# Pearson Edexcel Level 3 GCE

# Thursday 14 May 2020

Afternoon | Paper Reference **8FM0/27**

# Further Mathematics

**Advanced Subsidiary**
**Further Mathematics options**
**27: Decision Mathematics 1**
**(Part of options D, F, H and K)**

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator, D1 Answer Book (enclosed)

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the Answer Book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.
- Do not return the question paper with the D1 Answer Book.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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**1.** 3.7 2.5 5.4 1.9 2.7 3.2 3.1 2.7 4.2 2.0

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 8.5

**(3)**

The first-fit bin packing algorithm is to be used to pack $n$ numbers into bins. The number of comparisons is used to measure the order of the first-fit bin packing algorithm.

(b) By considering the worst case, determine the order of the first-fit bin packing algorithm in terms of $n$. You must make your method and working clear.

**(3)**

**(Total for Question 1 is 6 marks)**

**2.**

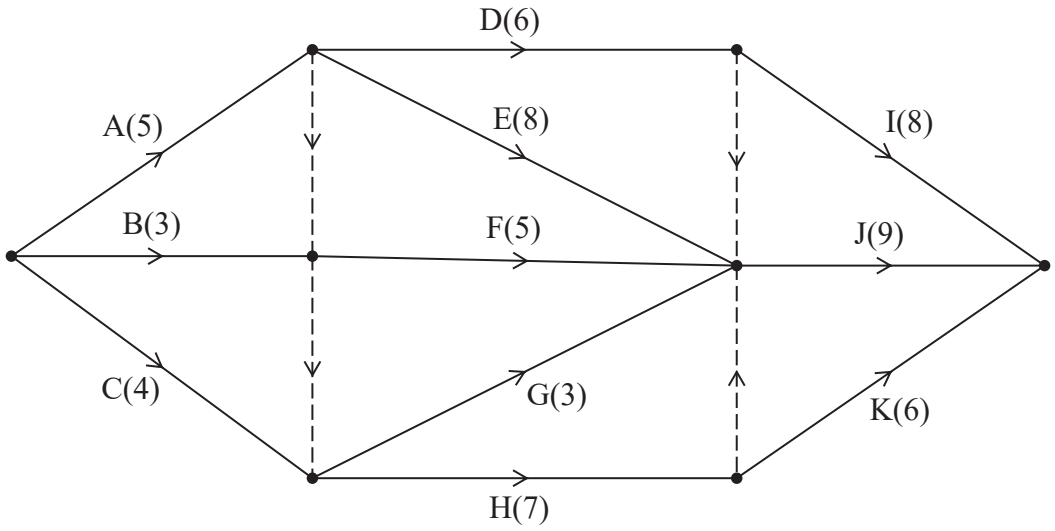



**Figure 1**

A project is modelled by the activity network shown in Figure 1. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, to complete the corresponding activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Complete the precedence table in the answer book.

**(2)**

(b) Complete Diagram 1 in the answer book to show the early event times and the late event times.

**(3)**

(c) (i) State the minimum project completion time.

(ii) List the critical activities.

**(2)**

(d) Calculate the maximum number of hours by which activity H could be delayed without affecting the shortest possible completion time of the project. You must make the numbers used in your calculation clear.

**(1)**

(e) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

**(2)**

(f) Draw a cascade chart for this project on Grid 1 in the answer book.

**(3)**

(g) Using the answer to (f), explain why it is not possible to complete the project in the shortest possible time using the number of workers found in (e).

