Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶

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1. (a) Express $\dfrac{2}{r(r^2-1)}$ in partial fractions.

(3)

(b) Hence find, in terms of n,

$$\sum_{r=2}^{n} \frac{1}{r(r^2-1)}$$

Give your answer in the form

$$\frac{n^2 + An + B}{Cn(n+1)}$$

where A, B and C are constants to be found.

(5)

(a)

$\dfrac{2}{r(r-1)(r+1)} \equiv \dfrac{A}{r-1} + \dfrac{B}{r} + \dfrac{C}{r+1}$

$2 \equiv Ar(r+1) + B(r-1)(r+1) + Cr(r-1)$

$r = 1$: $2 = 2A \Rightarrow A = 1$

$r = 0$: $2 = -B \Rightarrow B = -2$

$r = -1$: $2 = 2C \Rightarrow C = 1$

$\therefore \dfrac{2}{r(r^2-1)} = \dfrac{1}{r-1} - \dfrac{2}{r} + \dfrac{1}{r+1}$

(b)

$\displaystyle\sum_{r=2}^{n} \frac{2}{r(r^2-1)} = \sum_{r=2}^{n} \left(\frac{1}{r-1} - \frac{2}{r} + \frac{1}{r+1} \right)$

$r = 2$: $\dfrac{1}{1} - \dfrac{2}{2} + \dfrac{1}{3}$

$r = 3$: $\dfrac{1}{2} - \dfrac{2}{3} + \dfrac{1}{4}$

$r = 4$: $\dfrac{1}{3} - \dfrac{2}{4} + \dfrac{1}{5}$

$\vdots$

$r = n-1$: $\dfrac{1}{n-2} - \dfrac{2}{n-1} + \dfrac{1}{n}$

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Question 1 continued

$r = n$: $\quad \frac{1}{n-1} - \frac{2}{n} + \frac{1}{n+1}$

Adding, most terms cancel:

$$\sum_{r=2}^{n} \frac{2}{r(r^2-1)} = 1 - \frac{2}{2} + \frac{1}{2} + \frac{1}{n} - \frac{2}{n} + \frac{1}{n+1} = \frac{1}{2} - \frac{1}{n} + \frac{1}{n+1}$$

$$\sum_{r=2}^{n} \frac{1}{r(r^2-1)} = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{n} + \frac{1}{n+1}\right)$$

$$= \frac{n(n+1) - 2(n+1) + 2n}{4n(n+1)}$$

$$= \frac{n^2 + n - 2}{4n(n+1)}$$

$A = 1, \quad B = -2, \quad C = 4$

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Question 1 continued

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Question 1 continued

Q1

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2. The transformation T from the z-plane, where $z = x + iy$, to the w-plane, where $w = u + iv$, is given by

$$w = \frac{z + 2}{z - i} \quad z \neq i$$

The transformation T maps the circle $|z| = 2$ in the z-plane onto a circle C in the w-plane.

Find (i) the centre of C,

(ii) the radius of C.

(8)

$w(z - i) = z + 2 \Rightarrow wz - z = 2 + iw \Rightarrow z(w - 1) = 2 + iw$

$z = \frac{2 + iw}{w - 1}$

$|z| = 2 \Rightarrow \frac{|2 + iw|}{|w - 1|} = 2 \Rightarrow |2 + iw| = 2|w - 1|$

$w = u + iv$: $|2 + iu - v| = 2|u - 1 + iv|$

$(2 - v)^2 + u^2 = 4[(u - 1)^2 + v^2]$

$4 - 4v + v^2 + u^2 = 4u^2 - 8u + 4 + 4v^2$

$3u^2 + 3v^2 - 8u + 4v = 0$

$u^2 + v^2 - \frac{8}{3}u + \frac{4}{3}v = 0$

$\left(u - \frac{4}{3}\right)^2 + \left(v + \frac{2}{3}\right)^2 = \frac{16}{9} + \frac{4}{9} = \frac{20}{9}$

(i) Centre of C is $\left(\frac{4}{3}, -\frac{2}{3}\right)$ i.e. $w = \frac{4}{3} - \frac{2}{3}i$

(ii) Radius of $C = \sqrt{\left(\frac{20}{9}\right)} = \frac{2\sqrt{5}}{3}$

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Question 2 continued

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Question 2 continued

Q2

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3. The curve C, with pole O, has polar equation

$$r = 1 + \cos\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point A on C, the tangent to C is parallel to the initial line.

