Pearson Edexcel Level 3 GCE

Paper reference **8FM0/28**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
28: Decision Mathematics 2
(Part of option K only)

You must have: Mathematical Formulae and Statistical Tables (Green), calculator, D2 Answer Book (enclosed)

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the Answer Book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.
- Do not return the question paper with the D2 Answer Book.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ▶

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P 6 6 7 9 5 A

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Write your answers in the answer book provided.

1. Five workers, A, B, C, D and E, are available to complete four tasks, P, Q, R and S.

Each task must be assigned to exactly one worker and each worker can do at most one task.

Worker B cannot be assigned to task R.

The amount, in pounds, that each worker will earn if they are assigned to each task is shown in the table below.

	P	**Q**	**R**	**S**
A	55	56	58	57
B	60	61	–	64
C	59	60	62	63
D	64	66	71	69
E	65	68	72	66

The Hungarian algorithm is to be used to find the maximum total amount that can be earned by the five workers.

(a) Explain how the table should be modified to allow the Hungarian algorithm to be used, giving reasons for your answer.

(2)

(b) Reducing rows first, use the Hungarian algorithm to obtain the maximum possible total earnings. You should explain how any initial row and column reductions were made and how you determined if the table was optimal at each stage.

(7)

(Total for Question 1 is 9 marks)

2.

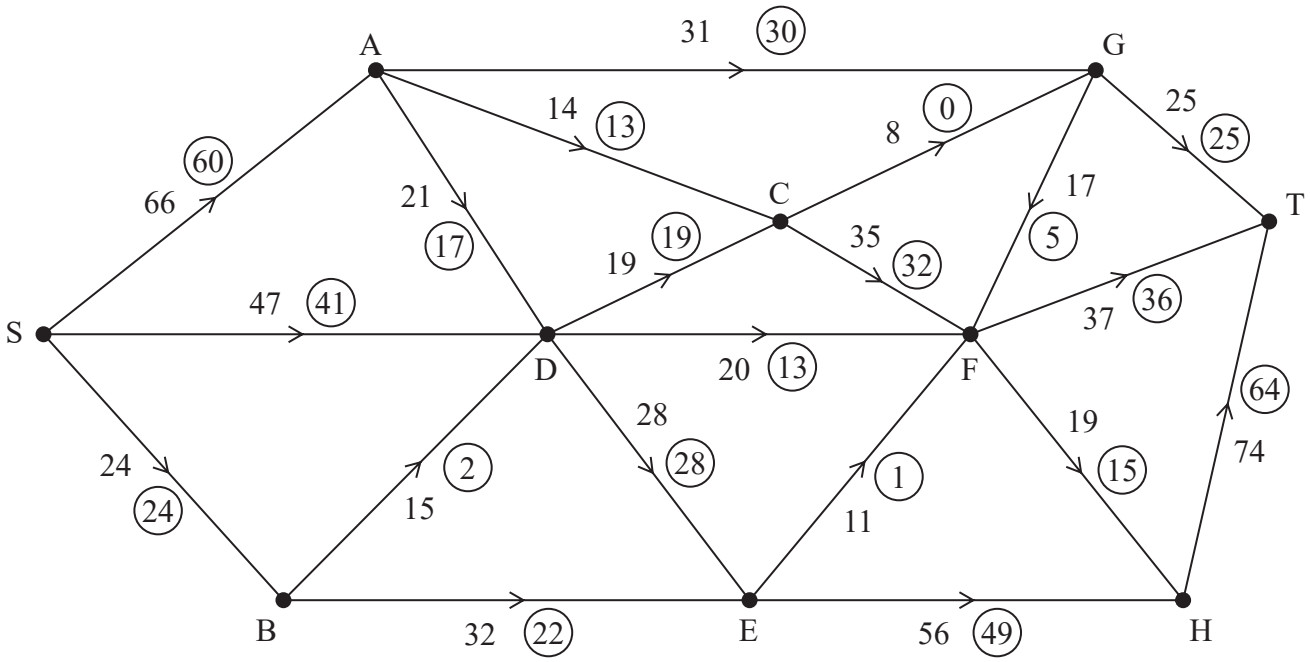


Figure 1

Figure 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow.

