Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Time 1 hour 40 minutes

Paper reference **8FM0/01**

Further Mathematics

Advanced Subsidiary

PAPER 1: Core Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ▶

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1. $\mathbf{P} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\mathbf{Q} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

(a) (i) Describe fully the single geometrical transformation P represented by the matrix **P**.

(ii) Describe fully the single geometrical transformation Q represented by the matrix **Q**.

(4)

The transformation P followed by the transformation Q is the transformation R, which is represented by the matrix **R**.

(b) Determine **R**.

(1)

(c) (i) Evaluate the determinant of **R**.

(ii) Explain how the value obtained in (c)(i) relates to the transformation R.

(2)

(a) (i) Rotation through 90° anticlockwise about the origin (0, 0)

(ii) Stretch, scale factor 3, parallel to the y-axis

(b) $R = QP = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 3 & 0 \end{pmatrix}$

(c) (i) $\det R = (0)(0) - (-1)(3) = 3$

(ii) $\det R = 3$ is the area scale factor of R: any shape transformed by R has its area multiplied by 3 (and orientation is preserved since $\det R > 0$).

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Question 1 continued

(Total for Question 1 is 7 marks)

2. The cubic equation

$$9x^3 - 5x^2 + 4x + 7 = 0$$

has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(3\alpha - 2)$, $(3\beta - 2)$ and $(3\gamma - 2)$, giving your answer in the form $aw^3 + bw^2 + cw + d = 0$, where a, b, c and d are integers to be determined.

(5)

$w = 3x - 2 \quad \Rightarrow \quad x = \frac{w + 2}{3}$

Substitute into $9x^3 - 5x^2 + 4x + 7 = 0$: $9\left(\frac{w + 2}{3}\right)^3 - 5\left(\frac{w + 2}{3}\right)^2 + 4\left(\frac{w + 2}{3}\right) + 7 = 0$

$\frac{1}{3}(w + 2)^3 - \frac{5}{9}(w + 2)^2 + \frac{4}{3}(w + 2) + 7 = 0$

Multiply by 9: $3(w + 2)^3 - 5(w + 2)^2 + 12(w + 2) + 63 = 0$

$3(w^3 + 6w^2 + 12w + 8) - 5(w^2 + 4w + 4) + 12w + 24 + 63 = 0$

$3w^3 + 18w^2 + 36w + 24 - 5w^2 - 20w - 20 + 12w + 87 = 0$

$3w^3 + 13w^2 + 28w + 91 = 0$

$\therefore \quad 3w^3 + 13w^2 + 28w + 91 = 0 \qquad (a = 3, \ b = 13, \ c = 28, \ d = 91)$

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Question 2 continued

(Total for Question 2 is 5 marks)

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3. (a) Use the standard results for summations to show that for all positive integers n

$$\sum_{r=1}^{n}(5r-2)^2 = \frac{1}{6}n(an^2+bn+c)$$

where a, b and c are integers to be determined.

(5)

(b) Hence determine the value of k for which

$$\sum_{r=1}^{k}(5r-2)^2 = 94k^2$$

(4)

(a) $(5r-2)^2 = 25r^2 - 20r + 4$

$$\sum_{r=1}^{n}(25r^2-20r+4) = 25\sum_{r=1}^{n}r^2 - 20\sum_{r=1}^{n}r + \sum_{r=1}^{n}4$$

$$= \frac{25}{6}n(n+1)(2n+1) - \frac{20}{2}n(n+1) + 4n$$

$$= \frac{1}{6}n[25(2n^2+3n+1) - 60(n+1) + 24]$$

$$= \frac{1}{6}n[50n^2+75n+25-60n-60+24]$$

$$= \frac{1}{6}n(50n^2+15n-11) \qquad \therefore \quad a=50,\ b=15,\ c=-11$$

(b) $\frac{1}{6}k(50k^2+15k-11) = 94k^2$

$50k^3+15k^2-11k = 564k^2$

$50k^3-549k^2-11k = 0 \Rightarrow k(50k^2-549k-11)=0$

$k \neq 0$: $50k^2-549k-11=0$

$(k-11)(50k+1)=0$

$k = 11$ or $k = -\frac{1}{50}$ (rejected: k must be a positive integer)

$\therefore \quad k = 11$

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Question 3 continued

(Total for Question 3 is 9 marks)

4. $\mathbf{M} = \begin{pmatrix} 2 & 1 & 4 \\ k & 2 & -2 \\ 4 & 1 & -2 \end{pmatrix}$ $\mathbf{N} = \begin{pmatrix} k-7 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}$

where k is a constant.