**(1)**

**(Total for Question 2 is 14 marks)**

**3.**

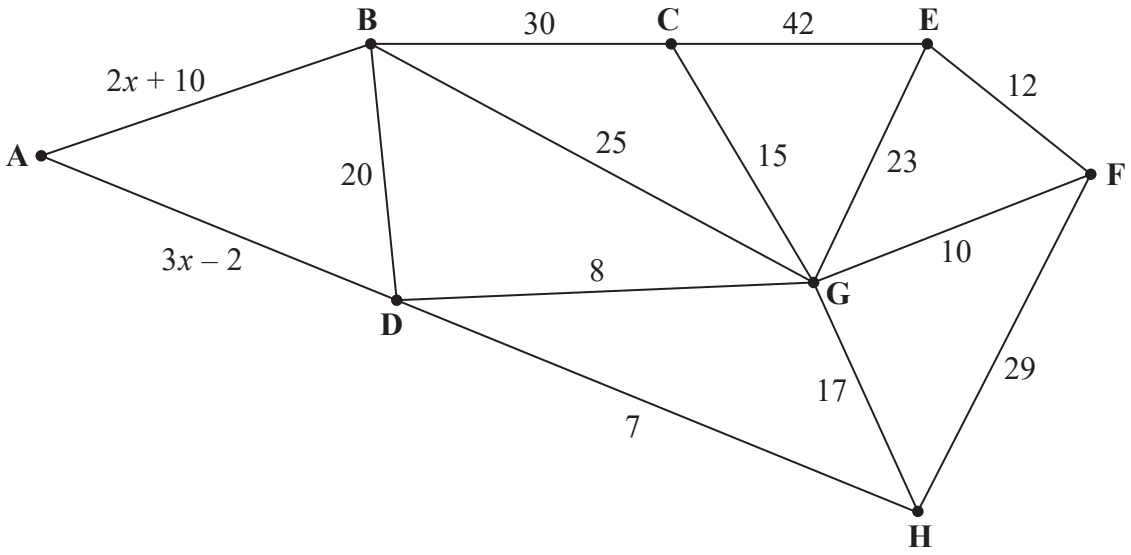



**Figure 2**

[*The weight of the network is* $5x + 246$]

(a) Explain why it is not possible to draw a graph with an odd number of vertices of odd valency.

**(2)**

Figure 2 represents a network of 14 roads in a town. The expression on each arc gives the time, in minutes, to travel along the corresponding road.

Prim's algorithm, starting at A, is applied to the network. The order in which the arcs are selected is AD, DH, DG, FG, EF, CG, BD. It is given that the order in which the arcs are selected is unique.

(b) Using this information, find the smallest possible range of values for $x$, showing your working clearly.

**(3)**

A route that minimises the total time taken to traverse each road at least once is required. The route must start and finish at the same vertex.

Given that the time taken to traverse this route is 318 minutes,

(c) use an appropriate algorithm to determine the value of $x$, showing your working clearly.