(1)

(b) Obtain the capacity of the cut that passes through the arcs AG, CG, GF, FT, FH and EH.

(1)

(c) Complete the initialisation of the labelling procedure on Diagram 1 in the answer book by entering values along SD, BD, BE and GF.

(2)

(d) Use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

(3)

(e) Use the answer to part (d) to add a maximum flow pattern to Diagram 2 in the answer book.

(1)

(f) Prove that your answer to part (e) is optimal.

(3)

(Total for Question 2 is 11 marks)

3. In your answer to this question you must show detailed reasoning.

A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays X	B plays Y
A plays Q	4	−3
A plays R	2	−1
A plays S	−3	5
A plays T	−1	3

(a) Verify that there is no stable solution to this game.

(2)

Player B plays their option X with probability p.

(b) Use a graphical method to find the optimal value of p and hence find the best strategy for player B.

(6)

(c) Find the value of the game to player A.

(1)

(d) Hence find the best strategy for player A.

(2)

(Total for Question 3 is 11 marks)

4. Sarah takes out a mortgage of £155 000 to buy a house. Interest is added each month on the outstanding balance at a constant rate of r % each month. Sarah makes fixed monthly repayments to reduce the amount owed.

Each month, interest is added, and then her monthly repayment is used to reduce the outstanding amount owed.

The recurrence relationship for the amount of the mortgage outstanding after $n + 1$ months is modelled by

$$u_{n+1} = 1.0025u_n - x \qquad n \geqslant 0$$

where £u_n is the amount of the mortgage outstanding after n months and £x is the monthly repayment.

(a) State the value of r.

(1)

(b) Solve the recurrence relation to find an expression for u_n in terms of x and n.

(5)

Given that the mortgage will be paid off in exactly 30 years,

(c) determine, to 2 decimal places, the least possible value of x.

(3)

(Total for Question 4 is 9 marks)

TOTAL FOR DECISION MATHEMATICS 2 IS 40 MARKS

END

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Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

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Answer Book

Do not return the question paper with the answer book.

Total Marks

Turn over ▶

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1. (a)

Subtract every entry from 72 (the largest value), so that the maximisation problem becomes a minimisation problem (minimise 72 − earnings).

There are 5 workers but only 4 tasks, so add a dummy column X (a dummy task) with all entries equal (e.g. 72) so that the table is square (5 × 5).

B cannot do task R, so put a large value (e.g. 100) in cell BR so that this allocation is never chosen.

	P	Q	R	S
A	55	56	58	57
B	60	61	–	64
C	59	60	62	63
D	64	66	71	69
E	65	68	72	66

You may not need to use all of these tables or all the rows/columns in each table.

	P	Q	R	S	X
A	17	16	14	15	72
B	12	11	100	8	72
C	13	12	10	9	72
D	8	6	1	3	72
E	7	4	0	6	72

(b) 72 − each entry, dummy column X with every entry 72, and 100 in cell BR.

	P	Q	R	S	X
A	3	2	0	1	58
B	4	3	92	0	64
C	4	3	1	0	63
D	7	5	0	2	71
E	7	4	0	6	72

Reduce rows: subtract the least value in each row from every entry in that row: row A − 14, row B − 8, row C − 9, row D − 1, row E − 0 (no reduction).

	P	Q	R	S	X
A	0	0	0	1	0
B	1	1	92	0	6
C	1	1	1	0	5
D	4	3	0	2	13
E	4	2	0	6	14

Reduce columns: subtract the least value in each column: column P − 3, column Q − 2, columns R and S − 0 (no reduction), X − 58.

The zeros can be covered by a minimum of 3 lines (row A, columns R and S).

3 < 5, so the table is not optimal.

Least uncovered entry = 1: subtract 1 from uncovered entries, add 1 to entries covered twice, others unchanged.

