

Please check the examination details below before entering your candidate information

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| Candidate surname |  |  |  |  | Other names      |  |  |  |  |
| Centre Number     |  |  |  |  | Candidate Number |  |  |  |  |
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**Pearson Edexcel International Advanced Level**

**Tuesday 11 June 2024**

Morning (Time: 1 hour 30 minutes) **Paper reference** **WFM03/01**

**Mathematics**

**International Advanced Subsidiary/ Advanced Level**

**Further Pure Mathematics F3**

**You must have:**  
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

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1. The hyperbola  $H$  has

- foci with coordinates  $\left(\pm \frac{13}{2}, 0\right)$
- directrices with equations  $x = \pm \frac{72}{13}$
- eccentricity  $e$

Determine

- (a) the value of  $e$  (3)
- (b) an equation for  $H$ , giving your answer in the form  $px^2 - qy^2 = r$ , where  $p$ ,  $q$  and  $r$  are integers. (3)

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**Question 1 continued**

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**(Total for Question 1 is 6 marks)**

2.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

$$\mathbf{M} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & -4 & -3 \\ 0 & -4 & 0 \end{pmatrix}$$

Given that  $\mathbf{M}$  has exactly two distinct eigenvalues  $\lambda_1$  and  $\lambda_2$  where  $\lambda_1 < \lambda_2$

- (a) determine a normalised eigenvector corresponding to the eigenvalue  $\lambda_1$  (6)

The line  $l_1$  has equation  $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ , where  $\mu$  is a scalar parameter.

The transformation  $T$  is represented by  $\mathbf{M}$ .

The line  $l_1$  is transformed by  $T$  to the line  $l_2$

- (b) Determine a vector equation for  $l_2$ , giving your answer in the form  $\mathbf{r} \times \mathbf{b} = \mathbf{c}$  where  $\mathbf{b}$  and  $\mathbf{c}$  are constant vectors. (3)

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## Question 2 continued

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**Question 2 continued**

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**(Total for Question 2 is 9 marks)**

3.  $y = \operatorname{arsinh}(\sqrt{x^2 - 1}) \quad x > 1$

(a) Prove that  $\frac{dy}{dx} = \frac{1}{\sqrt{x^2 - 1}}$  (3)

$$f(x) = \frac{1}{3} \operatorname{arsinh}(\sqrt{x^2 - 1}) - \arctan x \quad x > 1$$

(b) Determine the exact values of  $x$  for which  $f'(x) = 0$  (4)

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**Question 3 continued**

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**(Total for Question 3 is 7 marks)**

4. (a) Use the definitions of hyperbolic functions in terms of exponentials to show that

$$\sinh(A + B) \equiv \sinh A \cosh B + \cosh A \sinh B \quad (3)$$

- (b) Hence express  $10 \sinh x + 8 \cosh x$  in the form  $R \sinh(x + \alpha)$  where  $R > 0$ , giving  $\alpha$  in the form  $\ln p$  where  $p$  is an integer. (4)

- (c) Hence solve the equation

$$10 \sinh x + 8 \cosh x = 18\sqrt{7}$$

giving your answer in the form  $\ln(\sqrt{7} + q)$  where  $q$  is a rational number to be determined. (2)

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**Question 4 continued**

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**Question 4 continued**

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**(Total for Question 4 is 9 marks)**

5.

$$4x^2 + 4x + 17 \equiv (2x + p)^2 + q$$

where  $p$  and  $q$  are integers.

- (a) Determine the value of  $p$  and the value of  $q$  (2)

Given that

$$\frac{8x + 5}{\sqrt{4x^2 + 4x + 17}} \equiv \frac{1}{\sqrt{4x^2 + 4x + 17}} + \frac{Ax + B}{\sqrt{4x^2 + 4x + 17}}$$

where  $A$  and  $B$  are integers,

- (b) write down the value of  $A$  and the value of  $B$  (1)

- (c) Hence use algebraic integration to show that

$$\int_{\frac{1}{3}}^1 \frac{8x + 5}{\sqrt{4x^2 + 4x + 17}} dx = k + \frac{1}{2} \ln k$$

where  $k$  is a rational number to be determined. (5)

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**Question 5 continued**

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**Question 5 continued**

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**(Total for Question 5 is 8 marks)**

6. The ellipse  $E$  has equation

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

The line  $l$  is the normal to  $E$  at the point  $P(5 \cos \theta, 3 \sin \theta)$  where  $0 < \theta < \frac{\pi}{2}$

(a) Using calculus, show that an equation for  $l$  is

$$5x \sin \theta - 3y \cos \theta = 16 \sin \theta \cos \theta \quad (4)$$

Given that

- $l$  intersects the  $y$ -axis at the point  $Q$
  - the midpoint of the line segment  $PQ$  is  $M$
- (b) determine the exact maximum area of triangle  $OMP$  as  $\theta$  varies, where  $O$  is the origin.

You must justify your answer.

(5)

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**Question 6 continued**

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**Question 6 continued**

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7.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

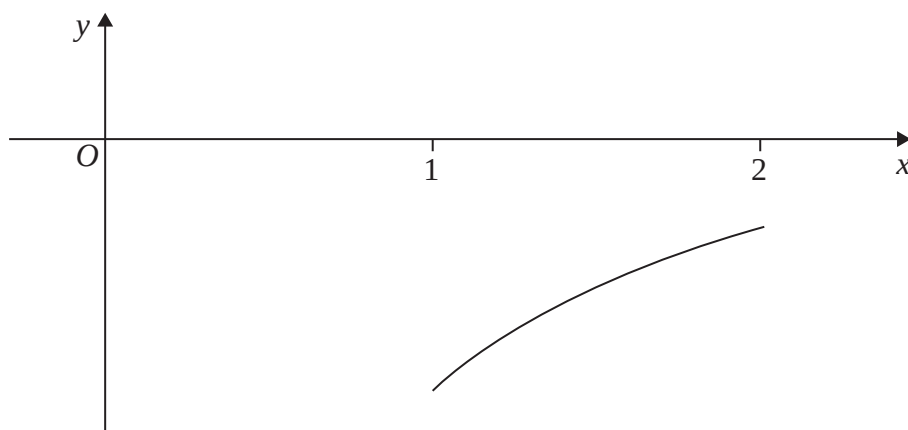


Figure 1

Figure 1 shows the curve with equation

$$y = \ln \left( \tanh \frac{x}{2} \right) \quad 1 \leq x \leq 2$$

(a) Show that the length,  $s$ , of the curve is given by

$$s = \int_1^2 \coth x \, dx \quad (4)$$

(b) Hence show that

$$s = \ln \left( e + \frac{1}{e} \right) \quad (4)$$

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**Question 7 continued**

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**(Total for Question 7 is 8 marks)**

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8. 
$$I_n = \int_0^k x^n (k-x)^{\frac{1}{2}} dx \quad n \geq 0$$

where  $k$  is a positive constant.

(a) Show that

$$I_n = \frac{2kn}{3+2n} I_{n-1} \quad n \geq 1 \quad (5)$$

Given that

$$\int_0^k x^2 (k-x)^{\frac{1}{2}} dx = \frac{9\sqrt{3}}{280}$$

(b) use the result in part (a) to determine the exact value of  $k$ . (4)

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**Question 8 continued**

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## Question 8 continued

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(Total for Question 8 is 9 marks)







**Question 9 continued**

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**(Total for Question 9 is 10 marks)****TOTAL FOR PAPER IS 75 MARKS**