**(6)**

**(Total for Question 3 is 11 marks)**

**4.**

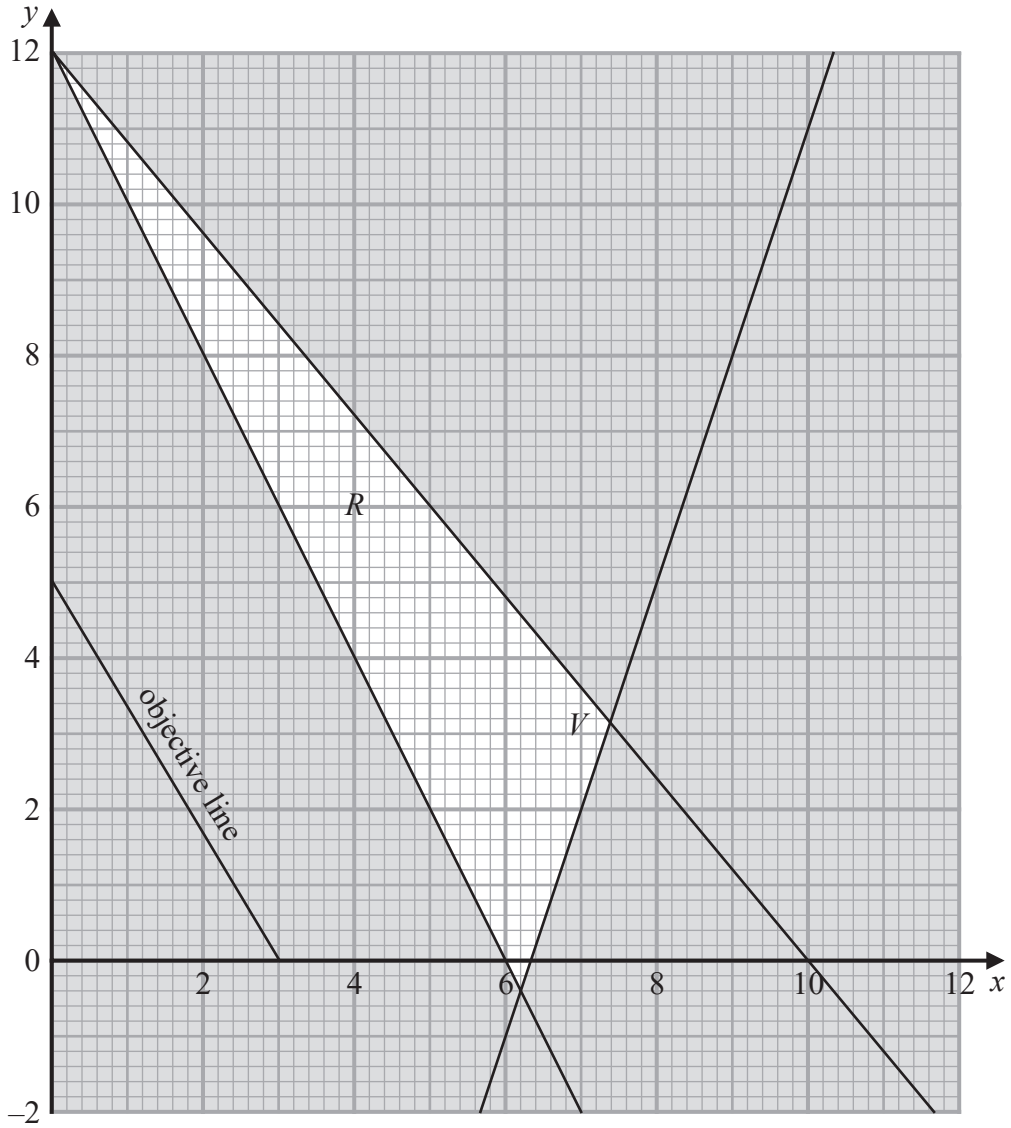



**Figure 3**

Figure 3 shows the constraints of a linear programming problem in $x$ and $y$, where $R$ is the feasible region. Figure 3 also shows an objective line for the problem and the optimal vertex, which is labelled as $V$.

The value of the objective at $V$ is 556

Express the linear programming problem in algebraic form. List the constraints as simplified inequalities with integer coefficients and determine the objective.

**(9)**

**(Total for Question 4 is 9 marks)**

**TOTAL FOR DECISION MATHEMATICS 1 IS 40 MARKS**

**END**

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Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

**Pearson Edexcel Level 3 GCE**

Centre Number | Candidate Number

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# Further Mathematics

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**Answer Book**
Do not return the question paper with the answer book.

Total Marks

*Turn over* ▶

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**1.**

3.7 2.5 5.4 1.9 2.7 3.2 3.1 2.7 4.2 2.0

(a) First-fit, bins of size 8.5:

Bin 1: 3.7 2.5 1.9 (total 8.1)

Bin 2: 5.4 2.7 (total 8.1)

Bin 3: 3.2 3.1 2.0 (total 8.3)

Bin 4: 2.7 4.2 (total 6.9)

(b) Worst case: each number must be compared with every bin used so far (each number goes into a new bin).

1st number: 0 comparisons

2nd number: compared with the 1st bin, 1 comparison

3rd number: compared with 2 bins, 2 comparisons

...

nth number: compared with $(n - 1)$ bins, $(n - 1)$ comparisons

Total comparisons $= 1 + 2 + 3 + \ldots + (n - 1)$

$= \frac{1}{2}(n - 1)\, n$

$= \frac{1}{2} n^2 - \frac{1}{2} n$

The highest power of n is $n^2$, so the order of the algorithm is quadratic, i.e. $O(n^2)$.