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Question 1 continued

	P	Q	R	S	X
A	0	0	1	2	0
B	0	0	92	0	5
C	0	0	1	0	4
D	3	2	0	2	12
E	3	1	0	6	13

Zeros now need a minimum of 4 lines to cover them (rows A, B, C and column R).

4 < 5, so the table is not optimal.

Least uncovered entry = 1: subtract 1 from uncovered entries, add 1 to entries covered twice (AR, BR, CR), others unchanged.

	P	Q	R	S	X
A	0	0	2	2	0
B	0	0	93	0	5
C	0	0	2	0	4
D	2	1	0	1	11
E	2	0	0	5	12

Zeros now need a minimum of 5 lines to cover them (row A, columns P, Q, R and S).

5 lines = 5 × 5 table, so this is optimal.

Allocation:

A – X (dummy, so A is not given a task)

B – S, C – P, D – R, E – Q

	P	Q	R	S	
A					
B					
C					
D					
E					

Maximum total earnings

= 64 + 59 + 71 + 68

= £262

	P	Q	R	S	
A					
B					
C					
D					
E					

	P	Q	R	S	
A					
B					
C					
D					
E					

(Total for Question 1 is 9 marks)

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2.

(a) Initial flow = 25 + 36 + 64 = 125

(b) Cut through AG, CG, GF, FT, FH, EH (GF goes from the sink side to the source side, so it contributes 0): 31 + 8 + 0 + 37 + 19 + 56 = 151

(d) Flow-augmenting routes:

S A D F H T – 4

S D B E H T – 2

S A C F T – 1

S A G F E H T – 1

Total increase in flow = 4 + 2 + 1 + 1 = 8

Maximum flow = 125 + 8 = 133

(c)

