Please check the examination details below before entering your candidate information

Candidate surname	Other names
StemUp Model Solution	

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Q:1/1/1/1/

P 7 2 4 0 2 A 0 1 3 2

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1. Given that

$$\frac{2n+1}{n^2(n+1)^2} \equiv \frac{A}{n^2} + \frac{B}{(n+1)^2}$$

(a) determine the value of A and the value of B

(1)

(b) Hence show that, for $n \geqslant 5$

$$\sum_{r=5}^{n} \frac{2r+1}{r^2(r+1)^2} = \frac{n^2 + an + b}{c(n+1)^2}$$

where a, b and c are integers to be determined.

(4)

(a) $2n + 1 \equiv A(n+1)^2 + Bn^2$

$n = 0$: $1 = A \Rightarrow A = 1$

$n = -1$: $-1 = B \Rightarrow B = -1$

$A = 1, \quad B = -1$

(b) $\sum_{r=5}^{n} \frac{2r+1}{r^2(r+1)^2} = \sum_{r=5}^{n} \left(\frac{1}{r^2} - \frac{1}{(r+1)^2} \right)$

$r = 5$: $\frac{1}{5^2} - \frac{1}{6^2}$

$r = 6$: $\frac{1}{6^2} - \frac{1}{7^2}$

⋮ ⋮

$r = n - 1$: $\frac{1}{(n-1)^2} - \frac{1}{n^2}$

$r = n$: $\frac{1}{n^2} - \frac{1}{(n+1)^2}$

All terms cancel except the first and the last:

$\sum_{r=5}^{n} \frac{2r+1}{r^2(r+1)^2} = \frac{1}{25} - \frac{1}{(n+1)^2}$

$= \frac{(n+1)^2 - 25}{25(n+1)^2} = \frac{n^2 + 2n + 1 - 25}{25(n+1)^2} = \frac{n^2 + 2n - 24}{25(n+1)^2}$

$a = 2, \quad b = -24, \quad c = 25$

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Question 1 continued

Question 1 continued

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Question 1 continued

(Total for Question 1 is 5 marks)

2. (a) Use algebra to determine the set of values of x for which

$$x - 5 < \frac{9}{x + 3}$$

(6)

(b) Hence, or otherwise, determine the set of values of x for which

$$x - 5 < \frac{9}{|x + 3|}$$

(2)

(a) $x - 5 < \frac{9}{x + 3}$ Multiply by $(x + 3)^2 > 0$:

$(x - 5)(x + 3)^2 < 9(x + 3)$

$(x - 5)(x + 3)^2 - 9(x + 3) < 0$

$(x + 3)[(x - 5)(x + 3) - 9] < 0$

$(x + 3)(x^2 - 2x - 15 - 9) < 0$

$(x + 3)(x^2 - 2x - 24) < 0$

$(x + 3)(x - 6)(x + 4) < 0$

Critical values: $x = -4$, $x = -3$, $x = 6$

Cubic with positive x^3 coefficient is negative for

$x < -4$ or $-3 < x < 6$

(b) For $x > -3$, $|x + 3| = x + 3$ so solutions as in (a): $-3 < x < 6$

For $x < -3$, $x - 5 < 0$ and $\frac{9}{|x + 3|} > 0$ so it is always true

$x < 6$, $x \neq -3$

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Question 2 continued

(Total for Question 2 is 8 marks)

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3. The transformation T from the z-plane to the w-plane is given by

$$w = \frac{z}{z + 4i} \qquad z \neq -4i$$

The circle with equation $|z| = 3$ is mapped by T onto the circle C

Determine

(i) a Cartesian equation of C

(ii) the centre and radius of C

(8)

(i) $w(z + 4i) = z \Rightarrow wz + 4iw = z$

$z(1 - w) = 4iw \Rightarrow z = \frac{4iw}{1 - w}$

$|z| = 3 \Rightarrow \left|\frac{4iw}{1 - w}\right| = 3$

$|4iw| = 3|1 - w| \Rightarrow 4|w| = 3|1 - w|$

Let $w = u + iv$:

