Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1 (a) Use the definitions of hyperbolic functions in terms of exponentials to prove that

$$8\cosh^4 x = \cosh 4x + p\cosh 2x + q$$

where p and q are constants to be determined.

(3)

(b) Hence, or otherwise, solve the equation

$$\cosh 4x - 17\cosh 2x + 9 = 0$$

giving your answers in exact simplified form in terms of natural logarithms.

(5)

(a) $8\cosh^4 x = 8\left(\frac{e^x + e^{-x}}{2}\right)^4 = \frac{8}{16}(e^{4x} + 4e^{2x} + 6 + 4e^{-2x} + e^{-4x})$

$= \frac{e^{4x} + e^{-4x}}{2} + 4\left(\frac{e^{2x} + e^{-2x}}{2}\right) + 3$

$= \cosh 4x + 4\cosh 2x + 3$

$\therefore p = 4,\ q = 3$

(b) $\cosh 4x = 8\cosh^4 x - 4\cosh 2x - 3$

$\Rightarrow 8\cosh^4 x - 4\cosh 2x - 3 - 17\cosh 2x + 9 = 0$

$\Rightarrow 8\cosh^4 x - 21\cosh 2x + 6 = 0$

using $\cosh 2x = 2\cosh^2 x - 1$:

$8\cosh^4 x - 21(2\cosh^2 x - 1) + 6 = 0$

$\Rightarrow 8\cosh^4 x - 42\cosh^2 x + 27 = 0$

$\Rightarrow (2\cosh^2 x - 9)(4\cosh^2 x - 3) = 0$

$\Rightarrow \cosh^2 x = \frac{9}{2}$ or $\cosh^2 x = \frac{3}{4}$ (reject since $\cosh^2 x \geq 1$)

$\cosh x = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$ $(\cosh x > 0)$

$\sinh^2 x = \cosh^2 x - 1 = \frac{7}{2} \Rightarrow \sqrt{\cosh^2 x - 1} = \frac{\sqrt{14}}{2}$

$x = \pm\operatorname{arcosh}\left(\frac{3\sqrt{2}}{2}\right) = \pm\ln\left(\frac{3\sqrt{2}}{2} + \frac{\sqrt{14}}{2}\right)$

$x = \pm\ln\left(\frac{3\sqrt{2} + \sqrt{14}}{2}\right)$

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Question 1 continued

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Question 1 continued

Q1

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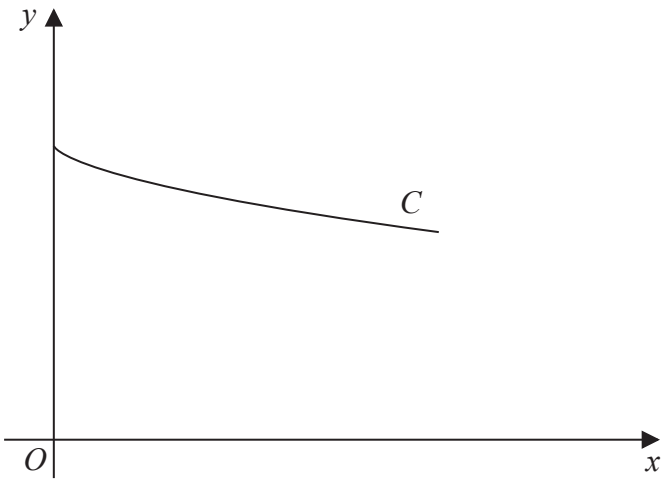


Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = \ln(\sec\theta + \tan\theta) - \sin\theta \qquad y = \cos\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{4}$$

The curve C is rotated through 2π radians about the x-axis and is used to form a solid of revolution S.

Using calculus, show that the **total** surface area of S is given by

$$\frac{\pi}{2}\left(p + q\sqrt{2}\right)$$

where p and q are integers to be determined.