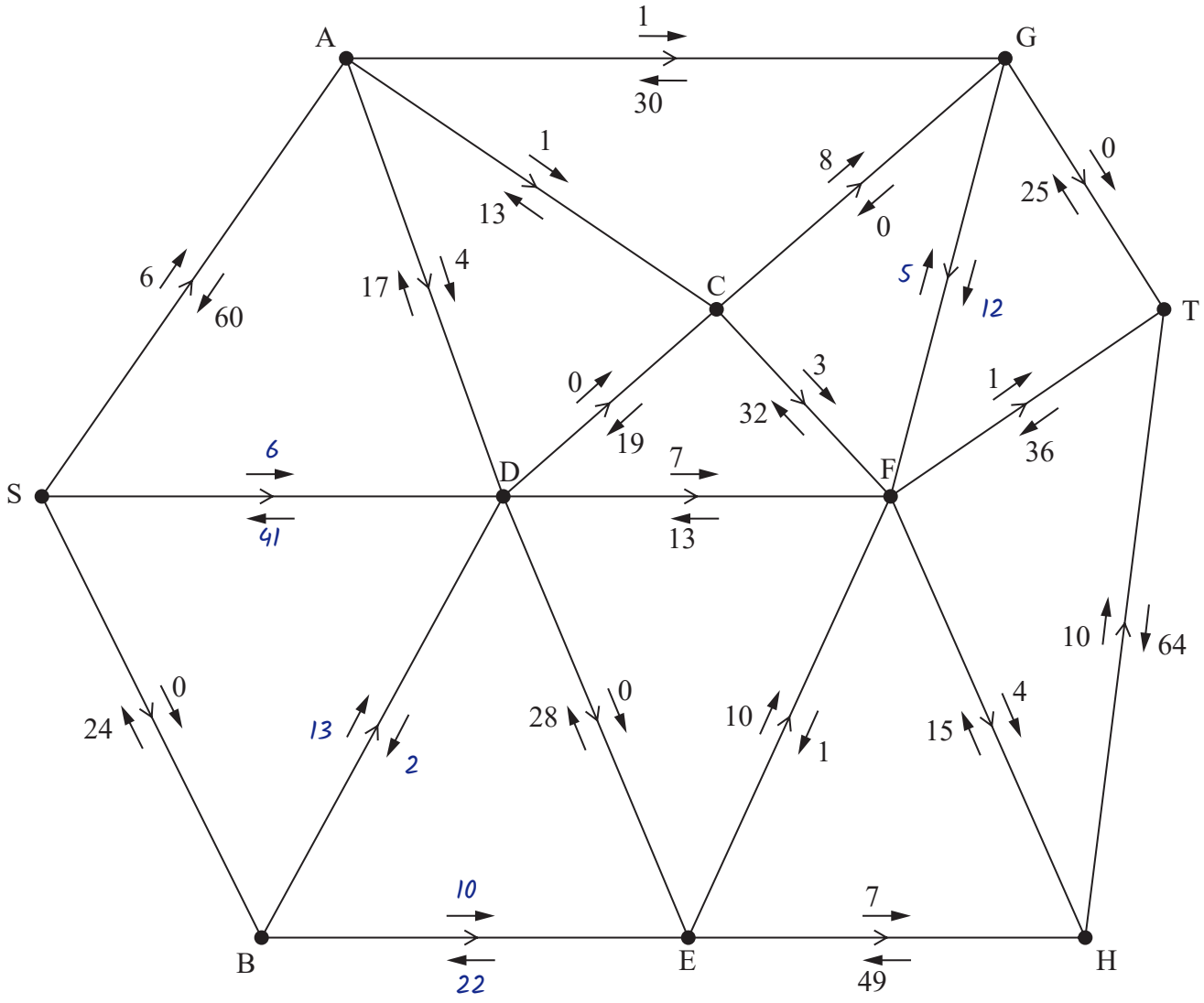


Diagram 1

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Question 2 continued

(e)

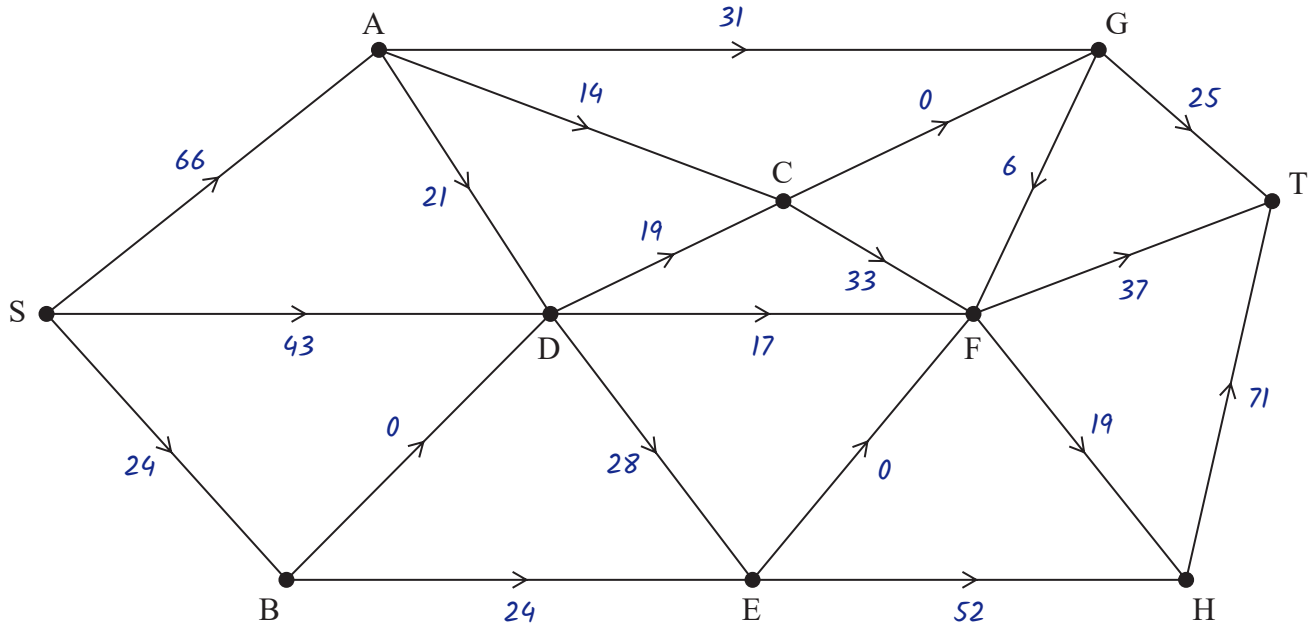


Diagram 2

(f) Max-flow min-cut theorem: if a flow equals the capacity of a cut, the flow is maximal. Consider the cut through GT, FT, FH, EF, DE, BD and SB (source side S, A, C, D, F, G; sink side B, E, H, T).

