Pearson Edexcel Level 3 GCE

Thursday 14 May 2020

Afternoon | Paper Reference **8FM0/28**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
28: Decision Mathematics 2
(Part of option K only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator, D2 Answer Book (enclosed)

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of the D2 Answer Book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D2 Answer Book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.
- Do not return the question paper with the D2 Answer Book.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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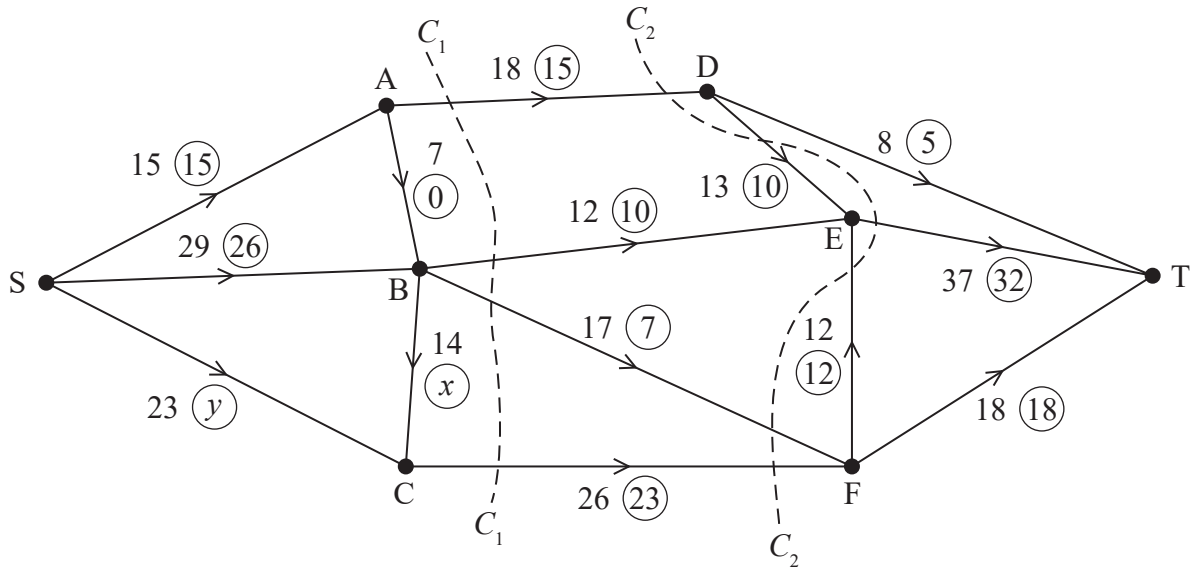


Figure 1

Figure 1 shows a capacitated, directed network of pipes. The number on each arc represents the capacity of the corresponding pipe. The numbers in circles represent a feasible flow from S to T.

(a) (i) Find the value of x.

(ii) Find the value of y.

(2)

(b) List the saturated arcs.

(1)

Two cuts, C_1 and C_2, are shown in Figure 1.

(c) Find the capacity of

(i) C_1

(ii) C_2

(2)

(d) Write down a flow-augmenting route, using the arc CF, that increases the flow by two units.

(1)

Given that the flow through the network is increased by two units using the route found in (d),

(e) prove that this new flow is maximal.

(3)

(Total for Question 1 is 9 marks)

2. Four workers, A, B, C and D, are each to be assigned to one of four tasks, P, Q, R and S.

Each worker must be assigned to one task, and each task must be done by exactly one worker.

Worker C cannot be assigned to task Q.

The amount, in pounds, that each worker would earn when assigned to each task is shown in the table below.

	P	Q	R	S
A	72	98	59	84
B	67	87	68	86
C	70	–	62	79
D	78	93	64	81

The Hungarian algorithm is to be used to find the maximum total amount that can be earned by the four workers.

(a) Explain how the table should be modified so that the Hungarian algorithm may be applied.

