Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Paper reference **8FM0/21**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
21: Further Pure Mathematics 1
(Part of options A, B, C and D)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ▶

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Pearson

1. Use algebra to determine the values of x for which

$$x(x-1) > \frac{x-1}{x}$$

giving your answer in set notation.

(6)

$x(x-1) - \frac{x-1}{x} > 0$

Multiply both sides by $x^2 > 0$:

$x^3(x-1) - x(x-1) > 0$

$x(x-1)(x^2-1) > 0$

$x(x-1)(x-1)(x+1) > 0 \Rightarrow x(x-1)^2(x+1) > 0$

Critical values: $x = -1$, $x = 0$, $x = 1$

Sign of $x(x-1)^2(x+1)$:

$x < -1$: $(-)(+)(-) > 0$ ✓ $\quad -1 < x < 0$: $(-)(+)(+) < 0$ ✗

$0 < x < 1$: $(+)(+)(+) > 0$ ✓ $\quad x > 1$: $(+)(+)(+) > 0$ ✓

($x = 1$ gives 0, so $x = 1$ is excluded; $x = 0$ is undefined)

$\{x \in \mathbb{R} : x < -1\} \cup \{x \in \mathbb{R} : 0 < x < 1\} \cup \{x \in \mathbb{R} : x > 1\}$

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Question 1 continued

(Total for Question 1 is 6 marks)

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2. The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} + 15\frac{dy}{dx} - 3y^2 = 2x$$

where $y = 1$ at $x = 0$ and where $y = 2$ at $x = 0.1$

Use the approximations

$$\left(\frac{d^2y}{dx^2}\right)_n \approx \frac{(y_{n+1} - 2y_n + y_{n-1})}{h^2} \text{ and } \left(\frac{dy}{dx}\right)_n \approx \frac{(y_{n+1} - y_{n-1})}{2h}$$

with $h = 0.1$ to find an estimate for the value of y when $x = 0.3$

(6)

$y_0 = 1$ $(x = 0)$, $y_1 = 2$ $(x = 0.1)$, $h = 0.1$

At $x = 0.1$ $(n = 1)$:

$\left(\frac{dy}{dx}\right)_1 \approx \frac{y_2 - y_0}{2(0.1)} = \frac{y_2 - 1}{0.2}$, $\left(\frac{d^2y}{dx^2}\right)_1 \approx \frac{y_2 - 2(2) + 1}{0.1^2}$

$$\frac{y_2 - 2(2) + 1}{0.1^2} + 15\left(\frac{y_2 - 1}{0.2}\right) - 3(2)^2 = 2(0.1)$$

$$100(y_2 - 3) + 75(y_2 - 1) - 12 = 0.2$$

$$175y_2 = 300 + 75 + 12.2 = 387.2$$

$$y_2 = \frac{387.2}{175} = \frac{1936}{875} = 2.212571\ldots \quad (y \text{ at } x = 0.2)$$

At $x = 0.2$ $(n = 2)$:

$$\frac{y_3 - 2(2.212571\ldots) + 2}{0.1^2} + 15\left(\frac{y_3 - 2}{0.2}\right) - 3(2.212571\ldots)^2 = 2(0.2)$$

$$100(y_3 - 4.425142\ldots + 2) + 75(y_3 - 2) - 14.686417\ldots = 0.4$$

$$175y_3 = 0.4 + 14.686417\ldots + 242.514285\ldots + 150$$

$$175y_3 = 407.600702\ldots$$

$$y_3 = 2.329147\ldots$$

At $x = 0.3$, $y \approx 2.33$ (3 s.f.)

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Question 2 continued

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 6 marks)

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3. On a particular day, the depth of water in a river estuary at a specific location is modelled by the equation

$$D = 2\sin\left(\frac{x}{3}\right) + 3\cos\left(\frac{x}{3}\right) + 6 \qquad 0 \leqslant x \leqslant 7\pi \qquad \text{(I)}$$

where the depth of water is D metres at time x hours after midnight on that day.

(a) Write down the depth of water at midnight, according to the model.

(1)

Using the substitution $t = \tan\left(\frac{x}{6}\right)$

(b) show that equation (I) can be re-written as

$$D = \frac{3t^2 + 4t + 9}{1 + t^2}$$

(3)

(c) Hence determine, according to the model, the time after midnight when the depth of water is 5 metres for the first time. Give your answer to the nearest minute.

(5)

(a) $x = 0$: $D = 2\sin 0 + 3\cos 0 + 6 = 0 + 3 + 6 = 9$ m

(b) $t = \tan\left(\frac{x}{6}\right)$: $\sin\left(\frac{x}{3}\right) = \frac{2t}{1+t^2}$, $\cos\left(\frac{x}{3}\right) = \frac{1-t^2}{1+t^2}$

$D = 2\left(\frac{2t}{1+t^2}\right) + 3\left(\frac{1-t^2}{1+t^2}\right) + 6$

$D = \frac{4t + 3 - 3t^2 + 6 + 6t^2}{1+t^2}$

$D = \frac{3t^2 + 4t + 9}{1+t^2}$ as required

(c) $\frac{3t^2 + 4t + 9}{1+t^2} = 5 \Rightarrow 3t^2 + 4t + 9 = 5 + 5t^2$

$2t^2 - 4t - 4 = 0 \Rightarrow t^2 - 2t - 2 = 0$

$t = \frac{2 \pm \sqrt{(4+8)}}{2} = 1 \pm \sqrt{3}$

$t = 1 + \sqrt{3}$: $\frac{x}{6} = \tan^{-1}(1+\sqrt{3}) = 1.21991\ldots$

$t = 1 - \sqrt{3} < 0$: $\frac{x}{6} = \tan^{-1}(1-\sqrt{3}) + \pi = 2.5051\ldots$ (later time)

