Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Paper reference **8FM0/25**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
25: Further Mechanics 1
(Part of options C, E, H and J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ▶

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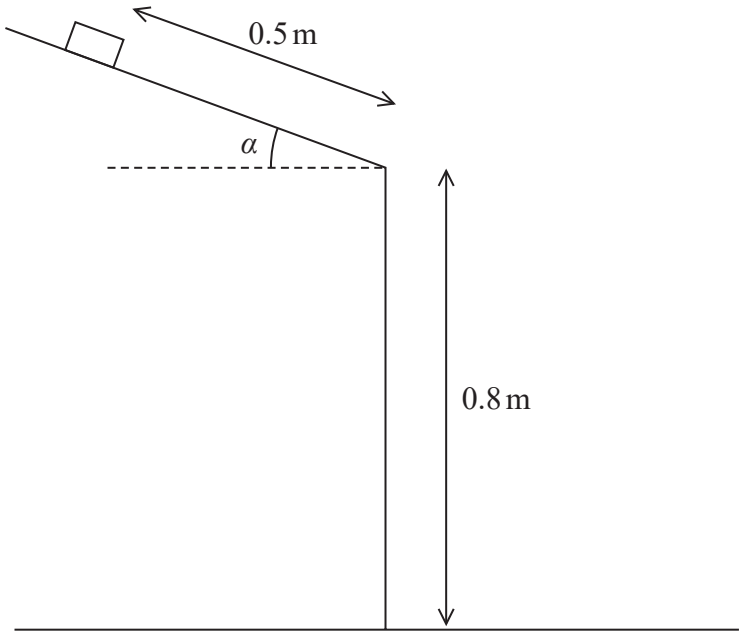


Figure 1

A small book of mass m is held on a rough straight desk lid which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The book is released from rest at a distance of 0.5 m from the edge of the desk lid, as shown in Figure 1. The book slides down the desk lid and then hits the floor that is 0.8 m below the edge of the desk lid. The coefficient of friction between the book and the desk lid is 0.4

The book is modelled as a particle which, after leaving the desk lid, is assumed to move freely under gravity.

(a) Find, in terms of m and g, the magnitude of the normal reaction on the book as it slides down the desk lid.

(2)

(b) Use the work-energy principle to find the speed of the book as it hits the floor.

(5)

(a) $\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$

Resolving perpendicular to the desk lid: $R = mg\cos\alpha$

$R = \frac{4}{5}mg$

(b) Friction $F = \mu R = 0.4 \times \frac{4}{5}mg = 0.32mg$

Work done against friction $= 0.32mg \times 0.5 = 0.16mg \quad (= 1.568m)$ J

Loss in PE $= mg \times 0.5\sin\alpha + mg \times 0.8 = 0.3mg + 0.8mg = 1.1mg$ J

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Question 1 continued

Gain in KE = $\frac{1}{2}mv^2$ (released from rest)

Work-energy principle: PE loss = work done against friction + KE gain

$1.1mg = 0.16mg + \frac{1}{2}mv^2$

$\frac{1}{2}v^2 = 0.94g = 9.212$

$v^2 = 18.424$

$v = \sqrt{18.424} = 4.2923...$

Speed as it hits the floor = 4.29 m s^{-1} (3 s.f.)

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Question 1 continued

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Question 1 continued

(Total for Question 1 is 7 marks)

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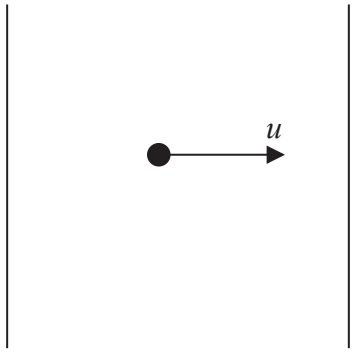


Figure 2

A particle of mass em is at rest on a smooth horizontal plane between two smooth fixed parallel vertical walls, as shown in the plan view in Figure 2. The particle is projected along the plane with speed u towards one of the walls and strikes the wall at right angles. The coefficient of restitution between the particle and each wall is e and air resistance is modelled as being negligible.

Using the model,

(a) find, in terms of m, u and e, an expression for the total loss in the kinetic energy of the particle as a result of the first two impacts.

(3)

Given that e can vary such that $0 < e < 1$ and using the model,

(b) find the value of e for which the total loss in the kinetic energy of the particle as a result of the first two impacts is a maximum,

(4)

(c) describe the subsequent motion of the particle.

(2)

(a) NLR at each wall: speed after 1st impact = eu

speed after 2nd impact = $e(eu) = e^2u$

KE loss = $\frac{1}{2}(em)u^2 - \frac{1}{2}(em)(e^2u)^2$

$= \frac{1}{2}emu^2 - \frac{1}{2}e^5mu^2$

Total loss in KE, $K = \frac{1}{2}mu^2(e - e^5)$

(b) $\frac{dK}{de} = \frac{1}{2}mu^2(1 - 5e^4)$

For a maximum, $\frac{dK}{de} = 0 \Rightarrow 1 - 5e^4 = 0$

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Question 2 continued

$e^4 = \frac{1}{5} \Rightarrow e = \left(\frac{1}{5}\right)^{1/4} = 0.66874... \quad (0 < e < 1)$

$\frac{d^2K}{de^2} = \frac{1}{2}mu^2(-20e^3) < 0$ for $e > 0$, so this is a maximum

$e = 0.669$ (3 s.f.)