Value of cut = 25 + 37 + 19 + 0 + 28 + 0 + 24 = 133

(EF and BD go from the sink side to the source side, so they contribute 0)

Value of flow = 133 = value of this cut, so by the max-flow min-cut theorem the flow is maximal (and the cut is a minimum cut).

(Total for Question 2 is 11 marks)

3.

	B plays X	B plays Y
A plays Q	4	−3
A plays R	2	−1
A plays S	−3	5
A plays T	−1	3

(a) Row minima: Q −3, R −1, S −3, T −1 ⇒ row maximin = −1

Column maxima: X 4, Y 5 ⇒ column minimax = 4

Row maximin ≠ column minimax (−1 ≠ 4), so there is no stable solution.

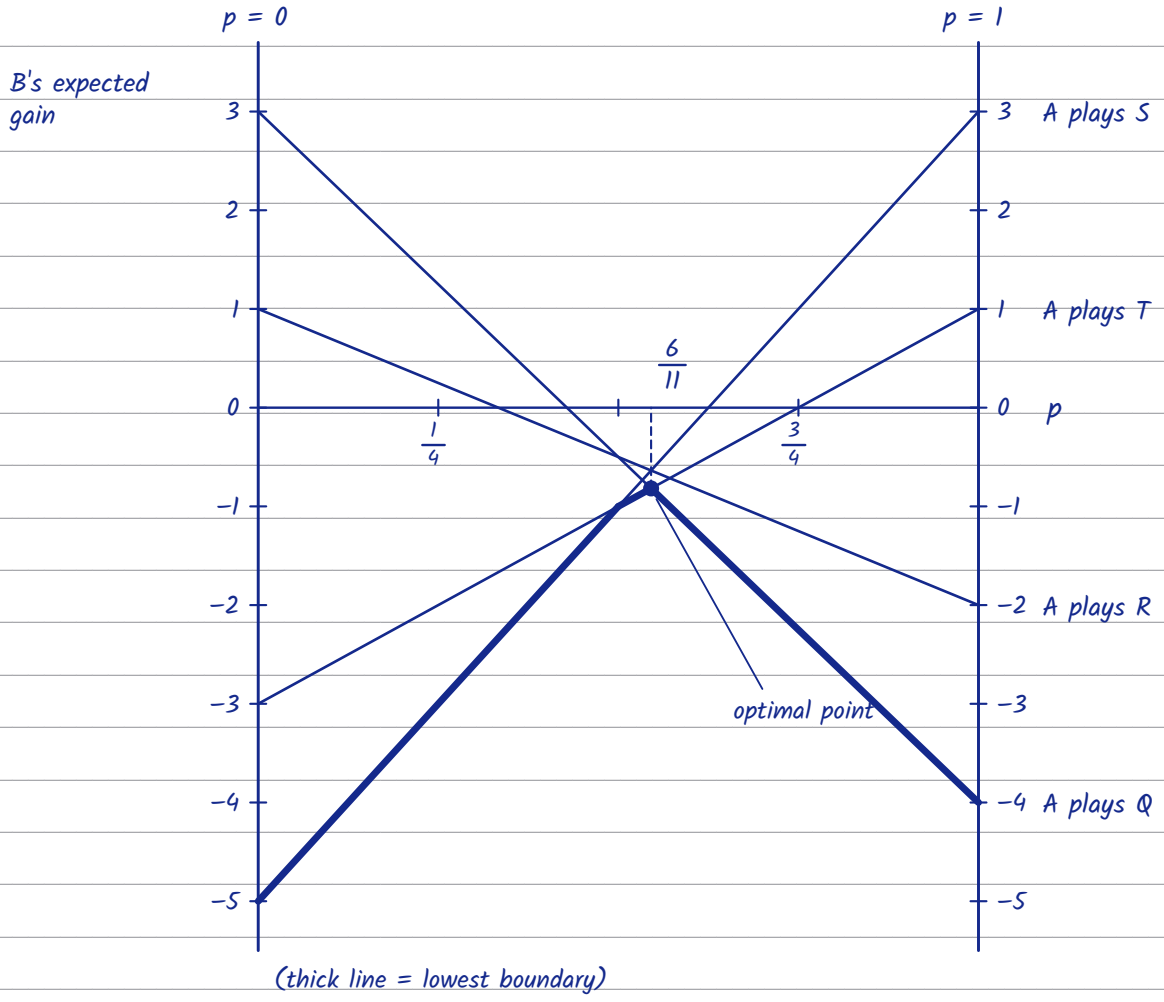
(b) B's pay-offs are the negatives of A's. B plays X with probability p and Y with probability 1 − p.

