Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Paper reference **8FM0/26**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ▶

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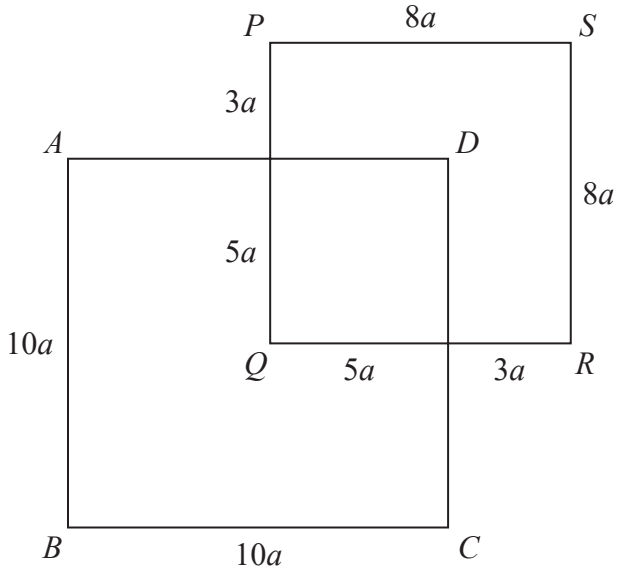


Figure 1

A uniform rod of length $72a$ is cut into pieces. The pieces are used to make two rigid squares, $ABCD$ and $PQRS$, with sides of length $10a$ and $8a$ respectively. The two squares are joined to form the rigid framework shown in Figure 1.

The squares both lie in the same plane with the rod AB parallel to the rod PQ.

Given that

- AD cuts PQ in the ratio 3 : 5
- DC cuts QR in the ratio 5 : 3

(a) explain why the centre of mass of square $ABCD$ is at Q.

(1)

(b) Find the distance of the centre of mass of the framework from B.

(5)

(a) ABCD is a uniform square framework, so by symmetry its centre of mass is at the centre of the square. Q is 5a from AB and 5a from BC (AQ = 5a, QC = 5a along the diagonal), so Q is the centre of ABCD.

(b) Masses are proportional to lengths:

ABCD : PQRS : framework = 40a : 32a : 72a = 40 : 32 : 72

Centre of PQRS is 4a from PQ and 4a from QR.

Distances from AB: ABCD 5a, PQRS 5a + 4a = 9a, framework $\bar{x}$

Moments about AB:

$40 \times 5a + 32 \times 9a = 72\bar{x}$

$488a = 72\bar{x}$

$\bar{x} = \frac{488a}{72} = \frac{61a}{9}$

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Question 1 continued

By symmetry the centre of mass is also $\frac{61a}{9}$ from BC (same working).

So the centre of mass lies on the diagonal through B:

distance from B = $\sqrt{(\bar{x}^2 + \bar{x}^2)} = \sqrt{2} \times \frac{61a}{9}$

$= \frac{61\sqrt{2}\,a}{9}$ $(= 9.585...a \approx 9.59a)$

(Total for Question 1 is 6 marks)

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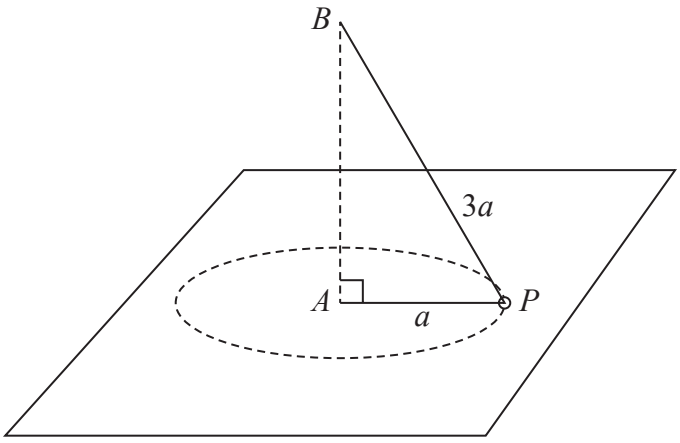


Figure 2

A small smooth ring P, of mass m, is threaded onto a light inextensible string of length $4a$.
One end of the string is attached to a fixed point A on a smooth horizontal table.
The other end of the string is attached to a fixed point B which is vertically above A.
The ring moves in a horizontal circle with centre A and radius a, as shown in Figure 2.