(8)

$\frac{dx}{d\theta} = \frac{\sec\theta\tan\theta + \sec^2\theta}{\sec\theta + \tan\theta} - \cos\theta = \frac{\sec\theta(\tan\theta + \sec\theta)}{\sec\theta + \tan\theta} - \cos\theta$

$= \sec\theta - \cos\theta = \frac{1 - \cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}, \qquad \frac{dy}{d\theta} = -\sin\theta$

$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = \frac{\sin^4\theta}{\cos^2\theta} + \sin^2\theta = \frac{\sin^2\theta(\sin^2\theta + \cos^2\theta)}{\cos^2\theta} = \tan^2\theta$

$\Rightarrow \sqrt{(dx/d\theta)^2 + (dy/d\theta)^2} = \tan\theta \quad (\tan\theta \geqslant 0 \text{ for } 0 \leqslant \theta \leqslant \pi/4)$

Curved surface: $S_c = 2\pi\int_0^{\pi/4} y\sqrt{(dx/d\theta)^2 + (dy/d\theta)^2}\,d\theta$

$= 2\pi\int_0^{\pi/4} \cos\theta\tan\theta\,d\theta = 2\pi\int_0^{\pi/4} \sin\theta\,d\theta$

$= 2\pi[-\cos\theta]_0^{\pi/4} = 2\pi\left(-\frac{1}{\sqrt{2}} + 1\right) = 2\pi - \sqrt{2}\pi$

Ends: at $\theta = 0$, $x = 0$, $y = 1$ $\Rightarrow$ disc area $= \pi \times 1^2 = \pi$

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Question 2 continued

at $\theta = \frac{\pi}{4}$, $y = \frac{1}{\sqrt{2}}$ $\Rightarrow$ disc area $= \pi\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{\pi}{2}$

Total surface area $= 2\pi - \sqrt{2}\pi + \pi + \frac{\pi}{2} = \frac{7\pi}{2} - \sqrt{2}\pi$

$= \frac{\pi}{2}(7 - 2\sqrt{2})$ $\quad p = 7,\ q = -2$

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Question 2 continued

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Question 2 continued

Q2

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3. (a) Given that $y = \text{arsech}\left(\frac{x}{2}\right)$, where $0 < x \leqslant 2$, show that

$$\frac{dy}{dx} = \frac{p}{x\sqrt{q - x^2}}$$

where p and q are constants to be determined.

(4)

In part (b) solutions based entirely on calculator technology are not acceptable.

$$f(x) = \text{artanh}(x) + \text{arsech}\left(\frac{x}{2}\right) \qquad 0 < x \leqslant 1$$

(b) Determine, in simplest form, the exact value of x for which $f'(x) = 0$

(5)

(a) $y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow \text{sech } y = \frac{x}{2}$

$\frac{dx}{dy} = -2\text{sech } y \tanh y$

$\tanh y = \sqrt{1 - \text{sech}^2 y} = \frac{\sqrt{4 - x^2}}{2} \qquad (y \geqslant 0)$

$\frac{dx}{dy} = -2 \times \frac{x}{2} \times \frac{\sqrt{4 - x^2}}{2} = -\frac{x\sqrt{4 - x^2}}{2}$

$\frac{dy}{dx} = \frac{-2}{x\sqrt{4 - x^2}} \qquad p = -2, \; q = 4$

(b) $f(x) = \frac{1}{1 - x^2} - \frac{2}{x\sqrt{4 - x^2}} = 0$

$\Rightarrow x\sqrt{4 - x^2} = 2(1 - x^2)$

$\Rightarrow x^2(4 - x^2) = 4(1 - x^2)^2$

$\Rightarrow 4x^2 - x^4 = 4 - 8x^2 + 4x^4$

$\Rightarrow 5x^4 - 12x^2 + 4 = 0$

$\Rightarrow (5x^2 - 2)(x^2 - 2) = 0$

$\Rightarrow x^2 = \frac{2}{5}$ or $x^2 = 2$ (reject since $0 < x \leqslant 1$)

$x = \frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$

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Question 3 continued

Q3

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4. $$\mathbf{M} = \begin{pmatrix} 6 & k & 2 \\ k & 5 & 0 \\ 2 & 0 & 7 \end{pmatrix}$$

where k is a constant.

Given that 3 is an eigenvalue of **M**,

(a) determine the possible values of k.

(3)

Given that $k < 0$

(b) determine the other eigenvalues of **M**.