(a) Determine, in simplest form in terms of k, the matrix $\mathbf{MN}$.

(2)

(b) Given that $k = 5$

(i) write down $\mathbf{MN}$

(ii) hence write down $\mathbf{M}^{-1}$

(2)

(c) Solve the simultaneous equations

$$\begin{aligned} 2x + y + 4z &= 2 \\ 5x + 2y - 2z &= 3 \\ 4x + y - 2z &= -1 \end{aligned}$$

(2)

(d) Interpret the answer to part (c) geometrically.

(1)

(a) $MN = \begin{pmatrix} 2 & 1 & 4 \\ k & 2 & -2 \\ 4 & 1 & -2 \end{pmatrix}\begin{pmatrix} k-7 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} 2k-14+2-12 & 12-20+8 & -20+24-4 \\ k^2-7k+4+6 & 6k-40-4 & -10k+48+2 \\ 4k-28+2+6 & 24-20-4 & -40+24+2 \end{pmatrix}$

$= \begin{pmatrix} 2k-24 & 0 & 0 \\ k^2-7k+10 & 6k-44 & -10k+50 \\ 4k-20 & 0 & -14 \end{pmatrix}$

(b) (i) $k = 5$: $MN = \begin{pmatrix} -14 & 0 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & -14 \end{pmatrix}$

(ii) $M^{-1} = -\frac{1}{14}\begin{pmatrix} -2 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}$

(c) The equations are $Mx = b$ with $k = 5$, so

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Question 4 continued

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M^{-1}\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = -\frac{1}{14}\begin{pmatrix} -2 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

$$= -\frac{1}{14}\begin{pmatrix} -4 + 18 + 10 \\ 4 - 60 - 24 \\ -6 + 6 + 1 \end{pmatrix} = -\frac{1}{14}\begin{pmatrix} 24 \\ -80 \\ 1 \end{pmatrix}$$

$x = -\frac{12}{7}$, $y = \frac{40}{7}$, $z = -\frac{1}{14}$

(d) The three equations represent three planes which meet at a single point, the point $\left(-\frac{12}{7}, \frac{40}{7}, -\frac{1}{14}\right)$.

Question 4 continued

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Question 4 continued

(Total for Question 4 is 7 marks)

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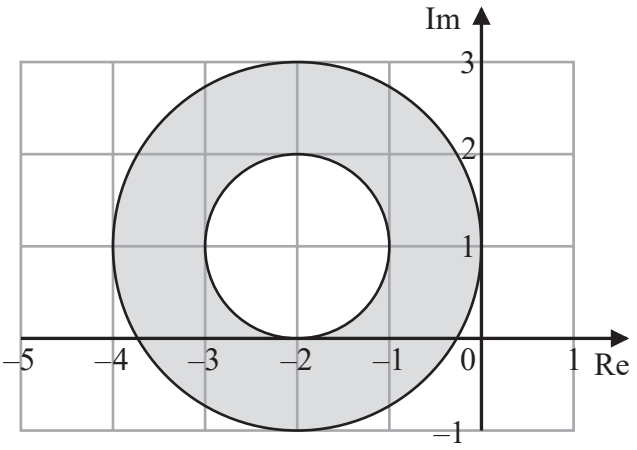


Figure 1

Figure 1 shows an Argand diagram.

The set P, of points that lie within the shaded region including its boundaries, is defined by

$$P = \{z \in \mathbb{C} : a \leqslant |z + b + c\mathrm{i}| \leqslant d\}$$

where a, b, c and d are integers.

(a) Write down the values of a, b, c and d.

(3)

The set Q is defined by

$$Q = \{z \in \mathbb{C} : a \leqslant |z + b + c\mathrm{i}| \leqslant d\} \cap \{z \in \mathbb{C} : |z - \mathrm{i}| \leqslant |z - 3\mathrm{i}|\}$$

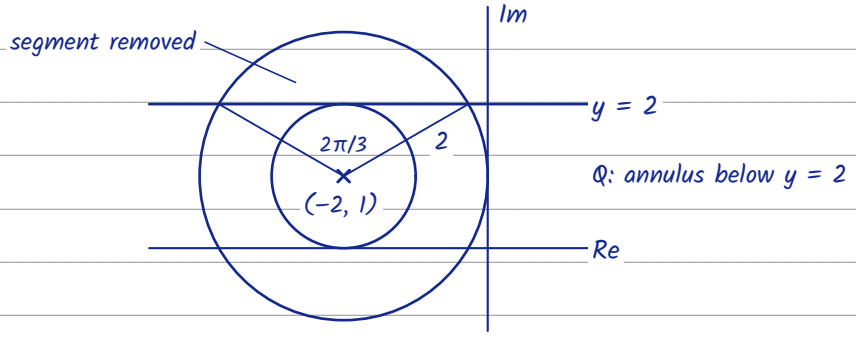
(b) Determine the exact area of the region defined by Q, giving your answer in simplest form.