$16(u^2 + v^2) = 9((1 - u)^2 + v^2)$

$16u^2 + 16v^2 = 9(1 - 2u + u^2 + v^2)$

$16u^2 + 16v^2 = 9 - 18u + 9u^2 + 9v^2$

$7u^2 + 7v^2 + 18u - 9 = 0$

C: $7u^2 + 7v^2 + 18u - 9 = 0$

(ii) $u^2 + v^2 + \frac{18}{7}u - \frac{9}{7} = 0$

$\left(u + \frac{9}{7}\right)^2 - \frac{81}{49} + v^2 - \frac{63}{49} = 0$

$\left(u + \frac{9}{7}\right)^2 + v^2 = \frac{144}{49}$

Centre $\left(-\frac{9}{7}, 0\right)$ Radius $\frac{12}{7}$

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 8 marks)

4. (a) Determine the general solution of the differential equation

$$\frac{dy}{dx} - 3y\tan x = e^{4x}\sec^3 x$$

giving your answer in the form $y = f(x)$

(5)

(b) Determine the particular solution for which $y = 4$ at $x = 0$

(2)

(a) $IF = e^{\int -3\tan x\,dx} = e^{3\ln\cos x} = e^{\ln\cos^3 x} = \cos^3 x$

$\cos^3 x\frac{dy}{dx} - 3y\sin x\cos^2 x = e^{4x}\sec^3 x\cos^3 x = e^{4x}$

$\frac{d}{dx}(y\cos^3 x) = e^{4x}$

$y\cos^3 x = \int e^{4x}\,dx$

$y\cos^3 x = \frac{1}{4}e^{4x} + c$

$y = \left(\frac{1}{4}e^{4x} + c\right)\sec^3 x$

(b) $y = 4,\ x = 0:\quad 4 = \left(\frac{1}{4} + c\right)\times 1 \quad\Rightarrow\quad c = \frac{15}{4}$

$y = \frac{1}{4}(e^{4x} + 15)\sec^3 x$

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 7 marks)

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5. Given that

$$y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 - 2y = 0 \qquad y > 0$$

(a) determine $\frac{d^3y}{dx^3}$ in terms of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y

(4)

Given that $y = 2$ and $\frac{dy}{dx} = 1$ at $x = 0$

(b) determine a series solution for y in ascending powers of x, up to and including the term in x^3, giving each coefficient in its simplest form.

(4)

(a) $\frac{d^2y}{dx^2} = 2 - \frac{2}{y}\left(\frac{dy}{dx}\right)^2$ (dividing by y)

$\frac{d^3y}{dx^3} = \frac{2}{y^2}\frac{dy}{dx}\left(\frac{dy}{dx}\right)^2 - \frac{2}{y} \times 2\frac{dy}{dx}\frac{d^2y}{dx^2}$

$\frac{d^3y}{dx^3} = \frac{2}{y^2}\left(\frac{dy}{dx}\right)^3 - \frac{4}{y}\frac{dy}{dx}\frac{d^2y}{dx^2}$

(b) $x = 0,\ y = 2, \frac{dy}{dx} = 1$: $\frac{d^2y}{dx^2} = 2 - \frac{2}{2} \times 1^2 = 1$

$\frac{d^3y}{dx^3} = \frac{2}{4} \times 1^3 - \frac{4}{2} \times 1 \times 1 = \frac{1}{2} - 2 = -\frac{3}{2}$

$y = 2 + 1 \times x + \frac{1}{2!}x^2 + \frac{-3/2}{3!}x^3 + \ldots$

$y = 2 + x + \frac{1}{2}x^2 - \frac{1}{4}x^3 + \ldots$

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 8 marks)

6.

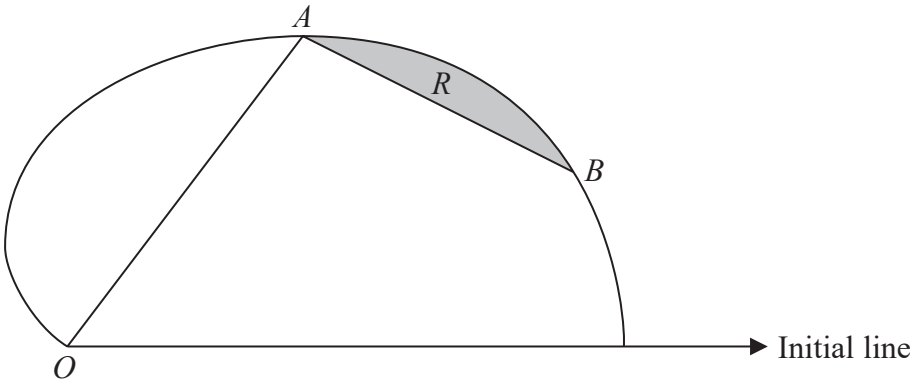


Figure 1

The curve shown in Figure 1 has polar equation

$$r = 4a(1 + \cos\theta) \qquad 0 \leqslant \theta < \pi$$

where a is a positive constant.