(2)

(b) Modify the table so that the Hungarian algorithm may be applied.

(1)

(c) Reducing rows first, use the Hungarian algorithm to obtain an allocation that maximises the total earnings. You should explain how any initial row and column reductions were made and also how you determined if the table was optimal at each stage.

(6)

(Total for Question 2 is 9 marks)

3. Two teams, A and B, each have three team members. One member of Team A will compete against one member of Team B for 10 rounds of a competition. None of the rounds can end in a draw.

Table 1 shows, for each pairing, the expected number of rounds that the member of Team A will win minus the expected number of rounds that the member of Team B will win. These numbers are the scores awarded to Team A. This competition between Teams A and B is a zero-sum game. Each team must choose one member to play. Each team wants to choose the member who will maximise its score.

		Team B		
		Paul	Qaasim	Rashid
Team A	Mischa	4	–6	2
	Noel	0	–2	6
	Olive	–6	2	0

Table 1

(a) (i) Find the number of rounds that Team A expects to win if Team A chooses Mischa and Team B chooses Paul.

(ii) Find the number of rounds that Team B expects to win if Team A chooses Noel and Team B chooses Qaasim.

(2)

Table 1 models this zero-sum game.

(b) (i) Find the play-safe strategies for the game.

(ii) Explain how you know that the game is not stable.

(4)

(c) Determine which team member Team B should choose if Team B thinks that Team A will play safe. Give a reason for your answer.

(1)

At the last minute, Rashid is ill and is therefore unavailable for selection by Team B.

(d) Find the best strategy for Team B, defining any variables you use.

(7)

(Total for Question 3 is 14 marks)

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4. A sequence $\{u_n\}$, where $n \geqslant 1$, satisfies the recurrence relation

$$2u_n = u_{n-1} - kn^2 \quad \text{where} \quad 4u_2 - u_0 = 27k^2$$

and k is a non-zero constant.

Show that, as n becomes large, u_n can be approximated by a quadratic function of the form $an^2 + bn + c$ where a, b and c are constants to be determined.

(8)

(Total for Question 4 is 8 marks)

TOTAL FOR DECISION MATHEMATICS 2 IS 40 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

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Total Marks

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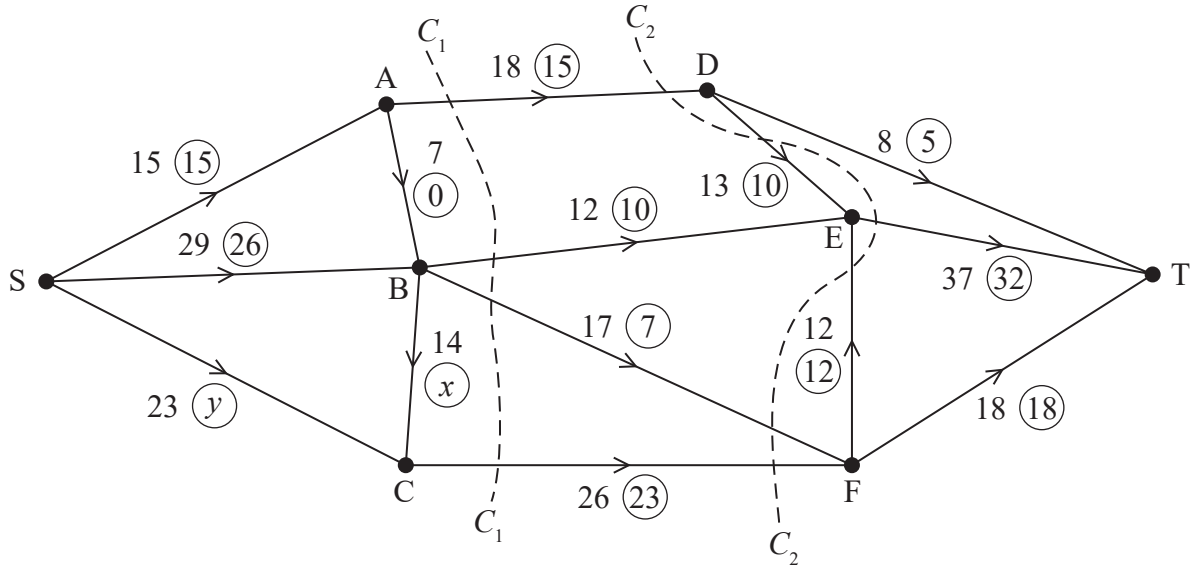