(7)

(a) Centre of both circles is −2 + i, inner radius 1, outer radius 2:

$|z + 2 - i|$ between 1 and 2 $\Rightarrow$ $a = 1$, $b = 2$, $c = -1$, $d = 2$

(b) $|z - i| \leqslant |z - 3i|$: points nearer to i than to 3i, boundary is the perpendicular bisector of (0, 1) and (0, 3), i.e. the line $y = 2$. Q is the part of the annulus with $y \leqslant 2$.



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Question 5 continued

The line $y = 2$ is 1 unit above the centre $(-2, 1)$: it touches the inner circle and cuts the outer circle (radius 2). The part of P removed is the minor segment of the outer circle above $y = 2$.

Half-angle at centre: $\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$, so angle subtended at centre $= \frac{2\pi}{3}$

Segment area $= \frac{1}{2} \times 2^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 2^2 \times \sin\left(\frac{2\pi}{3}\right) = \frac{4\pi}{3} - \sqrt{3}$

Annulus area $= \pi \times 2^2 - \pi \times 1^2 = 3\pi$

Area of Q $= 3\pi - \left(\frac{4\pi}{3} - \sqrt{3}\right)$

$= \frac{5\pi}{3} + \sqrt{3}$

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 10 marks)

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6. A mining company has identified a mineral layer below ground.

The mining company wishes to drill down to reach the mineral layer and models the situation as follows.

With respect to a fixed origin O,

- the ground is modelled as a horizontal plane with equation $z = 0$
- the mineral layer is modelled as part of the plane containing the points $A(10, 5, -50)$, $B(15, 30, -45)$ and $C(-5, 20, -60)$, where the units are in metres

(a) Determine an equation for the plane containing A, B and C, giving your answer in the form $\mathbf{r}.\mathbf{n} = d$

(5)

(b) Determine, according to the model, the acute angle between the ground and the plane containing the mineral layer. Give your answer to the nearest degree.

(3)

The mining company plans to drill vertically downwards from the point (5, 12, 0) on the ground to reach the mineral layer.

(c) Using the model, determine, in metres to 1 decimal place, the distance the mining company will need to drill in order to reach the mineral layer.

(2)

(d) State a limitation of the assumption that the mineral layer can be modelled as a plane.

(1)

(a) $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 25 \\ 5 \end{pmatrix} = 5\begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} -15 \\ 15 \\ -10 \end{pmatrix} = 5\begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$

Normal $\mathbf{n} = (\mathbf{i} + 5\mathbf{j} + \mathbf{k}) \times (-3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$

$= (5(-2) - 1(3))\mathbf{i} - (1(-2) - 1(-3))\mathbf{j} + (1(3) - 5(-3))\mathbf{k} = -13\mathbf{i} - \mathbf{j} + 18\mathbf{k}$

Check: $(-13\mathbf{i} - \mathbf{j} + 18\mathbf{k}) \cdot (\mathbf{i} + 5\mathbf{j} + \mathbf{k}) = -13 - 5 + 18 = 0$ ✓

$d = (-13\mathbf{i} - \mathbf{j} + 18\mathbf{k}) \cdot (10\mathbf{i} + 5\mathbf{j} - 50\mathbf{k}) = -130 - 5 - 900 = -1035$

$\therefore \quad \mathbf{r} \cdot (13\mathbf{i} + \mathbf{j} - 18\mathbf{k}) = 1035$

(b) Ground $z = 0$ has normal $\mathbf{k}$. $(13\mathbf{i} + \mathbf{j} - 18\mathbf{k}) \cdot \mathbf{k} = -18$

$\cos\alpha = \dfrac{-18}{\sqrt{(13^2 + 1^2 + 18^2)}} = \dfrac{-18}{\sqrt{494}} = -0.80986\ldots \Rightarrow \alpha = 144.08\ldots°$

acute angle between the planes $= 180° - 144.08\ldots° = 35.91\ldots° = 36°$ (nearest degree)

(c) Vertical line from (5, 12, 0): $\mathbf{r} = 5\mathbf{i} + 12\mathbf{j} + \lambda\mathbf{k}$ meets the plane when

$\begin{pmatrix} 5 \\ 12 \\ \lambda \end{pmatrix} \cdot \begin{pmatrix} 13 \\ 1 \\ -18 \end{pmatrix} = 1035 \Rightarrow 65 + 12 - 18\lambda = 1035 \Rightarrow \lambda = -\dfrac{479}{9} = -53.222\ldots$

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Question 6 continued

distance to drill = $|\lambda|$ = 53.2 m (1 d.p.)