The tangent to the curve at the point A is parallel to the initial line.

(a) Show that the polar coordinates of A are $\left(6a, \frac{\pi}{3}\right)$

(6)

The point B lies on the curve such that angle $AOB = \frac{\pi}{6}$

The finite region R, shown shaded in Figure 1, is bounded by the line AB and the curve.

(b) Use calculus to determine the area of the shaded region R, giving your answer in the form $a^2\left(n\pi + p\sqrt{3} + q\right)$, where n, p and q are integers.

(7)

(a) $r \sin\theta = 4a \sin\theta + 4a \sin\theta \cos\theta = 4a \sin\theta + 2a \sin 2\theta$

$\frac{d(r\sin\theta)}{d\theta} = 4a\cos\theta + 4a\cos 2\theta$

Tangent parallel to initial line: $4a\cos\theta + 4a\cos 2\theta = 0$

$\cos\theta + 2\cos^2\theta - 1 = 0 \Rightarrow 2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{2}$ ($\cos\theta = -1$ gives $\theta = \pi$, not in range) $\Rightarrow \theta = \frac{\pi}{3}$

$r = 4a\left(1 + \cos\frac{\pi}{3}\right) = 4a \times \frac{3}{2} = 6a$ so A is $\left(6a, \frac{\pi}{3}\right)$

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Question 6 continued

(b) Area under curve $= \frac{1}{2}\int_{\pi/6}^{\pi/3} 16a^2(1 + \cos\theta)^2\, d\theta$

$= 8a^2\int_{\pi/6}^{\pi/3} (1 + 2\cos\theta + \cos^2\theta)\, d\theta$

$= 8a^2\int_{\pi/6}^{\pi/3} \left(1 + 2\cos\theta + \frac{1}{2}(1 + \cos 2\theta)\right) d\theta$

$= 8a^2\left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_{\pi/6}^{\pi/3}$

$= 8a^2\left[\left(\frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8}\right) - \left(\frac{\pi}{4} + 1 + \frac{\sqrt{3}}{8}\right)\right]$

$= 8a^2\left(\frac{\pi}{4} + \sqrt{3} - 1\right) = 2\pi a^2 + 8\sqrt{3}a^2 - 8a^2$

$OA = 6a, \quad OB = 4a\left(1 + \cos\frac{\pi}{6}\right) = 4a + 2\sqrt{3}a$

Triangle $OAB = \frac{1}{2} \times 6a \times (4a + 2\sqrt{3}a) \times \sin\frac{\pi}{6} = 6a^2 + 3\sqrt{3}a^2$

Area $R = 2\pi a^2 + 8\sqrt{3}a^2 - 8a^2 - (6a^2 + 3\sqrt{3}a^2)$

Area $R = a^2(2\pi + 5\sqrt{3} - 14)$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 13 marks)

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7. (a) Show that the transformation $y = xv$ transforms the equation

$$3\frac{d^2y}{dx^2} - \frac{6}{x}\frac{dy}{dx} + \frac{6y}{x^2} + 3y = x^2 \qquad x \neq 0 \qquad \text{(I)}$$

into the equation

$$3\frac{d^2v}{dx^2} + 3v = x \qquad \text{(II)}$$

(6)

(b) Hence obtain the general solution of the differential equation (I), giving your answer in the form $y = f(x)$

(6)

(a) $y = xv \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\frac{d^2y}{dx^2} = \frac{dv}{dx} + \frac{dv}{dx} + x\frac{d^2v}{dx^2} = 2\frac{dv}{dx} + x\frac{d^2v}{dx^2}$

Substitute into (I):

$3\left(2\frac{dv}{dx} + x\frac{d^2v}{dx^2}\right) - \frac{6}{x}\left(v + x\frac{dv}{dx}\right) + \frac{6xv}{x^2} + 3xv = x^2$

$6\frac{dv}{dx} + 3x\frac{d^2v}{dx^2} - \frac{6v}{x} - 6\frac{dv}{dx} + \frac{6v}{x} + 3xv = x^2$