Figure 1

(a)(i) Flow into B = flow out of B

$26 + 0 = 10 + 7 + x \Rightarrow x = 9$

(ii) Flow into C = flow out of C

$y + x = 23 \Rightarrow y = 23 - 9 = 14$

(b) Saturated arcs: SA, FE, FT

(c)(i) $C_1 = AD + BE + BF + CF = 18 + 12 + 17 + 26 = 73$

(ii) $C_2 = AD + BF + CF + ET = 18 + 17 + 26 + 37 = 98$

(DE and FE cross C_2 from the sink side to the source side, so count 0)

(d) S – C – F – B – E – T

(spare: SC 9, CF 3, FB backflow 7, BE 2, ET 5, so increases flow by 2)

(e) Initial flow = 15 + 26 + 14 = 55, so new flow = 55 + 2 = 57

Consider the cut through SA, AB, BE, FE and FT

(separating {S, B, C, F} from {A, D, E, T}):

Capacity = 15 + 0 + 12 + 12 + 18 = 57

(AB goes from the sink side to the source side, so counts 0)

Value of flow = 57 = capacity of this cut

By the maximum flow – minimum cut theorem, the flow of 57 is maximal.

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Question 1 continued

(Total for Question 1 is 9 marks)

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2.

	P	Q	R	S
A	72	98	59	84
B	67	87	68	86
C	70	–	62	79
D	78	93	64	81

(a) Subtract every entry from a constant (e.g. 98, the largest value) to convert the maximisation problem into a minimisation problem. Put a large value (e.g. 78) in cell CQ to make it unattractive, so that worker C is not assigned to task Q.

You may not need to use all of these tables.

	P	Q	R	S
A	26	0	39	14
B	31	11	30	12
C	28	78	36	19
D	20	5	34	17

(b) 98 – each entry; cell CQ given the large value 78

	P	Q	R	S
A	26	0	39	14
B	20	0	19	1
C	9	59	17	0
D	15	0	29	12

(c) Reduce rows: subtract the least value in each row from every element of that row: A – 0, B – 11, C – 19, D – 5

	P	Q	R	S
A	17	0	22	14
B	11	0	2	1
C	~~0~~	~~59~~	~~0~~	~~0~~
D	6	0	12	12

Reduce columns: P – 9, R – 17, no reduction for columns Q and S. Zeros covered by 2 lines (column Q, row C). 2 < 4, so not optimal.

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Question 2 continued

	P	Q	R	S
A	16	0	21	13
B	~~10~~	~~0~~	~~1~~	~~0~~
C	~~0~~	~~60~~	~~0~~	~~0~~
D	5	0	11	11

Least uncovered element = 1: subtract 1 from uncovered elements, add 1 to CQ (covered twice), others unchanged. Zeros covered by 3 lines (rows B and C, column Q). 3 < 4, so not optimal.

	P	Q	R	S
A	11	0	16	8
B	10	5	1	0
C	~~0~~	~~65~~	~~0~~	~~0~~
D	0	0	6	6

Least uncovered element = 5: subtract 5 from uncovered elements, add 5 to BQ and CQ (covered twice), others unchanged. Now 4 lines are needed to cover the zeros (columns P, Q, S and row C). 4 = 4, so the table is optimal.