(d) The mineral layer will not be perfectly flat/smooth, so it will not form an exact plane (also the layer has a thickness/depth which a plane ignores).

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 11 marks)

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7. $f(z) = z^4 - 6z^3 + pz^2 + qz + r$

where p, q and r are real constants.

The roots of the equation $f(z) = 0$ are α, β, γ and δ where $\alpha = 3$ and $\beta = 2 + i$

Given that γ is a complex root of $f(z) = 0$

(a) (i) write down the root γ,

(ii) explain why δ must be real.

(2)

(b) Determine the value of δ.

(2)

(c) Hence determine the values of p, q and r.

(3)

(d) Write down the roots of the equation $f(-2z) = 0$

(2)

(a) (i) $\gamma = 2 - i$

(ii) $f(z)$ has real coefficients, so complex roots occur in conjugate pairs. β and γ are a conjugate pair and α is real, so if δ were complex its conjugate would also have to be a root, giving 5 roots for a quartic. Hence δ must be real.

(b) $\alpha + \beta + \gamma + \delta = -\frac{-6}{1} = 6$

$3 + (2 + i) + (2 - i) + \delta = 6 \Rightarrow 7 + \delta = 6 \Rightarrow \delta = -1$

(c) $f(z) = (z - 3)(z + 1)(z - (2 + i))(z - (2 - i))$

$(z - 3)(z + 1) = z^2 - 2z - 3$, $\quad (z - 2 - i)(z - 2 + i) = (z - 2)^2 + 1 = z^2 - 4z + 5$

$f(z) = (z^2 - 2z - 3)(z^2 - 4z + 5)$

$= z^4 - 4z^3 + 5z^2 - 2z^3 + 8z^2 - 10z - 3z^2 + 12z - 15$

$= z^4 - 6z^3 + 10z^2 + 2z - 15$

$\therefore \quad p = 10, \quad q = 2, \quad r = -15$

(d) $f(-2z) = 0 \Rightarrow -2z \in \{3, -1, 2 + i, 2 - i\} \Rightarrow z = -\frac{w}{2}$ for each root w

$z = -\frac{3}{2}, \quad z = \frac{1}{2}, \quad z = -1 - \frac{i}{2}, \quad z = -1 + \frac{i}{2}$

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 9 marks)

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8. (a) Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n} r(r+1)(2r+1) = \frac{1}{2}n(n+1)^2(n+2)$$

(6)

(b) Hence, show that, for all positive integers n,

$$\sum_{r=n}^{2n} r(r+1)(2r+1) = \frac{1}{2}n(n+1)(an+b)(cn+d)$$

where a, b, c and d are integers to be determined.

(3)

(a) $n = 1$: LHS $= 1(2)(3) = 6$, RHS $= \frac{1}{2}(1)(2)^2(3) = 6$ so true for $n = 1$

Assume true for $n = k$: $\sum_{r=1}^{k} r(r+1)(2r+1) = \frac{1}{2}k(k+1)^2(k+2)$

Then $\sum_{r=1}^{k+1} r(r+1)(2r+1) = \frac{1}{2}k(k+1)^2(k+2) + (k+1)(k+2)(2k+3)$

$= \frac{1}{2}(k+1)(k+2)[k(k+1) + 2(2k+3)]$

$= \frac{1}{2}(k+1)(k+2)(k^2+5k+6) = \frac{1}{2}(k+1)(k+2)(k+2)(k+3)$

$= \frac{1}{2}(k+1)(k+2)^2(k+3) = \frac{1}{2}(k+1)((k+1)+1)^2((k+1)+2)$

which is the result for $n = k + 1$.

If the statement is true for $n = k$ then it has been shown to be true for $n = k + 1$,

and as it is true for $n = 1$, the statement is true for all positive integers n.

