\dfrac{15x}{-x + 4} = x - 2 \Rightarrow 15x = -x^2 + 6x - 8 \Rightarrow x^2 + 9x + 8 = 0$

$(x + 8)(x + 1) = 0 \Rightarrow x = -8,\ x = -1$

From Figure 2, l lies above C for x between -8 and -1, and to the right of $x = \dfrac{13 + \sqrt{201}}{2}$

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Question 8 continued

$-8 < x < -1$ or $x > \frac{13 + \sqrt{201}}{2}$

(b) Also need the intersections of $y = -(x - 2)$ with C:

$\frac{15x}{x + 4} = -x + 2 \Rightarrow 15x = -x^2 - 2x + 8 \Rightarrow x^2 + 17x - 8 = 0$

$x = \frac{-17 \pm \sqrt{(289 + 32)}}{2} = \frac{-17 \pm \sqrt{321}}{2} \Rightarrow x = \frac{-17 + \sqrt{321}}{2}$ $(= 0.458..., x > 0)$

(For $x < 0$: $\frac{15x}{-x + 4} = 2 - x \Rightarrow x^2 - 21x + 8 = 0$, both roots positive, so rejected)

$|x - 2| > \left|\frac{15x}{|x| + 4}\right|$:

$x < -8$ or $-1 < x < \frac{-17 + \sqrt{321}}{2}$ or $x > \frac{13 + \sqrt{201}}{2}$



Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

9: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Use de Moivre's theorem to show that

$$\sin 5\theta \equiv a\sin^5\theta + b\sin^3\theta + c\sin\theta$$

where a, b and c are integers to be determined.

(5)

(b) Hence determine the possible exact values of $\sin^2\left(\frac{k\pi}{5}\right)$ where $k \in \mathbb{Z}$

(4)

(a) $\cos 5\theta + i\sin 5\theta = (\cos\theta + i\sin\theta)^5$ Let $c = \cos\theta$, $s = \sin\theta$

$= c^5 + 5c^4(is) + 10c^3(is)^2 + 10c^2(is)^3 + 5c(is)^4 + (is)^5$

$= c^5 + 5ic^4s - 10c^3s^2 - 10ic^2s^3 + 5cs^4 + is^5$

Imaginary parts: $\sin 5\theta = 5c^4s - 10c^2s^3 + s^5$

$= 5s(1 - s^2)^2 - 10s^3(1 - s^2) + s^5$

$= 5s - 10s^3 + 5s^5 - 10s^3 + 10s^5 + s^5$

$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$

$a = 16, \quad b = -20, \quad c = 5$

(b) $\theta = \frac{k\pi}{5} \Rightarrow \sin 5\theta = \sin k\pi = 0$

$16\sin^5\theta - 20\sin^3\theta + 5\sin\theta = 0 \Rightarrow \sin\theta(16\sin^4\theta - 20\sin^2\theta + 5) = 0$

$\sin\theta = 0$ or $\sin^2\theta = \frac{20 \pm \sqrt{(400 - 320)}}{32} = \frac{20 \pm \sqrt{80}}{32} = \frac{20 \pm 4\sqrt{5}}{32} = \frac{5 \pm \sqrt{5}}{8}$

$\sin^2\left(\frac{k\pi}{5}\right) = 0, \quad \frac{5 + \sqrt{5}}{8}, \quad \frac{5 - \sqrt{5}}{8}$

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Question 9 continued

Question 9 continued

(Total for Question 9 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

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