Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Friday 24 January 2025

Afternoon (Time: 1 hour 30 minutes) | Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Pearson

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Solve the equation

$$2\sinh^2 x + 3\cosh x = 7$$

Give your answers as simplified natural logarithms.

(6)

$2\sinh^2 x + 3\cosh x = 7$

$2(\cosh^2 x - 1) + 3\cosh x = 7$

$2\cosh^2 x + 3\cosh x - 9 = 0$

$(2\cosh x - 3)(\cosh x + 3) = 0$

$\cosh x = 3/2$ ($\cosh x = -3$ is rejected since $\cosh x \geq 1$)

$x = \pm\operatorname{arcosh}(3/2) = \pm\ln(3/2 + \sqrt{(9/4 - 1)})$

$x = \pm\ln(3/2 + \sqrt{5}/2) = \pm\ln((3 + \sqrt{5})/2)$

$-\ln((3 + \sqrt{5})/2) = \ln(2/(3 + \sqrt{5})) = \ln(2(3 - \sqrt{5})/(9 - 5)) = \ln((3 - \sqrt{5})/2)$

$\therefore\ x = \ln((3 + \sqrt{5})/2)$ or $x = \ln((3 - \sqrt{5})/2)$

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Question 1 continued

(Total for Question 1 is 6 marks)

2. The hyperbola H has equation

$$kx^2 - y^2 = 9$$

where k is a positive constant.

Given that the point (6, 0) is a focus of H

(a) determine the value of k.

(4)

(b) Hence determine an equation of the corresponding directrix of H

(2)

(a) $x^2/(9/k) - y^2/9 = 1 \Rightarrow a^2 = 9/k,\ b^2 = 9$

Focus $(6, 0) \Rightarrow ae = 6$

$b^2 = a^2(e^2 - 1) = a^2e^2 - a^2$

$9 = 36 - a^2 \Rightarrow a^2 = 27$

$9/k = 27 \Rightarrow k = 1/3$

(b) $a = \sqrt{27} = 3\sqrt{3},\ e = 6/a = 6/(3\sqrt{3}) = 2/\sqrt{3}$

directrix $x = a/e = 3\sqrt{3} \div (2/\sqrt{3}) = 9/2$

Equation of directrix: $x = 9/2$

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Question 2 continued

(Total for Question 2 is 6 marks)

3. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Determine

$$\int \frac{1}{4x^2 + 12x + 25}\,\mathrm{d}x$$

(4)

(ii) Show that

$$\int_3^9 \frac{1}{\sqrt{x^2 + 4x - 17}}\,\mathrm{d}x = \ln a$$

where a is an integer to be determined.

(6)

(i) $4x^2 + 12x + 25 = (2x + 3)^2 + 16$

$\int 1/(4x^2 + 12x + 25)\,dx = \int 1/((2x + 3)^2 + 4^2)\,dx$

let $u = 2x + 3$, $du = 2\,dx$: $= \frac{1}{2} \int 1/(u^2 + 4^2)\,du = \frac{1}{2} \times \frac{1}{4} \arctan(u/4) + c$

$= (1/8) \arctan((2x + 3)/4) + c$

(ii) $x^2 + 4x - 17 = (x + 2)^2 - 4 - 17 = (x + 2)^2 - 21$

$\int 1/\sqrt{(x^2 + 4x - 17)}\,dx = \int 1/\sqrt{((x + 2)^2 - 21)}\,dx = \operatorname{arcosh}((x + 2)/\sqrt{21})$

$\int_3^9 1/\sqrt{(x^2 + 4x - 17)}\,dx = [\operatorname{arcosh}((x + 2)/\sqrt{21})]_3^9$

$= \operatorname{arcosh}(11/\sqrt{21}) - \operatorname{arcosh}(5/\sqrt{21})$

$= \ln(11/\sqrt{21} + \sqrt{(121/21 - 1)}) - \ln(5/\sqrt{21} + \sqrt{(25/21 - 1)})$

$= \ln((11 + \sqrt{100})/\sqrt{21}) - \ln((5 + \sqrt{4})/\sqrt{21})$

$= \ln(21/\sqrt{21}) - \ln(7/\sqrt{21}) = \ln(21/7)$

$= \ln 3$, so $a = 3$

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

$$\mathbf{M} = \begin{pmatrix} 4 & 0 & 2 \\ 0 & 4 & a \\ 2 & a & \frac{20}{3} \end{pmatrix}$$

where a is a constant.

Given that $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is an eigenvector of **M**,

(a) determine the corresponding eigenvalue for this eigenvector.

(2)

(b) Hence show that $a = 4$

(2)

Determine

(c) the other two eigenvalues of **M**,

(3)

(d) a normalised eigenvector for each eigenvalue of **M**.