	P	Q	R	S
A				
B				
C				
D				

Allocation:

A – Q, B – S, C – R, D – P

Maximum total earnings

= 98 + 86 + 62 + 78 = £324

	P	Q	R	S
A				
B				
C				
D				

	P	Q	R	S
A				
B				
C				
D				

(Total for Question 2 is 9 marks)

3.

		Team B		
		Paul	Qaasim	Rashid
Team A	Mischa	4	–6	2
	Noel	0	–2	6
	Olive	–6	2	0

Table 1

(a)(i) Let Team A win w rounds: $w - (10 - w) = 4$

$2w = 14 \Rightarrow$ Team A expects to win 7 rounds

(ii) Let Team B win b rounds: $(10 - b) - b = -2$

$2b = 12 \Rightarrow$ Team B expects to win 6 rounds

(b)(i) Row minima: Mischa −6, Noel −2, Olive −6 ⇒ row maximin = −2

Column maxima: Paul 4, Qaasim 2, Rashid 6 ⇒ column minimax = 2

Play-safe strategies: Team A plays Noel, Team B plays Qaasim

(ii) Row maximin (−2) ≠ column minimax (2), so the game is not stable.

(c) Qaasim. If Team A plays safe (Noel), Team B gains 0 by choosing Paul, gains 2 by choosing Qaasim and loses 6 by choosing Rashid, so choosing Qaasim gives Team B the greatest gain (2).

(d) Without Rashid, B's gains are the negatives of the Paul and Qaasim columns.

Let Team B play Paul with probability q and Qaasim with probability $1 - q$.

If A plays Mischa, B's expected gain $= -(4q - 6(1 - q)) = 6 - 10q$

If A plays Noel, B's expected gain $= -(0q - 2(1 - q)) = 2 - 2q$

If A plays Olive, B's expected gain $= -(-6q + 2(1 - q)) = 8q - 2$

(graph on the next page)

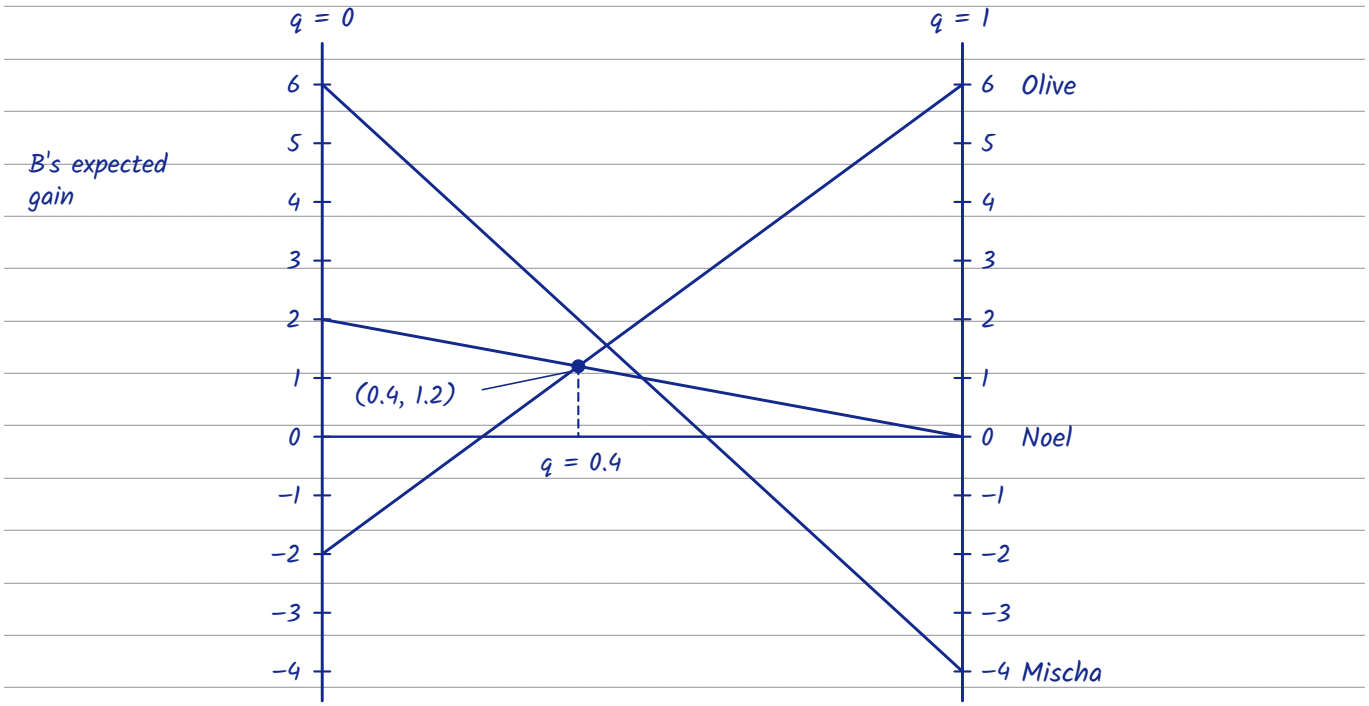
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Question 3 continued