(a) Find the polar coordinates of A.

(4)

(b) Find the finite area enclosed by the initial line, the line OA and the curve C, giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are rational constants to be found.

(6)

(a)

$y = r\sin\theta = (1 + \cos\theta)\sin\theta = \sin\theta + \frac{1}{2}\sin 2\theta$

$\frac{dy}{d\theta} = \cos\theta + \cos 2\theta$

Tangent parallel to initial line: $\frac{dy}{d\theta} = 0$

$\cos\theta + 2\cos^2\theta - 1 = 0 \Rightarrow (2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{2}$ ($\cos\theta = -1$ gives $\theta = \pi$, outside the range)

$\theta = \frac{\pi}{3}, \quad r = 1 + \frac{1}{2} = \frac{3}{2}$

A is $\left(\frac{3}{2}, \frac{\pi}{3}\right)$

(b)

$\text{Area} = \frac{1}{2}\int_0^{\pi/3} r^2\, d\theta = \frac{1}{2}\int_0^{\pi/3} (1 + \cos\theta)^2\, d\theta$

$= \frac{1}{2}\int_0^{\pi/3} (1 + 2\cos\theta + \cos^2\theta)\, d\theta$

$= \frac{1}{2}\int_0^{\pi/3} \left(1 + 2\cos\theta + \frac{1}{2}(\cos 2\theta + 1)\right) d\theta$

$= \frac{1}{2}\int_0^{\pi/3} \left(\frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta\right) d\theta$

$= \frac{1}{2}\left(\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right)\Big]_0^{\pi/3}$

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Question 3 continued

$$= \frac{1}{2}\left(\frac{\pi}{2} + 2 \times \frac{\sqrt{3}}{2} + \frac{1}{4} \times \frac{\sqrt{3}}{2}\right) - 0$$

$$= \frac{1}{2}\left(\frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8}\right)$$

$$= \frac{\pi}{4} + \frac{9\sqrt{3}}{16} \qquad a = \frac{1}{4},\ b = \frac{9}{16}$$

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Question 3 continued

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Question 3 continued

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Q3

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4. Given that

$$y\frac{d^2y}{dx^2} - 4\left(\frac{dy}{dx}\right)^2 + 3y = 0$$

(a) show that

$$\frac{d^3y}{dx^3} = \frac{28}{y^2}\left(\frac{dy}{dx}\right)^3 - \frac{24}{y}\left(\frac{dy}{dx}\right)$$

(5)

Given also that $y = 8$ and $\frac{dy}{dx} = 1$ at $x = 0$

(b) find a series solution for y in ascending powers of x, up to and including the term in x^3, simplifying the coefficients where possible.

(4)

(a)

$\frac{d^2y}{dx^2} = \frac{4}{y}\left(\frac{dy}{dx}\right)^2 - 3$

$\frac{d^3y}{dx^3} = -\frac{4}{y^2}\left(\frac{dy}{dx}\right)^3 + \frac{8}{y}\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}$

$= -\frac{4}{y^2}\left(\frac{dy}{dx}\right)^3 + \frac{8}{y}\left(\frac{dy}{dx}\right)\left(\frac{4}{y}\left(\frac{dy}{dx}\right)^2 - 3\right)$

$= -\frac{4}{y^2}\left(\frac{dy}{dx}\right)^3 + \frac{32}{y^2}\left(\frac{dy}{dx}\right)^3 - \frac{24}{y}\left(\frac{dy}{dx}\right)$