(3)

(c) Determine a normalised eigenvector corresponding to the eigenvalue 3

(3)

(a) $\det(M - 3I) = \begin{vmatrix} 3 & k & 2 \\ k & 2 & 0 \\ 2 & 0 & 4 \end{vmatrix} = 0$

$3(8 - 0) - k(4k - 0) + 2(0 - 4) = 0$

$\Rightarrow 24 - 4k^2 - 8 = 0 \Rightarrow 4k^2 = 16$

$\Rightarrow k = 2$ or $k = -2$

(b) $k = -2$: $\begin{vmatrix} 6 - \lambda & -2 & 2 \\ -2 & 5 - \lambda & 0 \\ 2 & 0 & 7 - \lambda \end{vmatrix} = 0$

$(6 - \lambda)(5 - \lambda)(7 - \lambda) + 2(-2(7 - \lambda)) + 2(0 - 2(5 - \lambda)) = 0$

$(6 - \lambda)(35 - 12\lambda + \lambda^2) - 28 + 4\lambda - 20 + 4\lambda = 0$

$\Rightarrow \lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0$

$\Rightarrow (\lambda - 3)(\lambda^2 - 15\lambda + 54) = 0 \Rightarrow (\lambda - 3)(\lambda - 6)(\lambda - 9) = 0$

Other eigenvalues: $\lambda = 6$ and $\lambda = 9$

(c) $\begin{pmatrix} 3 & -2 & 2 \\ -2 & 2 & 0 \\ 2 & 0 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$3x - 2y + 2z = 0, \quad -2x + 2y = 0, \quad 2x + 4z = 0$

$\Rightarrow y = x, \quad z = -\frac{1}{2}x$

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Question 4 continued

Eigenvector: $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ magnitude $\sqrt{2^2 + 2^2 + 1^2} = 3$

Normalised eigenvector: $\frac{1}{3}\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

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Question 4 continued

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Question 4 continued

Q4

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5. Determine

(i) $\int \frac{1}{\sqrt{x^2 - 3x + 5}} \, dx$

(3)

(ii) $\int \frac{1}{\sqrt{63 + 4x - 4x^2}} \, dx$

(4)

(i) $x^2 - 3x + 5 = \left(x - \frac{3}{2}\right)^2 + \frac{11}{4}$

$\int \frac{1}{\sqrt{(x - 3/2)^2 + 11/4}} \, dx = \operatorname{arsinh}\left(\frac{x - 3/2}{\sqrt{11}/2}\right) + c$

$= \operatorname{arsinh}\left(\frac{2x - 3}{\sqrt{11}}\right) + c$

(ii) $63 + 4x - 4x^2 = -4\left[x^2 - x - \frac{63}{4}\right] = -4\left[\left(x - \frac{1}{2}\right)^2 - 16\right]$

$= 64 - 4\left(x - \frac{1}{2}\right)^2 = 64 - (2x - 1)^2$

$\int \frac{1}{\sqrt{64 - (2x - 1)^2}} \, dx = \frac{1}{2} \arcsin\left(\frac{2x - 1}{8}\right) + c$

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Question 5 continued

Q5

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6. $$I_n = \int e^x \sin^n x \, dx \qquad n \in \mathbb{Z} \qquad n \geqslant 0$$

(a) Show that

$$I_n = \frac{e^x \sin^{n-1} x}{n^2+1}(\sin x - n\cos x) + \frac{n(n-1)}{n^2+1} I_{n-2} \qquad n \geqslant 2$$

(6)

(b) Hence find the exact value of

$$\int_0^{\frac{\pi}{2}} e^x \sin^4 x \, dx$$

giving your answer in the form $Ae^{\frac{\pi}{2}} + B$ where A and B are rational numbers to be determined.