(5)

(e) Hence determine a matrix **P** and a diagonal matrix **D** such that

$$\mathbf{P}^{\mathrm{T}}\mathbf{MP} = \mathbf{D}$$

(2)

(a) $\begin{pmatrix} 4 & 0 & 2 \\ 0 & 4 & a \\ 2 & a & 20/3 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4+0+6 \\ 0+8+3a \\ 2+2a+20 \end{pmatrix} = \begin{pmatrix} 10 \\ 8+3a \\ 22+2a \end{pmatrix} = \lambda\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

Top row: $10 = \lambda \times 1 \Rightarrow$ eigenvalue $\lambda = 10$

(b) Second row: $8 + 3a = 2\lambda = 20 \Rightarrow 3a = 12 \Rightarrow a = 4$

(check third row: $22 + 2(4) = 30 = 3\lambda$ ✓) Hence $a = 4$

(c) $\begin{vmatrix} 4-\lambda & 0 & 2 \\ 0 & 4-\lambda & 4 \\ 2 & 4 & 20/3-\lambda \end{vmatrix} = (4-\lambda)[(4-\lambda)(20/3-\lambda) - 16] + 2[0 - 2(4-\lambda)]$

$= (4-\lambda)[\lambda^2 - (32/3)\lambda + 80/3 - 16 - 4] = (4-\lambda)(\lambda^2 - (32/3)\lambda + 20/3)$

$= (1/3)(4-\lambda)(3\lambda^2 - 32\lambda + 20) = (1/3)(4-\lambda)(3\lambda - 2)(\lambda - 10) = 0$

$\Rightarrow \lambda = 4, 2/3, (10)$ Other eigenvalues: $\lambda = 4$ and $\lambda = 2/3$

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Question 4 continued

(d) $\lambda = 2/3$: $(10/3)x + 2z = 0$, $(10/3)y + 4z = 0$, $2x + 4y + 6z = 0$

$\Rightarrow z = -(5/3)x$, $y = -(6/5)z = 2x$. $x = 3$: $y = 6$, $z = -5$

(check: $2(3) + 4(6) + 6(-5) = 0$ ✓)

$$\text{eigenvector} \begin{pmatrix} 3 \\ 6 \\ -5 \end{pmatrix}, \quad |v| = \sqrt{(9 + 36 + 25)} = \sqrt{70}$$

$\lambda = 4$: $0x + 2z = 0$, $0y + 4z = 0$, $2x + 4y + (8/3)z = 0$

$\Rightarrow z = 0$, $x = -2y$. $y = -1$: $x = 2$

$$\text{eigenvector} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}, \quad |v| = \sqrt{(4 + 1 + 0)} = \sqrt{5}$$

$\lambda = 10$: eigenvector $(1, 2, 3)$, $|v| = \sqrt{(1 + 4 + 9)} = \sqrt{14}$

Normalised eigenvectors:

$$\lambda = 10 : \begin{pmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{pmatrix} \quad \lambda = 4 : \begin{pmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \\ 0 \end{pmatrix} \quad \lambda = 2/3 : \begin{pmatrix} 3/\sqrt{70} \\ 6/\sqrt{70} \\ -5/\sqrt{70} \end{pmatrix}$$

(e)

$$P = \begin{pmatrix} 1/\sqrt{14} & 2/\sqrt{5} & 3/\sqrt{70} \\ 2/\sqrt{14} & -1/\sqrt{5} & 6/\sqrt{70} \\ 3/\sqrt{14} & 0 & -5/\sqrt{70} \end{pmatrix}, \quad D = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2/3 \end{pmatrix}$$

Question 4 continued

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Question 4 continued

(Total for Question 4 is 14 marks)

5.

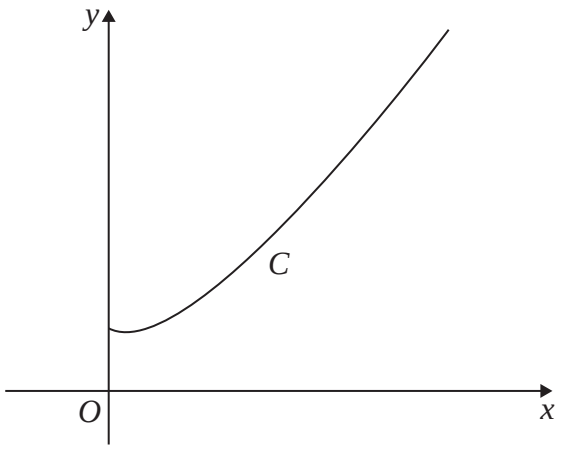


Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve C defined by the parametric equations

$$x = (2t + 3)^{\frac{3}{2}} \qquad y = \frac{3}{2}t^2 + 3t + 6 \qquad -\frac{3}{2} \leqslant t \leqslant 3$$

(a) Show that

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = a(t + 2)^2$$

where a is an integer to be determined.