$\frac{d^3y}{dx^3} = \frac{28}{y^2}\left(\frac{dy}{dx}\right)^3 - \frac{24}{y}\left(\frac{dy}{dx}\right)$ as required

(b)

At $x = 0$: $y = 8$, $\frac{dy}{dx} = 1$

$\frac{d^2y}{dx^2} = \frac{4}{8}(1)^2 - 3 = -\frac{5}{2}$

$\frac{d^3y}{dx^3} = \frac{28}{64}(1)^3 - \frac{24}{8}(1) = \frac{7}{16} - 3 = -\frac{41}{16}$

$y = y_0 + x\left(\frac{dy}{dx}\right)_0 + \frac{x^2}{2!}\left(\frac{d^2y}{dx^2}\right)_0 + \frac{x^3}{3!}\left(\frac{d^3y}{dx^3}\right)_0 + \ldots$

$y = 8 + x - \frac{5}{2} \times \frac{x^2}{2} - \frac{41}{16} \times \frac{x^3}{6} + \ldots$

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Question 4 continued

$y = 8 + x - \frac{5}{4}x^2 - \frac{41}{96}x^3 + \ldots$

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Question 4 continued

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Question 4 continued

Q4

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5. Use algebra to find the set of values of x for which

$$|2x^2 + x - 3| > 3(1 - x)$$

(7)

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

Critical values:

When $2x^2 + x - 3 \geqslant 0$:

$2x^2 + x - 3 = 3(1 - x) \Rightarrow 2x^2 + 4x - 6 = 0$

$x^2 + 2x - 3 = 0 \Rightarrow (x + 3)(x - 1) = 0 \Rightarrow x = -3,\ x = 1$

When $2x^2 + x - 3 < 0$:

$-(2x^2 + x - 3) = 3(1 - x) \Rightarrow -2x^2 - x + 3 = 3 - 3x$

$2x^2 - 2x = 0 \Rightarrow 2x(x - 1) = 0 \Rightarrow x = 0,\ x = 1$

Critical values: $x = -3,\ 0,\ 1$

Check regions:

$x = -4$: $|25| = 25 > 15$ true

$x = -1$: $|-2| = 2 > 6$ false

$x = 0.5$: $|-2| = 2 > 1.5$ true

$x = 1$: $0 > 0$ false; $x = 2$: $|7| = 7 > -3$ true

$x < -3$ or $0 < x < 1$ or $x > 1$

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Question 5 continued

Q5

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6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 2x^2 + x$$

(8)

(b) Find the particular solution of this differential equation for which $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$

(5)

(a)

$m^2 - 6m + 8 = 0 \Rightarrow (m-2)(m-4) = 0 \Rightarrow m = 2,\ 4$

CF: $y = Ae^{2x} + Be^{4x}$

PI: $y = \lambda x^2 + \mu x + \nu$, $\frac{dy}{dx} = 2\lambda x + \mu$, $\frac{d^2y}{dx^2} = 2\lambda$

$2\lambda - 6(2\lambda x + \mu) + 8(\lambda x^2 + \mu x + \nu) = 2x^2 + x$

x^2: $8\lambda = 2 \Rightarrow \lambda = \frac{1}{4}$

x: $-12\lambda + 8\mu = 1 \Rightarrow -3 + 8\mu = 1 \Rightarrow \mu = \frac{1}{2}$

const: $2\lambda - 6\mu + 8\nu = 0 \Rightarrow \frac{1}{2} - 3 + 8\nu = 0 \Rightarrow \nu = \frac{5}{16}$

$y = Ae^{2x} + Be^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16}$

(b)

$x = 0,\ y = 1$: $1 = A + B + \frac{5}{16} \Rightarrow A + B = \frac{11}{16}$

$\frac{dy}{dx} = 2Ae^{2x} + 4Be^{4x} + \frac{1}{2}x + \frac{1}{2}$

$x = 0,\ \frac{dy}{dx} = 0$: $0 = 2A + 4B + \frac{1}{2} \Rightarrow 2A + 4B = -\frac{1}{2}$