B wants to maximise its minimum gain: the optimal point is the highest point of the lowest boundary (marked on the graph), where the Noel and Olive lines meet:

$2 - 2q = 8q - 2 \Rightarrow 10q = 4 \Rightarrow q = 0.4$

(value of the game to Team B = $2 - 2(0.4) = 1.2$)

Team B should choose Paul with probability 0.4 and choose Qaasim with probability 0.6.

Question 3 continued

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Question 3 continued

(Total for Question 3 is 14 marks)

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4.

Auxiliary equation: $2m - 1 = 0 \quad \Rightarrow \quad m = \frac{1}{2}$

Complementary function: $u_n = A\left(\frac{1}{2}\right)^n$

Particular solution: try $u_n = \lambda n^2 + \beta n + \alpha$

$2(\lambda n^2 + \beta n + \alpha) = \lambda(n-1)^2 + \beta(n-1) + \alpha - kn^2$

$2\lambda n^2 + 2\beta n + 2\alpha = (\lambda - k)n^2 + (-2\lambda + \beta)n + (\lambda - \beta + \alpha)$

Comparing coefficients:

n^2: $2\lambda = \lambda - k \quad \Rightarrow \quad \lambda = -k$

n: $2\beta = -2\lambda + \beta \quad \Rightarrow \quad \beta = -2\lambda = 2k$

constant: $2\alpha = \lambda - \beta + \alpha \quad \Rightarrow \quad \alpha = \lambda - \beta = -k - 2k = -3k$

General solution: $u_n = A\left(\frac{1}{2}\right)^n - kn^2 + 2kn - 3k$

$u_0 = A - 3k$

$u_2 = \frac{1}{4}A - 4k + 4k - 3k = \frac{1}{4}A - 3k$

$4u_2 - u_0 = 27k^2 \quad \Rightarrow \quad 4\left(\frac{1}{4}A - 3k\right) - (A - 3k) = 27k^2$

$A - 12k - A + 3k = 27k^2 \quad \Rightarrow \quad 27k^2 + 9k = 0$

$9k(3k + 1) = 0 \text{ and } k \neq 0 \quad \Rightarrow \quad k = -\frac{1}{3}$

$u_n = A\left(\frac{1}{2}\right)^n + \frac{1}{3}n^2 - \frac{2}{3}n + 1$

As n becomes large, $\left(\frac{1}{2}\right)^n \to 0$, so $A\left(\frac{1}{2}\right)^n \to 0$

Hence, for large n, $u_n \approx \frac{1}{3}n^2 - \frac{2}{3}n + 1$

i.e. $a = \frac{1}{3}, \quad b = -\frac{2}{3}, \quad c = 1$

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Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

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