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**Question 1 continued**

**(Total for Question 1 is 6 marks)**

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**2.** (a)

| Activity | Immediately preceding activities |
|---|---|
| A | – |
| B | – |
| C | – |
| D | A |

| Activity | Immediately preceding activities |
|---|---|
| E | A |
| F | A, B |
| G | A, B, C |
| H | A, B, C |

| Activity | Immediately preceding activities |
|---|---|
| I | D |
| J | D, E, F, G, H |
| K | H |

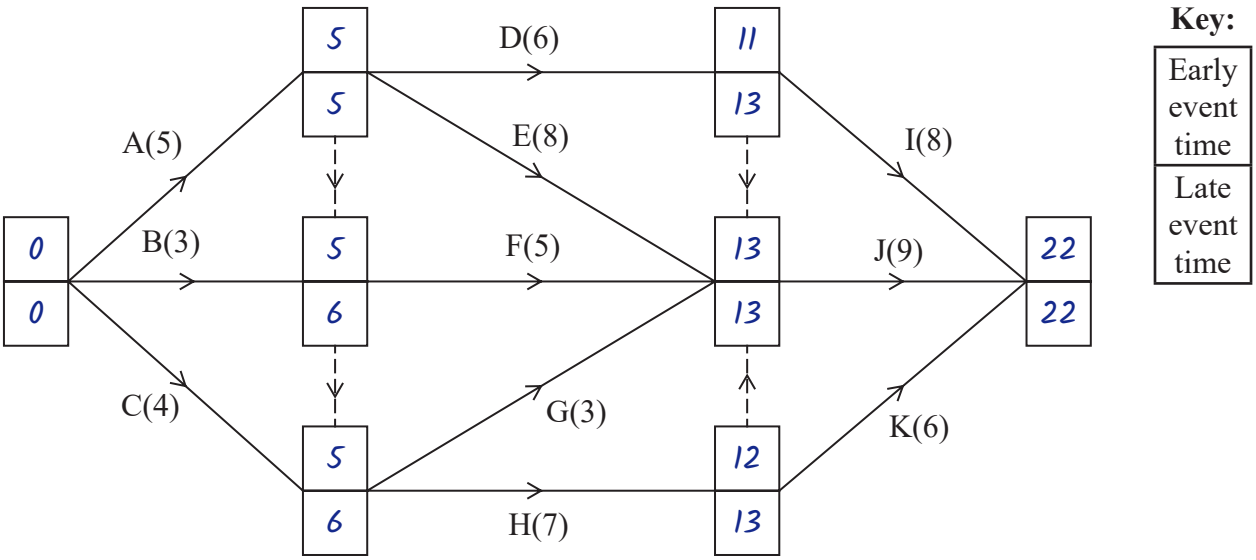



**Diagram 1**

(c)(i) Minimum project completion time = 22 hours

(ii) Critical activities: A, E and J

(d) Float on H = 13 – 5 – 7 = 1

H could be delayed by 1 hour

(e) Total activity time = 5 + 3 + 4 + 6 + 8 + 5 + 3 + 7 + 8 + 9 + 6 = 64

Lower bound = 64 ÷ 22 = 2.909...