The ring moves with constant angular speed $\sqrt{\frac{2g}{3a}}$ about AB.

The string remains taut throughout the motion.

(a) Find, in terms of m and g, the magnitude of the normal reaction between P and the table.

(6)

The angular speed of P is now gradually increased.

(b) Find, in terms of a and g, the angular speed of P at the instant when it loses contact with the table.

(3)

(c) Explain how you have used the fact that P is smooth.

(1)

(a) $AP = a$, so $BP = 4a - a = 3a$. Let θ = angle BPA.

$\cos\theta = \frac{a}{3a} = \frac{1}{3}$, $AB = \sqrt{(9a^2 - a^2)} = 2\sqrt{2}\,a$, $\sin\theta = \frac{2\sqrt{2}}{3}$

P is smooth, so the tension T is the same in AP and in BP.

Resolve vertically (N = normal reaction):

$T\sin\theta + N = mg$

Resolve horizontally (towards A), N2L:

$T + T\cos\theta = ma\omega^2 = ma \times \frac{2g}{3a}$

$\frac{4}{3}T = \frac{2}{3}mg \quad \Rightarrow \quad T = \frac{1}{2}mg$

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$\frac{2\sqrt{2}}{3} \times \frac{1}{2} mg + N = mg$

$N = mg\left(1 - \frac{\sqrt{2}}{3}\right)$ $(\approx 0.529mg)$

(b) P loses contact with the table when $N = 0$:

$T \sin\theta = mg \Rightarrow \frac{2\sqrt{2}}{3} T = mg \Rightarrow T = \frac{3mg}{2\sqrt{2}}$

Horizontally: $\frac{4}{3} T = ma\omega^2$

$\frac{4}{3} \times \frac{3mg}{2\sqrt{2}} = ma\omega^2 \Rightarrow \sqrt{2}\, mg = ma\omega^2$

$\omega^2 = \frac{\sqrt{2}\, g}{a}$

$\omega = \sqrt{\frac{\sqrt{2}\, g}{a}}$ $\left(\approx 1.19\sqrt{\frac{g}{a}}\right)$

(c) Because P is smooth, the tension in the string is the same on either side of P (in AP and in BP), so T is used in both resolutions.

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Question 2 continued

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Question 2 continued

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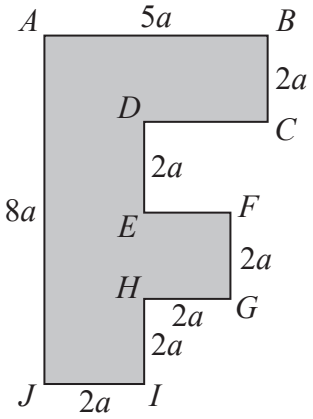


Figure 3

The uniform lamina $ABCDEFGHIJ$ is shown in Figure 3.

The lamina has $AJ = 8a$, $AB = 5a$ and $BC = DE = EF = FG = GH = HI = IJ = 2a$.

All the corners are right angles.

(a) Show that the distance of the centre of mass of the lamina from AJ is $\frac{49}{26}a$

(5)

A light inextensible rope is attached to the lamina at A and another light inextensible rope is attached to the lamina at B. The lamina hangs in equilibrium with both ropes vertical and AB horizontal. The weight of the lamina is W.

(b) Find, in terms of W, the tension in the rope attached to the lamina at B.

(3)

The rope attached to B breaks and subsequently the lamina hangs freely in equilibrium, suspended from A.

(c) Find the size of the angle between AJ and the downward vertical.