If A plays Q, B's expected gain = −4p + 3(1 − p) = 3 − 7p

If A plays R, B's expected gain = −2p + 1(1 − p) = 1 − 3p

If A plays S, B's expected gain = 3p − 5(1 − p) = 8p − 5

If A plays T, B's expected gain = p − 3(1 − p) = 4p − 3



(thick line = lowest boundary)

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Question 3 continued

B wants to maximise their minimum expected gain, so the optimal point is the highest point on the lowest boundary, where the lines for Q and T intersect:

$3 - 7p = 4p - 3 \quad \Rightarrow \quad 11p = 6$

$p = \frac{6}{11}$

B should play option X with probability $\frac{6}{11}$ and option Y with probability $\frac{5}{11}$

(c) Value to B $= 3 - 7 \times \frac{6}{11} = -\frac{9}{11}$

Value of the game to A $= \frac{9}{11}$

(d) A only uses options Q and T (the lines meeting at the optimal point).
A plays Q with probability q and T with probability $1 - q$.

If B plays X: $4q + (-1)(1 - q) = \frac{9}{11}$

$5q - 1 = \frac{9}{11} \quad \Rightarrow \quad 5q = \frac{20}{11} \quad \Rightarrow \quad q = \frac{4}{11}$

(Check, B plays Y: $-3 \times \frac{4}{11} + 3 \times \frac{7}{11} = \frac{9}{11}$ ✓)

A should play option Q with probability $\frac{4}{11}$, option T with probability $\frac{7}{11}$,

and should never play options R or S.

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 11 marks)

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4.

(a) $1.0025 = 1 + r/100 \quad \Rightarrow \quad r = 0.25$

(b) $u_{n+1} - 1.0025u_n = -x$

Auxiliary equation: $m - 1.0025 = 0 \quad \Rightarrow \quad m = 1.0025$

Complementary function: $u_n = A(1.0025)^n$

Particular solution: try $u_n = \lambda$

$\lambda = 1.0025\lambda - x \quad \Rightarrow \quad -0.0025\lambda = -x \quad \Rightarrow \quad \lambda = 400x$

General solution: $u_n = A(1.0025)^n + 400x$

When $n = 1$, $u_1 = 155\,000$:

$1.0025A + 400x = 155\,000$

$$A = \frac{155\,000 - 400x}{1.0025}$$

$$u_n = (155\,000 - 400x)(1.0025)^{n-1} + 400x$$

(c) 30 years = 360 months, so $u_{360} = 0$

$(155\,000 - 400x)(1.0025)^{359} + 400x = 0$

$155\,000(1.0025)^{359} = 400x((1.0025)^{359} - 1)$

$$x = \frac{155\,000(1.0025)^{359}}{400((1.0025)^{359} - 1)}$$

$$x = \frac{379\,860.89\ldots}{580.286\ldots} = 654.6095\ldots$$

Least possible monthly repayment: $x = £654.61$ (2 d.p.)

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

TOTAL FOR DECISION MATHEMATICS 2 IS 40 MARKS

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