(4)

Hence, using algebraic integration, determine

(b) the exact length of C,

(3)

(c) the exact area of the surface generated when C is rotated through 360° about the x-axis, giving your answer in the form $k\pi$ where k is a rational number.

(4)

(a) $dx/dt = (3/2)(2t + 3)^{1/2} \times 2 = 3(2t + 3)^{1/2}$, $dy/dt = 3t + 3$

$(dx/dt)^2 + (dy/dt)^2 = 9(2t + 3) + (3t + 3)^2$

$= 18t + 27 + 9t^2 + 18t + 9$

$= 9t^2 + 36t + 36 = 9(t^2 + 4t + 4)$

$= 9(t + 2)^2$, so $a = 9$

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Question 5 continued

(b) $s = \int_{-3/2}^{3} \sqrt{(9(t+2)^2)}\, dt = \int_{-3/2}^{3} 3(t+2)\, dt \qquad (t+2 > 0)$

$= 3[\, t^2/2 + 2t \,]_{-3/2}^{3}$

$= 3[\, (9/2 + 6) - (9/8 - 3) \,]$

$= 3(\, 21/2 + 15/8 \,) = 3 \times 99/8$

$= 297/8$

(c) $S = 2\pi \int y \sqrt{((dx/dt)^2 + (dy/dt)^2)}\, dt$

$= 2\pi \int_{-3/2}^{3} (\, (3/2)t^2 + 3t + 6 \,) \times 3(t+2)\, dt$

$= 6\pi \int_{-3/2}^{3} (\, (3/2)t^3 + 6t^2 + 12t + 12 \,)\, dt$

$= 6\pi \, [\, (3/8)t^4 + 2t^3 + 6t^2 + 12t \,]_{-3/2}^{3}$

$t = 3: \quad 243/8 + 54 + 54 + 36 = 1395/8$

$t = -3/2: \quad 243/128 - 27/4 + 27/2 - 18 = -1197/128$

$S = 6\pi(\, 1395/8 + 1197/128 \,) = 6\pi \times 23517/128$

$= 70551\pi/64$

Question 5 continued

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Question 5 continued

(Total for Question 5 is 11 marks)

6. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Determine a vector perpendicular to Π

(2)

The line l passes through the point $A(1, 7, 3)$ and meets Π at the point $B(-1, 5, 2)$

The acute angle between Π and l is α

(b) Determine the value of α to the nearest degree.

(4)

(c) Determine the exact shortest distance from A to Π

(4)

(a) $n = \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix} \times \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0\times3 - 7\times(-2) \\ 7\times3 - 2\times3 \\ 2\times(-2) - 0\times3 \end{pmatrix} = \begin{pmatrix} 14 \\ 15 \\ -4 \end{pmatrix}$

(b) direction of l : $\begin{pmatrix} 1 \\ 7 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

$\cos\beta = (14\times2 + 15\times2 - 4\times1)/(\sqrt{(14^2 + 15^2 + 4^2)} \times \sqrt{(2^2 + 2^2 + 1^2)})$

$= 54/(\sqrt{437} \times \sqrt{9}) = 18/\sqrt{437} \Rightarrow \beta = 30.56...°$

$\alpha = 90° - \beta = 59.43...°$

$\alpha = 59°$ (nearest degree)

(c) Plane: $r\cdot(14, 15, -4) = (-1, 5, 2)\cdot(14, 15, -4) = -14 + 75 - 8 = 53$

distance $= |14\times1 + 15\times7 - 4\times3 - 53| / \sqrt{437}$

$= |107 - 53| / \sqrt{437}$

$= 54/\sqrt{437}$

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 10 marks)

7. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a > b > 0$$

The point $P(a\cos\theta, b\sin\theta)$ lies on E where $0 < \theta < \frac{\pi}{2}$

(a) Use calculus to show that an equation of the normal to E at P is

$$by = ax\tan\theta + (b^2 - a^2)\sin\theta$$

(4)

The normal to E at P meets E again on the y-axis at the point B.

Given that O is the origin and that the area of triangle OBP is $\frac{3b^2}{4}$

(b) show that $\sin\theta = \frac{1}{2}$

(5)

(c) determine, in terms of a only, the exact coordinates of the point P.