(5)

(a) Split into: rectangle along AJ (8a × 2a), top bar (3a × 2a),
small square EFGH (2a × 2a). Measure x from AJ, y from AB.

	rectangle	top bar	square	lamina
mass ratio	16	6	4	26
x from AJ	a	3.5a	3a	$\bar{x}$
y from AB	4a	a	5a	$\bar{y}$

Moments about AJ:

$16 \times a + 6 \times 3.5a + 4 \times 3a = 26\bar{x}$

$16a + 21a + 12a = 49a = 26\bar{x}$

$\bar{x} = \frac{49}{26}a$ as required

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Question 3 continued

(b) Taking moments about A (T = tension at B, $AB = 5a$):

$$T \times 5a = W \times \frac{49}{26}a$$

$$T = \frac{49}{130}W \quad (\approx 0.377W)$$

(c) Moments about AB:

$$16 \times 4a + 6 \times a + 4 \times 5a = 26\bar{y}$$

$$64a + 6a + 20a = 90a = 26\bar{y}$$

$$\bar{y} = \frac{90}{26}a = \frac{45}{13}a$$

Hanging from A, G is vertically below A. With θ = angle between AJ and the downward vertical:

$$\tan\theta = \frac{\bar{x}}{\bar{y}} = \frac{49/26}{90/26} = \frac{49}{90}$$

$$\theta = 28.5658...^\circ = 28.6^\circ \text{ (3 s.f.)}$$

Question 3 continued

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Question 3 continued

(Total for Question 3 is 13 marks)

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4. A particle P moves on the x-axis. At time t seconds, $t \geqslant 0$, P is x metres from the origin O and moving with velocity $v\,\text{m s}^{-1}$ in the direction of x increasing, where

$$v = 5\sin 2t$$

When $t = 0$, $x = 1$ and P is at rest.

(a) Find the magnitude and direction of the acceleration of P at the instant when P is next at rest.

(4)

(b) Show that $1 \leqslant x \leqslant 6$

(4)

(c) Find the total time, in the first 4π seconds of the motion, for which P is more than 3 metres from O

(3)

(a) At rest: $v = 5\sin 2t = 0 \quad \Rightarrow \quad 2t = \pi \quad \Rightarrow \quad t = \pi/2$ (next time)

$a = \dfrac{dv}{dt} = 10\cos 2t$

$t = \pi/2$: $\quad a = 10\cos\pi = -10$

Magnitude 10 m s^{-2}, directed towards O (in the negative x direction).

(b) $x = \int 5\sin 2t\,dt = -\dfrac{5}{2}\cos 2t + C$

$t = 0, x = 1$: $\quad 1 = -\dfrac{5}{2} + C \quad \Rightarrow \quad C = \dfrac{7}{2}$

$x = \dfrac{7}{2} - \dfrac{5}{2}\cos 2t$

Since $-1 \leqslant \cos 2t \leqslant 1$:

minimum $x = \dfrac{7}{2} - \dfrac{5}{2} = 1$, $\quad$ maximum $x = \dfrac{7}{2} + \dfrac{5}{2} = 6$

$\therefore \quad 1 \leqslant x \leqslant 6 \qquad$ as required

(c) $x \geqslant 1 > 0$ always, so P is more than 3 m from O when $x > 3$.

$x = 3$: $\quad \dfrac{7}{2} - \dfrac{5}{2}\cos 2t = 3 \quad \Rightarrow \quad \cos 2t = \dfrac{1}{5}$

$2t = 1.36943\ldots \quad \Rightarrow \quad t = 0.68471\ldots$

x has period π. In each period $x > 3$ for $0.68471\ldots < t < \pi - 0.68471\ldots$,

i.e. for a time of $\pi - 2(0.68471\ldots) = \pi - 1.36943\ldots$

In 4π seconds there are 4 periods:

total time $= 4\pi - 8(0.68471\ldots) = 7.08861\ldots$

$= 7.09$ s (3 s.f.)

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Question 4 continued

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(Total for Question 4 is 11 marks)

TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS

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