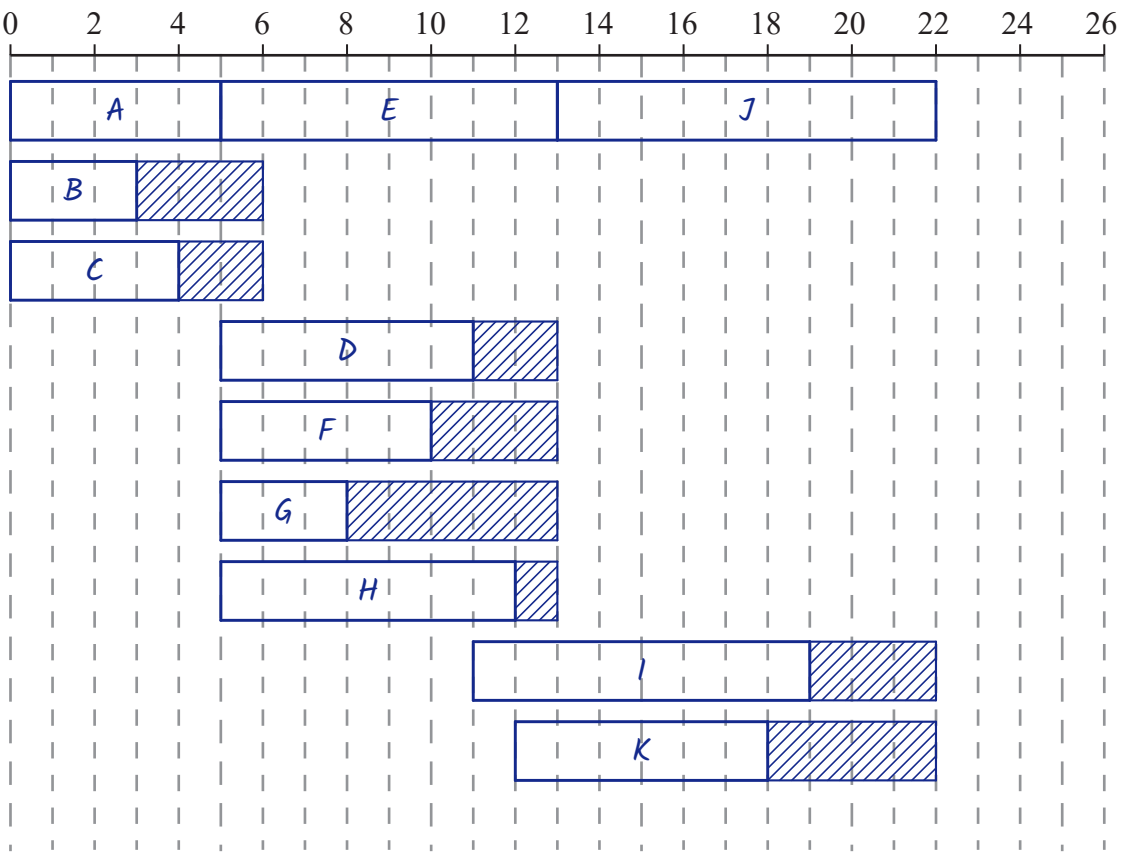
⇒ lower bound of 3 workers

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**Question 2 continued**

(f)




**Grid 1**

(g) From the cascade chart, at time 8.5 activities D, E, F and H must all be happening (E is critical, and D, F and H must start by times 7, 8 and 6 and last until at least 11, 10 and 12), so 4 workers are needed at 8.5. So the project cannot be completed in 22 hours with only 3 workers.