(c) The particle continues to bounce back and forth between the two walls indefinitely, moving perpendicular to the walls. Its speed decreases after every impact (by a factor e each time), so the speed tends to zero.

Question 2 continued

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Question 2 continued

(Total for Question 2 is 9 marks)

3. The total mass of a cyclist and his bicycle is 100 kg.

In all circumstances, the magnitude of the resistance to the motion of the cyclist from non-gravitational forces is modelled as being kv^2 N, where v m s^{-1} is the speed of the cyclist.

The cyclist can freewheel, without pedalling, down a slope that is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{35}$, at a constant speed of V m s^{-1}

When he is pedalling up a slope that is inclined to the horizontal at an angle β, where $\sin\beta = \frac{1}{70}$, and he is moving at the same constant speed V m s^{-1}, he is working at a constant rate of P watts.

(a) Find P in terms of V.

(7)

If he pedals and works at a rate of $35V$ watts on a horizontal road, he moves at a constant speed of U m s^{-1}

(b) Find U in terms of V.

(4)

(a) Freewheeling down at constant speed V (a = 0):

Resolving parallel to the slope: $100g \sin\alpha - kV^2 = 0$

$kV^2 = 100g \times \frac{1}{35} = \frac{100g}{35}$ $(= 28)$

Pedalling up at constant speed V, driving force F:

Resolving parallel to the slope: $F - 100g \sin\beta - kV^2 = 0$

Using $F = \frac{P}{V}$: $\frac{P}{V} = 100g \sin\beta + kV^2 = \frac{100g}{70} + \frac{100g}{35}$

$\frac{P}{V} = \frac{300g}{70} = \frac{30 \times 9.8}{7} = 42$

$P = 42V$

(b) On the horizontal road, driving force $= \frac{35V}{U}$

Constant speed $\Rightarrow \frac{35V}{U} - kU^2 = 0$

From (a), $k = \frac{100g}{35V^2} = \frac{28}{V^2}$

$\frac{35V}{U} = \frac{28U^2}{V^2} \Rightarrow 35V^3 = 28U^3$

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Question 3 continued

$U^3 = \frac{5}{4}V^3 \Rightarrow U = \left(\frac{5}{4}\right)^{1/3}V = 1.0772...V$

$U = 1.08V$ (3 s.f.)

Question 3 continued

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Question 3 continued

(Total for Question 3 is 11 marks)

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4. Two particles, P and Q, have masses m and em respectively. The particles are moving on a smooth horizontal plane in the same direction along the same straight line when they collide directly. The coefficient of restitution between P and Q is e, where $0 < e < 1$

Immediately before the collision the speed of P is u and the speed of Q is eu.

(a) Show that the speed of Q immediately after the collision is u.

(6)

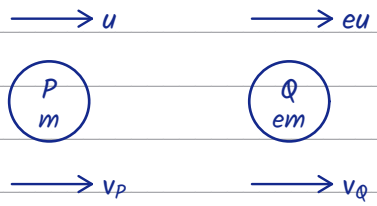
(b) Show that the direction of motion of P is unchanged by the collision.

(3)

The magnitude of the impulse on Q in the collision is $\frac{2}{9}mu$

(c) Find the possible values of e.

(4)



(a) Taking the direction of motion as positive.

CLM: $mu + em(eu) = mv_P + emv_Q$

$u + e^2u = v_P + ev_Q$ ...(1)

NLR: $v_Q - v_P = e(u - eu) = eu - e^2u$ ...(2)

(1) + (2): $u + eu = v_P + ev_Q + v_Q - v_P = (1 + e)v_Q$

$u(1 + e) = v_Q(1 + e)$ and $1 + e \neq 0$

$\therefore$ $v_Q = u$, so the speed of Q after the collision is u as required

(b) From (2): $v_P = v_Q - eu + e^2u = u - eu + e^2u$

$v_P = u(e^2 - e + 1)$

$= u\left[(e - \frac{1}{2})^2 + \frac{3}{4}\right]$

$(e - \frac{1}{2})^2 \geq 0$, so $v_P \geq \frac{3}{4}u > 0$

$v_P > 0$, so P is still moving in its original direction:

the direction of motion of P is unchanged by the collision.

(c) Impulse on Q = change in momentum of Q

$I = em(v_Q - eu) = em(u - eu) = mu(e - e^2)$

$mu(e - e^2) = \frac{2}{9}mu$

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Question 4 continued

$e - e^2 = \frac{2}{9} \Rightarrow 9e^2 - 9e + 2 = 0$

$(3e - 1)(3e - 2) = 0$

$e = \frac{1}{3}$ or $e = \frac{2}{3}$ (both satisfy $0 < e < 1$)

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Question 4 continued

(Total for Question 4 is 13 marks)

TOTAL FOR FURTHER MECHANICS 1 IS 40 MARKS

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