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Question 3 continued

First time: $x = 6 \times 1.21991... = 7.31950...$ hours

$0.31950... \times 60 = 19.17...$ minutes

Time = 07:19 (7:19 am)

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 9 marks)

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4. With respect to a fixed origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = 18\mathbf{i} - 14\mathbf{j} - 2\mathbf{k} \quad \overrightarrow{OB} = -7\mathbf{i} - 5\mathbf{j} + 3\mathbf{k} \quad \overrightarrow{OC} = -2\mathbf{i} - 9\mathbf{j} - 6\mathbf{k}$$

The points O, A, B and C form the vertices of a tetrahedron.

(a) Show that the area of the triangular face ABC of the tetrahedron is 108 to 3 significant figures.

(3)

(b) Find the volume of the tetrahedron.

(4)

An oak wood block is made in the shape of the tetrahedron, with centimetres taken for the units.

The density of oak is $0.85\,\text{g}\,\text{cm}^{-3}$

(c) Determine the mass of the block, giving your answer in kg.

(2)

(a) $AB = \begin{pmatrix} -7-18 \\ -5-(-14) \\ 3-(-2) \end{pmatrix} = \begin{pmatrix} -25 \\ 9 \\ 5 \end{pmatrix}$, $AC = \begin{pmatrix} -2-18 \\ -9-(-14) \\ -6-(-2) \end{pmatrix} = \begin{pmatrix} -20 \\ 5 \\ -4 \end{pmatrix}$

$AB \times AC = \begin{pmatrix} (9)(-4)-(5)(5) \\ (5)(-20)-(-25)(-4) \\ (-25)(5)-(9)(-20) \end{pmatrix} = \begin{pmatrix} -61 \\ -200 \\ 55 \end{pmatrix}$

Area $ABC = \frac{1}{2}|AB \times AC| = \frac{1}{2}\sqrt{(61^2 + 200^2 + 55^2)} = \frac{1}{2}\sqrt{46746}$

$= 108.104\ldots = 108$ (3 s.f.) as required

(b) Volume $= \frac{1}{6}|OA \cdot (OB \times OC)|$

$OA \cdot (OB \times OC) = \begin{vmatrix} 18 & -14 & -2 \\ -7 & -5 & 3 \\ -2 & -9 & -6 \end{vmatrix}$

$= 18(30+27) + 14(42+6) - 2(63-10)$

$= 18(57) + 14(48) - 2(53) = 1026 + 672 - 106 = 1592$

Volume $= \frac{1592}{6} = \frac{796}{3} = 265.33\ldots$

(c) Mass $= \frac{796}{3}\ \text{cm}^3 \times 0.85\ \text{g cm}^{-3} = 225.533\ldots\ \text{g}$

$= 225.533\ldots \div 1000 = 0.226$ kg (3 s.f.)

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

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5. The point $P\ (ap^2, 2ap)$, where a is a positive constant, lies on the parabola with equation

$$y^2 = 4ax$$

The normal to the parabola at P meets the parabola again at the point $Q\ (aq^2, 2aq)$

(a) Show that

$$q = \frac{-p^2 - 2}{p}$$

(5)

(b) Hence show that

$$PQ^2 = \frac{ka^2}{p^4}\left(p^2 + 1\right)^n$$

where k and n are integers to be determined.

(5)

(a) $y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y} = \frac{2a}{2ap} = \frac{1}{p}$

Gradient of normal at $P = -p$

Normal: $y - 2ap = -p(x - ap^2)$

Q lies on the normal: $2aq - 2ap = -p(aq^2 - ap^2)$

$\div a$: $2q - 2p = -pq^2 + p^3 \Rightarrow pq^2 + 2q - 2p - p^3 = 0$

$q = p$ is a root: $(q - p)(pq + p^2 + 2) = 0$

$q \neq p \Rightarrow pq + p^2 + 2 = 0 \Rightarrow q = \frac{-p^2 - 2}{p}$ as required

(b) $PQ^2 = (ap^2 - aq^2)^2 + (2ap - 2aq)^2$

$= a^2(p - q)^2(p + q)^2 + 4a^2(p - q)^2$

$= a^2(p - q)^2[(p + q)^2 + 4]$

$p - q = p + \frac{p^2 + 2}{p} = \frac{2p^2 + 2}{p} = \frac{2(p^2 + 1)}{p}$, $p + q = \frac{p^2 - p^2 - 2}{p} = -\frac{2}{p}$

$PQ^2 = a^2\left(\frac{2(p^2 + 1)}{p}\right)^2\left[\frac{4}{p^2} + 4\right] = \frac{4a^2(p^2 + 1)^2}{p^2} \times \frac{4(1 + p^2)}{p^2}$

$PQ^2 = \frac{16a^2}{p^4}(p^2 + 1)^3$

$k = 16, \quad n = 3$

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 10 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 1 IS 40 MARKS

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