$3x\frac{d^2v}{dx^2} + 3xv = x^2$

$\div x \ (x \neq 0): \quad 3\frac{d^2v}{dx^2} + 3v = x \quad (II)$ as required

(b) $3\lambda^2 + 3 = 0 \Rightarrow \lambda^2 = -1 \Rightarrow \lambda = \pm i$

CF: $v = A\cos x + B\sin x$

PI: try $v = kx + l$, $\frac{dv}{dx} = k$, $\frac{d^2v}{dx^2} = 0$

$3 \times 0 + 3(kx + l) = x$

$3k = 1, \ 3l = 0 \Rightarrow k = \frac{1}{3}, \ l = 0$

$v = A\cos x + B\sin x + \frac{1}{3}x$

$y = xv = x\left(A\cos x + B\sin x + \frac{1}{3}x\right)$

$y = Ax\cos x + Bx\sin x + \frac{1}{3}x^2$

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 12 marks)

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8. (a) Use de Moivre's theorem to show that

$$\sin 5\theta \equiv 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

(5)

(b) Hence determine the five distinct solutions of the equation

$$16x^5 - 20x^3 + 5x + \frac{1}{5} = 0$$

giving your answers to 3 decimal places.

(5)

(c) Use the identity given in part (a) to show that

$$\int_0^{\frac{\pi}{4}} \left(4\sin^5\theta - 5\sin^3\theta - 6\sin\theta\right) \mathrm{d}\theta = a\sqrt{2} + b$$

where a and b are rational numbers to be determined.

(4)

(a) $\cos 5\theta + i \sin 5\theta = (\cos\theta + i \sin\theta)^5$ (de Moivre)

Let $c = \cos\theta$, $s = \sin\theta$:

$(c + is)^5 = c^5 + 5c^4(is) + 10c^3(is)^2 + 10c^2(is)^3 + 5c(is)^4 + (is)^5$

$= c^5 + 5ic^4s - 10c^3s^2 - 10ic^2s^3 + 5cs^4 + is^5$

Equating imaginary parts:

$\sin 5\theta = 5c^4s - 10c^2s^3 + s^5$

$= 5(1 - s^2)^2 s - 10(1 - s^2)s^3 + s^5$

$= 5(1 - 2s^2 + s^4)s - 10s^3 + 10s^5 + s^5$

$= 5s - 10s^3 + 5s^5 - 10s^3 + 11s^5$

$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$ as required

(b) $16x^5 - 20x^3 + 5x + \frac{1}{5} = 0 \Rightarrow 16x^5 - 20x^3 + 5x = -\frac{1}{5}$

Let $x = \sin\theta$: $\sin 5\theta = -\frac{1}{5}$

$5\theta = -11.537...°, -168.463...°, 191.537...°, 348.463...°, -371.537...°$

$\theta = -2.3074...°, -33.6926...°, 38.3074...°, 69.6926...°, -74.3074...°$

$x = \sin\theta = -0.040260..., -0.554737..., 0.619880...,$

$0.937844..., -0.962727...$

$x = -0.963, -0.555, -0.040, 0.620, 0.938$

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Question 8 continued

(c) From (a): $4\sin^5\theta - 5\sin^3\theta = \frac{1}{4}(\sin 5\theta - 5\sin\theta)$

$$\int_0^{\pi/4} (4\sin^5\theta - 5\sin^3\theta - 6\sin\theta)\, d\theta$$

$$= \int_0^{\pi/4} \left(\frac{1}{4}\sin 5\theta - \frac{5}{4}\sin\theta - 6\sin\theta\right) d\theta$$

$$= \int_0^{\pi/4} \left(\frac{1}{4}\sin 5\theta - \frac{29}{4}\sin\theta\right) d\theta = \left[-\frac{1}{20}\cos 5\theta + \frac{29}{4}\cos\theta\right]_0^{\pi/4}$$

$$= \left(-\frac{1}{20}\cos\frac{5\pi}{4} + \frac{29}{4}\cos\frac{\pi}{4}\right) - \left(-\frac{1}{20} + \frac{29}{4}\right)$$

$$= \frac{1}{20}\times\frac{\sqrt{2}}{2} + \frac{29}{4}\times\frac{\sqrt{2}}{2} + \frac{1}{20} - \frac{29}{4} = \frac{\sqrt{2}}{40} + \frac{145\sqrt{2}}{40} - \frac{144}{20}$$

$$= \frac{73}{20}\sqrt{2} - \frac{36}{5} \qquad a = \frac{73}{20},\quad b = -\frac{36}{5}$$

Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

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