(4)

(a) $I_n = \int \sin^n x\, e^x\, dx = e^x \sin^n x - \int n \sin^{n-1} x \cos x\, e^x\, dx$

$\int e^x \sin^{n-1} x \cos x\, dx = e^x \sin^{n-1} x \cos x$

$- \int e^x [(n-1)\sin^{n-2} x \cos^2 x - \sin^n x]\, dx$

$I_n = e^x \sin^n x - n e^x \sin^{n-1} x \cos x + n \int e^x [(n-1)\sin^{n-2} x \cos^2 x - \sin^n x]\, dx$

using $\cos^2 x = 1 - \sin^2 x$:

$I_n = e^x \sin^n x - n e^x \sin^{n-1} x \cos x + n \int e^x [(n-1)\sin^{n-2} x - n \sin^n x]\, dx$

$I_n = e^x \sin^n x - n e^x \sin^{n-1} x \cos x + n(n-1) I_{n-2} - n^2 I_n$

$(n^2+1) I_n = e^x \sin^{n-1} x (\sin x - n \cos x) + n(n-1) I_{n-2}$

$I_n = \frac{e^x \sin^{n-1} x}{n^2+1} (\sin x - n \cos x) + \frac{n(n-1)}{n^2+1} I_{n-2}$

(b) Let $J_n = \int_0^{\pi/2} e^x \sin^n x\, dx$. At $x = \frac{\pi}{2}$: $\sin x = 1$, $\cos x = 0$

at $x = 0$: $\sin x = 0$, so for $n \geq 2$ the lower limit gives 0

$J_n = \frac{e^{\pi/2}(1-0)}{n^2+1} - 0 + \frac{n(n-1)}{n^2+1} J_{n-2}$

$J_4 = \frac{e^{\pi/2}}{17} + \frac{12}{17} J_2, \qquad J_2 = \frac{e^{\pi/2}}{5} + \frac{2}{5} J_0$

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Question 6 continued

$$J_0 = \int_0^{\pi/2} e^x \, dx = [e^x]_0^{\pi/2} = e^{\pi/2} - 1$$

$$J_2 = \frac{e^{\pi/2}}{5} + \frac{2}{5}(e^{\pi/2} - 1) = \frac{3}{5}e^{\pi/2} - \frac{2}{5}$$

$$J_4 = \frac{e^{\pi/2}}{17} + \frac{12}{17}\left(\frac{3}{5}e^{\pi/2} - \frac{2}{5}\right) = \frac{41}{85}e^{\pi/2} - \frac{24}{85}$$

$$A = \frac{41}{85}, \quad B = -\frac{24}{85}$$

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Question 6 continued

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Question 6 continued

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7. The line l_1 has equation

$$\frac{x-3}{4} = \frac{y-5}{-2} = \frac{z-4}{7}$$

The plane Π has equation

$$2x + 4y - z = 1$$

The line l_1 intersects the plane Π at the point P

(a) Determine the coordinates of P

(3)

The acute angle between l_1 and Π is θ degrees.

(b) Determine, to one decimal place, the value of θ

(3)

The line l_2 lies in Π and passes through P

Given that the acute angle between l_1 and l_2 is also θ degrees,

(c) determine a vector equation for l_2

(5)

(a) l_1: $\mathbf{r} = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -2 \\ 7 \end{pmatrix}$

$2(3 + 4\lambda) + 4(5 - 2\lambda) - (4 + 7\lambda) = 1$

$\Rightarrow 22 - 7\lambda = 1 \Rightarrow \lambda = 3$

P is $(15, -1, 25)$

(b) $\begin{pmatrix} 4 \\ -2 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} = 8 - 8 - 7 = -7$

$\sin\theta = \frac{|-7|}{\sqrt{69}\sqrt{21}} = \frac{7}{\sqrt{1449}} = 0.18389...$

$\theta = 10.5965... \Rightarrow \theta = 10.6$

(c) l_2 is the projection of l_1 onto Π (it gives the same angle θ with l_1):

$\mathbf{n} \times \mathbf{d} = \begin{vmatrix} i & j & k \\ 2 & 4 & -1 \\ 4 & -2 & 7 \end{vmatrix} = \begin{pmatrix} 26 \\ -18 \\ -20 \end{pmatrix} = 2\begin{pmatrix} 13 \\ -9 \\ -10 \end{pmatrix}$