(3)

(a) $x = a\cos\theta$, $y = b\sin\theta$: $dx/d\theta = -a\sin\theta$, $dy/d\theta = b\cos\theta$

$dy/dx = (b\cos\theta)/(-a\sin\theta)$ ⇒ gradient of normal $= (a\sin\theta)/(b\cos\theta)$

$y - b\sin\theta = ((a\sin\theta)/(b\cos\theta))(x - a\cos\theta)$

$by\cos\theta - b^2\sin\theta\cos\theta = ax\sin\theta - a^2\sin\theta\cos\theta$

$\div\cos\theta$: $by - b^2\sin\theta = ax\tan\theta - a^2\sin\theta$

$\therefore\ by = ax\tan\theta + (b^2 - a^2)\sin\theta$

(b) B is on the y-axis and on E, below O, so B is $(0, -b)$

$x = 0, y = -b$: $-b^2 = (b^2 - a^2)\sin\theta$ (1)

Area OBP = ½ × OB × (x-coordinate of P) = ½ × b × a cos θ = $3b^2/4$

⇒ $2a\cos\theta = 3b$ ⇒ $b = (2a/3)\cos\theta$ ⇒ $b^2 = (4a^2/9)\cos^2\theta$

Sub into (1): $-(4a^2/9)\cos^2\theta = ((4a^2/9)\cos^2\theta - a^2)\sin\theta$

$\times 9/a^2$: $-4\cos^2\theta = (4\cos^2\theta - 9)\sin\theta$

$4(1 - \sin^2\theta) = (9 - 4(1 - \sin^2\theta))\sin\theta = (5 + 4\sin^2\theta)\sin\theta$

$4\sin^3\theta + 4\sin^2\theta + 5\sin\theta - 4 = 0$

$(2\sin\theta - 1)(2\sin^2\theta + 3\sin\theta + 4) = 0$

$2\sin^2\theta + 3\sin\theta + 4 = 0$ has discriminant $9 - 32 < 0$, so no real roots

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Question 7 continued

(b) continued: $\therefore \sin\theta = 1/2$

(c) $\sin\theta = 1/2,\ 0 < \theta < \pi/2 \Rightarrow \theta = \pi/6,\ \cos\theta = \sqrt{3}/2$

$b = (2a/3)\cos\theta = (2a/3)(\sqrt{3}/2) = (\sqrt{3}/3)\,a$

$P = (a\cos\theta, b\sin\theta) = \left((\sqrt{3}/2)\,a,\ (\sqrt{3}/3)\,a \times \tfrac{1}{2}\right)$

P is $\left((\sqrt{3}/2)\,a,\ (\sqrt{3}/6)\,a\right)$

Question 7 continued

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Question 7 continued

(Total for Question 7 is 12 marks)

8. Given that

$$y = e^{3x}\cosh 2x$$

prove by induction that for $n \in \mathbb{N}$

$$\frac{d^n y}{dx^n} = e^{3x}\left(\frac{5^n + 1}{2}\cosh 2x + \frac{5^n - 1}{2}\sinh 2x\right)$$

(6)

$n = 1$: $dy/dx = 3e^{3x}\cosh 2x + 2e^{3x}\sinh 2x = e^{3x}(3\cosh 2x + 2\sinh 2x)$

RHS with $n = 1$: $e^{3x}(((5 + 1)/2)\cosh 2x + ((5 - 1)/2)\sinh 2x) = e^{3x}(3\cosh 2x + 2\sinh 2x)$

so the result is true for $n = 1$

Assume true for $n = k$: $d^k y/dx^k = e^{3x}(((5^k + 1)/2)\cosh 2x + ((5^k - 1)/2)\sinh 2x)$

Then $d^{k+1}y/dx^{k+1} = 3e^{3x}(((5^k + 1)/2)\cosh 2x + ((5^k - 1)/2)\sinh 2x)$

$+ e^{3x}((5^k + 1)\sinh 2x + (5^k - 1)\cosh 2x)$

$= e^{3x}[(3(5^k + 1)/2 + 5^k - 1)\cosh 2x + (3(5^k - 1)/2 + 5^k + 1)\sinh 2x]$

$= e^{3x}[((3\times5^k + 3 + 2\times5^k - 2)/2)\cosh 2x + ((3\times5^k - 3 + 2\times5^k + 2)/2)\sinh 2x]$

$= e^{3x}(((5\times5^k + 1)/2)\cosh 2x + ((5\times5^k - 1)/2)\sinh 2x)$

$= e^{3x}(((5^{k+1} + 1)/2)\cosh 2x + ((5^{k+1} - 1)/2)\sinh 2x)$

So if the result is true for $n = k$, then it is true for $n = k + 1$.

As the result is true for $n = 1$, by mathematical induction it is true for all $n \in \mathbb{N}$.

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Question 8 continued

Question 8 continued

(Total for Question 8 is 6 marks)

TOTAL FOR PAPER IS 75 MARKS

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