**(Total for Question 2 is 14 marks)**

**3.**

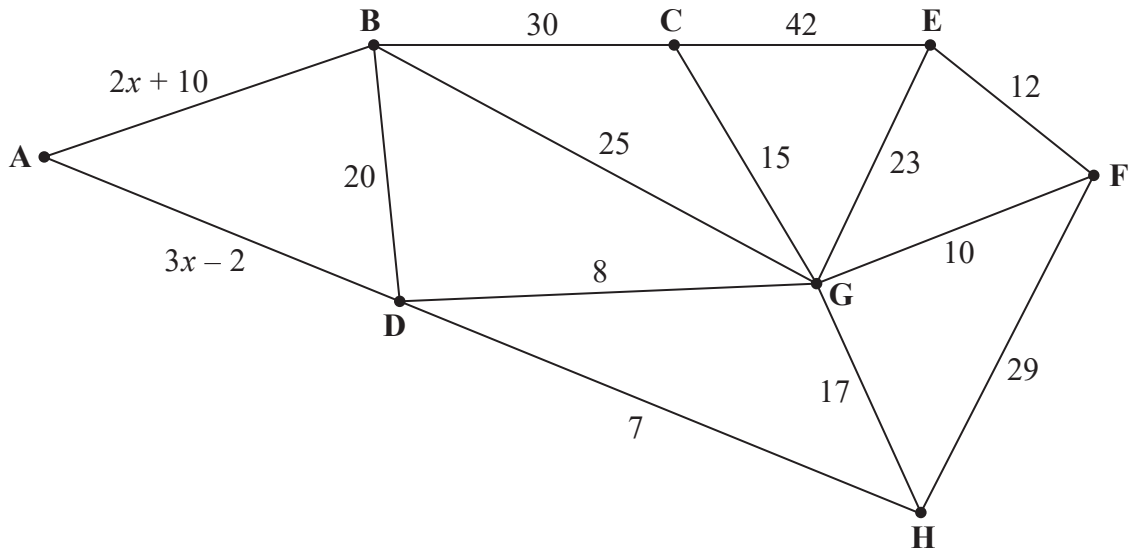



**Figure 2**

[*The weight of the network is* $5x + 246$]

(a) Each arc contributes 1 to the order (valency) of each of the two vertices it joins, so the sum of the orders of all the vertices = 2 × (number of arcs), which is even.
The vertices of even order contribute an even total, so the vertices of odd order must also have an even sum, which needs an even number of odd vertices. Hence a graph cannot have an odd number of vertices of odd valency.

(b) AD is chosen before AB, so $3x - 2 < 2x + 10 \Rightarrow x < 12$
BD is chosen to join B (not AB), so $20 < 2x + 10 \Rightarrow x > 5$
(strict inequalities as the order of selection is unique)
$\therefore 5 < x < 12$

(c) Orders: A 2, B 4, C 3, D 4, E 3, F 3, G 6, H 3
Odd vertices: C, E, F and H
CE + FH = 37 (CGFE) + 25 (FGDH) = 62
CF + EH = 25 (CGF) + 37 (EFGDH) = 62
CH + EF = 30 (CGDH) + 12 (EF) = 42 ← minimum
Repeat the arcs CG, GD, DH and EF, adding 42 to the weight.
Length of route = $5x + 246 + 42 = 318$
$5x + 288 = 318$
$5x = 30 \Rightarrow x = 6$