(b) $\sum_{r=n}^{2n} r(r+1)(2r+1) = \sum_{r=1}^{2n} r(r+1)(2r+1) - \sum_{r=1}^{n-1} r(r+1)(2r+1)$

$= \frac{1}{2}(2n)(2n+1)^2(2n+2) - \frac{1}{2}(n-1)n^2(n+1)$

$= \frac{1}{2}n(n+1)[4(2n+1)^2 - n(n-1)]$

$= \frac{1}{2}n(n+1)[16n^2 + 16n + 4 - n^2 + n]$

$= \frac{1}{2}n(n+1)(15n^2 + 17n + 4)$

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Question 8 continued

$= \frac{1}{2}n(n+1)(3n+1)(5n+4)$ $\quad \therefore \quad a = 3, \; b = 1, \; c = 5, \; d = 4$

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Question 8 continued

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Question 8 continued

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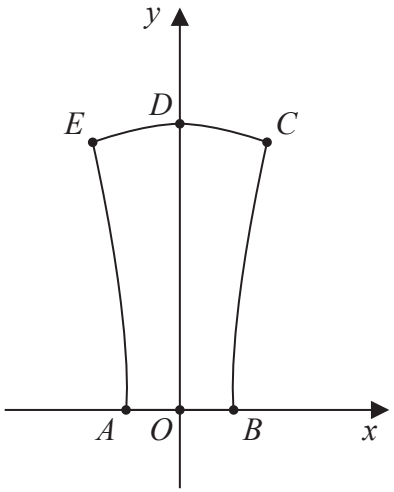


Figure 2

Figure 2 shows the vertical cross-section, *AOBCDE*, through the centre of a wax candle.

In a model, the candle is formed by rotating the region bounded by the *y*-axis, the line *OB*, the curve *BC*, and the curve *CD* through 360° about the *y*-axis.

The point *B* has coordinates (3, 0) and the point *C* has coordinates (5, 15).

The units are in centimetres.

The curve *BC* is represented by the equation

$$y = \frac{\sqrt{225x^2 - 2025}}{a} \qquad 3 \leqslant x < 5$$

where *a* is a constant.

(a) Determine the value of *a* according to this model.

(2)

The curve *CD* is represented by the equation

$$y = 16 - 0.04x^2 \qquad 0 \leqslant x < 5$$

(b) Using algebraic integration, determine, according to the model, the exact volume of wax that would be required to make the candle.

(9)

(c) State a limitation of the model.

(1)

When the candle was manufactured, 700 cm^3 of wax were required.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning.

(1)

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Question 9 continued

(a) C(5, 15): $15 = \frac{\sqrt{(225 \times 5^2 - 2025)}}{a} = \frac{\sqrt{3600}}{a} = \frac{60}{a} \Rightarrow a = 4$

(b) BC: $y = \frac{\sqrt{(225x^2 - 2025)}}{4} \Rightarrow 16y^2 = 225x^2 - 2025 \Rightarrow x^2 = \frac{16y^2 + 2025}{225} = \frac{16}{225}y^2 + 9$

CD: $y = 16 - 0.04x^2 \Rightarrow 0.04x^2 = 16 - y \Rightarrow x^2 = 25(16 - y) = 400 - 25y$

At C, $y = 15$; at D $(x = 0)$, $y = 16$. Volume $= \pi \int x^2 \, dy$

$V_1 = \pi \int_0^{15} \left(\frac{16}{225}y^2 + 9\right) dy = \pi \left[\frac{16y^3}{675} + 9y\right]_0^{15} = \pi\left(\frac{16 \times 3375}{675} + 135\right) - 0$

$= \pi(80 + 135) = 215\pi$

$V_2 = \pi \int_{15}^{16} (400 - 25y) \, dy = \pi \left[400y - \frac{25y^2}{2}\right]_{15}^{16}$

$= \pi[(6400 - 3200) - (6000 - 2812.5)] = \pi(3200 - 3187.5) = \frac{25}{2}\pi$

$V = V_1 + V_2 = 215\pi + \frac{25}{2}\pi = \frac{455}{2}\pi \text{ cm}^3$

(c) The sides of the candle will not be perfectly smooth/curved, and there will be a hole in the middle for the wick, so the curves may not be a suitable model.

(d) $\frac{455}{2}\pi = 714.7\ldots \text{ cm}^3$; 700 cm^3 is only about 15 cm^3 (about 2%) less than this, so the model gives a good estimate of the wax required (a slight overestimate).

Question 9 continued

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Question 9 continued

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Question 9 continued

(Total for Question 9 is 13 marks)

TOTAL FOR CORE PURE MATHEMATICS IS 80 MARKS

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