$2A + 4B - 2(A + B) = -\frac{1}{2} - \frac{11}{8} \Rightarrow 2B = -\frac{15}{8}$

$B = -\frac{15}{16}$, $A = \frac{11}{16} + \frac{15}{16} = \frac{13}{8}$

$y = \frac{13}{8}e^{2x} - \frac{15}{16}e^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16}$

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Question 6 continued

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7. (a) Use de Moivre's theorem to show that

$$\tan 4\theta \equiv \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$$

(6)

(b) Use the identity given in part (a) to find the 2 positive roots of

$$x^4 + 2x^3 - 6x^2 - 2x + 1 = 0$$

giving your answers to 3 significant figures.

(3)

(a)

By de Moivre: $(\cos\theta + i\sin\theta)^4 = \cos 4\theta + i\sin 4\theta$

Let $c = \cos\theta$, $s = \sin\theta$:

$(c + is)^4 = c^4 + 4c^3(is) + 6c^2(is)^2 + 4c(is)^3 + (is)^4$

$= c^4 + 4ic^3s - 6c^2s^2 - 4ics^3 + s^4$

Real parts: $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

Imaginary parts: $\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$

$\tan 4\theta = \frac{\sin 4\theta}{\cos 4\theta} = \frac{4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta}{\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta}$

Divide top and bottom by $\cos^4\theta$: $\tan 4\theta \equiv \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$ as required

(b)

$x^4 + 2x^3 - 6x^2 - 2x + 1 = 0 \Rightarrow x^4 - 6x^2 + 1 = 2x - 2x^3$

$\frac{2x - 2x^3}{1 - 6x^2 + x^4} = 1$

Let $x = \tan\theta$: $\frac{1}{2} \times \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta} = 1 \Rightarrow \frac{1}{2}\tan 4\theta = 1$

$\tan 4\theta = 2 \Rightarrow 4\theta = 1.10714..., 1.10714... + \pi, ...$

$\theta = 0.276787..., 1.062185...$ (for $\tan\theta > 0$)

$x = \tan(0.276787...) = 0.284079... = 0.284$

$x = \tan(1.062185...) = 1.793604... = 1.79$

Positive roots: $x = 0.284$, $x = 1.79$

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Question 7 continued

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8. (a) Show that the substitution $v = y^{-2}$ transforms the differential equation

$$\frac{dy}{dx} + 6xy = 3xe^{x^2}y^3 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{dv}{dx} - 12vx = -6xe^{x^2} \qquad x > 0 \qquad \text{(II)}$$

(5)

(b) Hence find the general solution of the differential equation (I), giving your answer in the form $y^2 = f(x)$.

(6)

(a)

$v = y^{-2} \Rightarrow \frac{dv}{dy} = -2y^{-3}$

$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx} = -\frac{y^3}{2}\frac{dv}{dx}$

Sub into (I): $-\frac{y^3}{2}\frac{dv}{dx} + 6xy = 3xe^{x^2}y^3$

Divide by $-\frac{y^3}{2}$: $\frac{dv}{dx} - 12xy^{-2} = -6xe^{x^2}$

$\frac{dv}{dx} - 12vx = -6xe^{x^2}$ (II) as required

(b)

Integrating factor $= e^{\int -12x\,dx} = e^{-6x^2}$

$\frac{d}{dx}(ve^{-6x^2}) = -6xe^{x^2} \times e^{-6x^2} = -6xe^{-5x^2}$

$ve^{-6x^2} = \int -6xe^{-5x^2}\,dx = \frac{3}{5}e^{-5x^2} + c$

$v = \frac{3}{5}e^{x^2} + ce^{6x^2}$

$\frac{1}{y^2} = \frac{3}{5}e^{x^2} + ce^{6x^2}$

$y^2 = \frac{1}{(3/5)e^{x^2} + ce^{6x^2}}$ i.e. $y^2 = \frac{5}{3e^{x^2} + ke^{6x^2}}$

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Question 8 continued

Q8

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TOTAL FOR PAPER: 75 MARKS

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