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**Question 3 continued**

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**Question 3 continued**

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**Question 3 continued**

**(Total for Question 3 is 11 marks)**

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**4.**

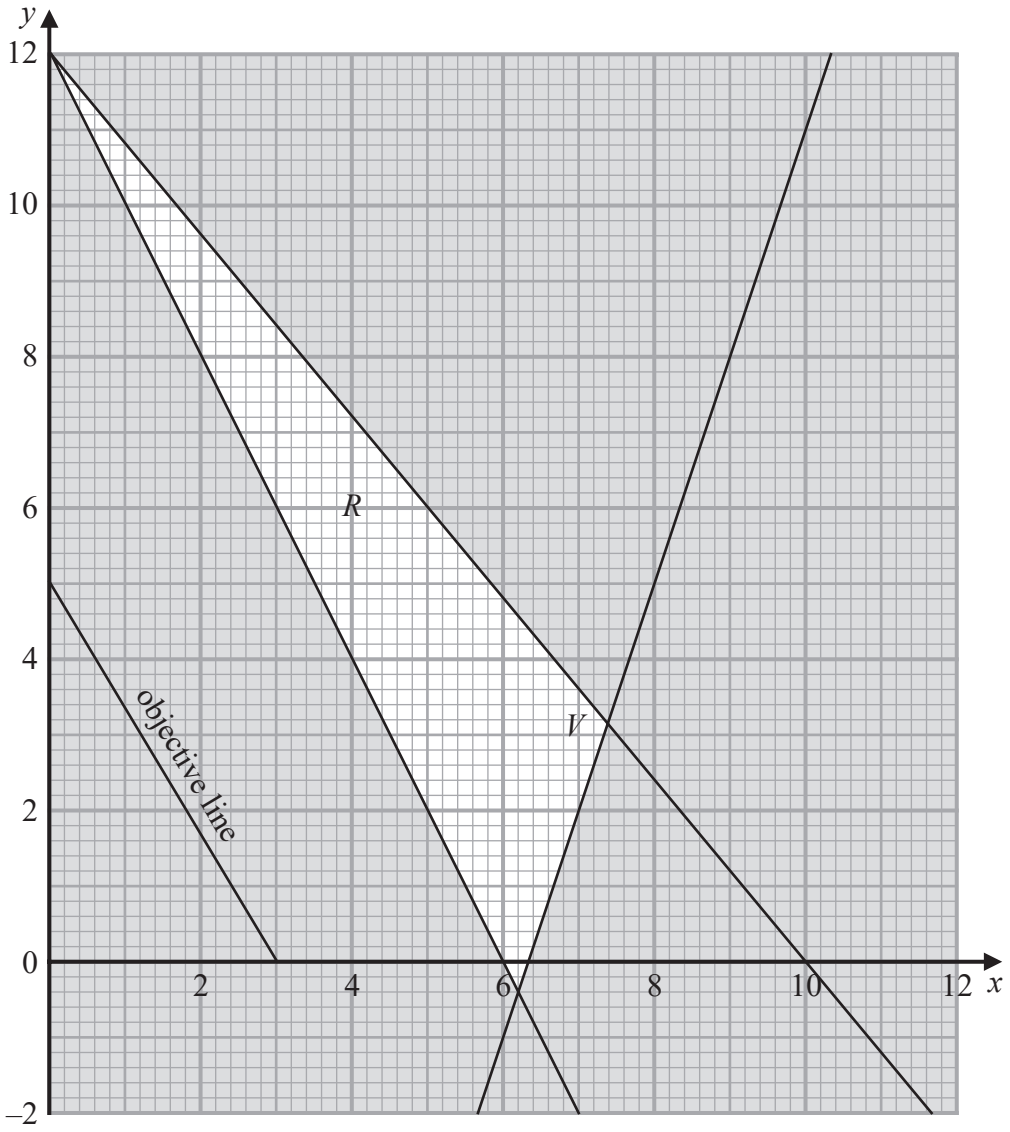



**Figure 3**

Line through (0, 12) and (6, 0): $y = 12 - 2x \Rightarrow 2x + y = 12$

Line through (0, 12) and (10, 0): $y = 12 - 6x/5 \Rightarrow 6x + 5y = 60$

Line through (7, 2) and (9, 8): gradient $= 6 \div 2 = 3$

$y - 2 = 3(x - 7) \Rightarrow y = 3x - 19 \Rightarrow 3x - y = 19$

R is the unshaded region, so the constraints are:

$2x + y \geqslant 12, \quad 6x + 5y \leqslant 60, \quad 3x - y \leqslant 19$

V: solve $6x + 5y = 60$ and $3x - y = 19$ simultaneously

$y = 3x - 19 \Rightarrow 6x + 5(3x - 19) = 60$

$21x - 95 = 60 \Rightarrow x = 155/21, \quad y = 3(155/21) - 19 = 22/7$

$V = (155/21, 22/7)$

Objective line through (0, 5) and (3, 0): $5x + 3y = 15$

so objective is $P = k(5x + 3y)$

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**Question 4 continued**

At V: $5(155/21) + 3(22/7) = 775/21 + 198/21 = 973/21$

$k \times 973/21 = 556 \Rightarrow k = 556 \times 21 \div 973 = 12$

$P = 12(5x + 3y) = 60x + 36y$

V is the last vertex reached as the objective line moves away from the origin, so the objective is to maximise.

Maximise $P = 60x + 36y$

subject to: $2x + y \geq 12$

$6x + 5y \leq 60$

$3x - y \leq 19$

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**Question 4 continued**

**(Total for Question 4 is 9 marks)**

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