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Question 7 continued

Direction of l_2 is perpendicular to n and to $n \times d$:

$$\begin{pmatrix} 13 \\ -9 \\ -10 \end{pmatrix} \times \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} = \begin{vmatrix} i & j & k \\ 13 & -9 & -10 \\ 2 & 4 & -1 \end{vmatrix} = \begin{pmatrix} 49 \\ -7 \\ 70 \end{pmatrix} = 7\begin{pmatrix} 7 \\ -1 \\ 10 \end{pmatrix}$$

$$l_2: \quad r = \begin{pmatrix} 15 \\ -1 \\ 25 \end{pmatrix} + \mu\begin{pmatrix} 7 \\ -1 \\ 10 \end{pmatrix}$$

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Question 7 continued

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Question 7 continued

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8. The ellipse E has equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

(a) Determine the eccentricity of E

(2)

(b) Hence, for this ellipse, determine

(i) the coordinates of the foci,

(ii) the equations of the directrices.

(2)

The point P lies on E and has coordinates $(3\cos\theta, 2\sin\theta)$.

The line l_1 is the tangent to E at the point P

(c) Using calculus, show that an equation for l_1 is

$$2x\cos\theta + 3y\sin\theta = 6$$

(3)

The line l_2 passes through the origin and is perpendicular to l_1

The line l_1 intersects the line l_2 at the point Q

(d) Determine the coordinates of Q

(3)

(e) Show that, as θ varies, the point Q lies on the curve with equation

$$(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$$

where α and β are constants to be determined.

(3)

(a) $b^2 = a^2(1 - e^2) \Rightarrow 4 = 9(1 - e^2) \Rightarrow e^2 = \frac{5}{9} \Rightarrow e = \frac{\sqrt{5}}{3}$

(b)(i) foci $(\pm ae, 0)$: $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$

(ii) directrices $x = \pm\frac{a}{e}$: $x = \frac{9}{\sqrt{5}} = \frac{9\sqrt{5}}{5}$ and $x = -\frac{9\sqrt{5}}{5}$

(c) $\frac{dx}{d\theta} = -3\sin\theta$, $\frac{dy}{d\theta} = 2\cos\theta \Rightarrow \frac{dy}{dx} = \frac{2\cos\theta}{-3\sin\theta}$

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Question 8 continued

$y - 2\sin\theta = -\frac{2\cos\theta}{3\sin\theta}(x - 3\cos\theta)$

$3y\sin\theta - 6\sin^2\theta = -2x\cos\theta + 6\cos^2\theta$

$2x\cos\theta + 3y\sin\theta = 6(\sin^2\theta + \cos^2\theta) = 6$

$\therefore\ 2x\cos\theta + 3y\sin\theta = 6$

(d) gradient of $l_1 = -\frac{2\cos\theta}{3\sin\theta} \Rightarrow l_2$: $y = \frac{3\sin\theta}{2\cos\theta}x$

$2x\cos\theta + 3\sin\theta \times \frac{3\sin\theta}{2\cos\theta}x = 6 \Rightarrow x(4\cos^2\theta + 9\sin^2\theta) = 12\cos\theta$

$x = \frac{12\cos\theta}{4\cos^2\theta + 9\sin^2\theta}, \quad y = \frac{3\sin\theta}{2\cos\theta}x = \frac{18\sin\theta}{4\cos^2\theta + 9\sin^2\theta}$

$Q\left(\frac{12\cos\theta}{4\cos^2\theta + 9\sin^2\theta}, \frac{18\sin\theta}{4\cos^2\theta + 9\sin^2\theta}\right)$

(e) Let $D = 4\cos^2\theta + 9\sin^2\theta$

$x^2 + y^2 = \frac{144\cos^2\theta + 324\sin^2\theta}{D^2} = \frac{36(4\cos^2\theta + 9\sin^2\theta)}{D^2} = \frac{36}{D}$

$\Rightarrow (x^2 + y^2)^2 = \frac{1296}{D^2}$

$9x^2 + 4y^2 = \frac{9 \times 144\cos^2\theta + 4 \times 324\sin^2\theta}{D^2} = \frac{1296(\cos^2\theta + \sin^2\theta)}{D^2} = \frac{1296}{D^2}$

$\therefore\ (x^2 + y^2)^2 = 9x^2 + 4y^2 \qquad \alpha = 9,\ \beta = 4$

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Question 8 continued

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Q8

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TOTAL FOR PAPER: 75 MARKS

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