Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 14 May 2020

Afternoon | Paper Reference **8FM0/26**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 3 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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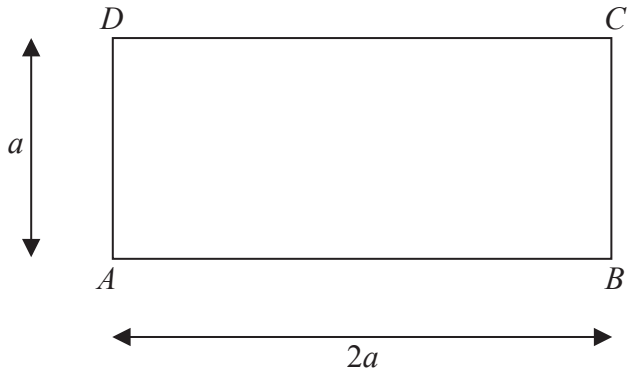


Figure 1

Figure 1 shows a uniform rectangular lamina $ABCD$ with $AB = 2a$ and $AD = a$ The mass of the lamina is $6m$.

A particle of mass $2m$ is attached to the lamina at A, a particle of mass m is attached to the lamina at B and a particle of mass $3m$ is attached to the lamina at D, to form a loaded lamina L of total mass $12m$.

(a) Write down the distance of the centre of mass of L from AB. You must give a reason for your answer.

(2)

(b) Show that the distance of the centre of mass of L from AD is $\frac{2a}{3}$

(3)

A particle of mass km is now also attached to L at D to form a new loaded lamina N.

(c) Show that the distance of the centre of mass of N from AB is $\frac{(k+6)a}{(k+12)}$

(4)

When N is freely suspended from A and is hanging in equilibrium, the side AB makes an angle α with the vertical, where $\tan\alpha = \frac{3}{2}$

(d) Find the value of k.

(6)

(a) Distance from AB = $\frac{1}{2}a$

Masses on AB: 2m + m = 3m; mass on DC: 3m (at D), and the lamina is uniform. The loaded lamina has a mass distribution which is symmetrical about the perpendicular bisector of AD, so G lies on it.

(b) Moments about AD:

$6m \times a + m \times 2a + 2m \times 0 + 3m \times 0 = 12m\,\bar{x}$

$8ma = 12m\,\bar{x}$

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Question 1 continued

$\bar{x} = \frac{8a}{12} = \frac{2a}{3}$ as required

(c) Moments about AB (total mass of N is $(12 + k)m$):

$12m \times \frac{1}{2}a + km \times a = (k + 12)m\,\bar{y}$

$6ma + kma = (k + 12)m\,\bar{y}$

$\bar{y} = \frac{(k + 6)a}{(k + 12)}$ as required

(d) Moments about AD for N:

$12m \times \frac{2a}{3} + km \times 0 = (k + 12)m\,\bar{x}_1$

$\bar{x}_1 = \frac{8a}{(k + 12)}$

Suspended from A, so AG is vertical and AB makes angle α with AG:

$\tan\alpha = \frac{\bar{y}}{\bar{x}_1} = \frac{(k + 6)a}{(k + 12)} \div \frac{8a}{(k + 12)} = \frac{k + 6}{8}$

$\frac{k + 6}{8} = \frac{3}{2}$

$k + 6 = 12$

$k = 6$

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Question 1 continued

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Question 1 continued

(Total for Question 1 is 15 marks)

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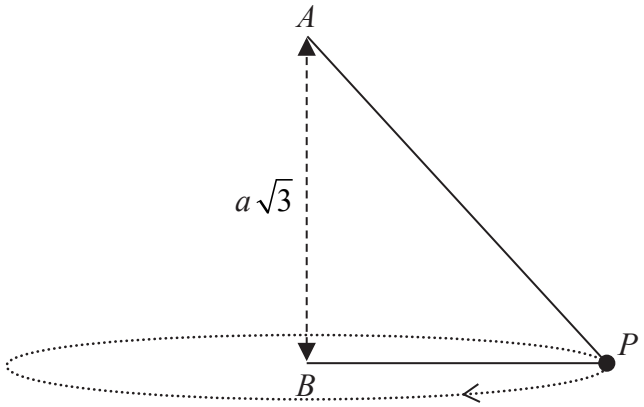


Figure 2

One end of a string of length $3a$ is attached to a point A and the other end is attached to a point B on a smooth horizontal table. The point B is vertically below A with $AB = a\sqrt{3}$ A small smooth bead, P, of mass m is threaded on to the string. The bead P moves on the table in a horizontal circle, with centre B, with constant speed U. Both portions, AP and BP, of the string are taut, as shown in Figure 2.

The string is modelled as being light and inextensible and the bead is modelled as a particle.

(a) Show that $AP = 2a$

(2)

(b) Find, in terms of m, U and a, the tension in the string.

(4)

(c) Show that $U^2 < ag\sqrt{3}$

(5)

(d) Describe what would happen if $U^2 > ag\sqrt{3}$

(1)

(e) State briefly how the tension in the string would be affected if the string were not modelled as being light.

(1)

(a) Let AP = x, then BP = 3a − x. Angle ABP = 90°, so by Pythagoras:

$(a\sqrt{3})^2 + (3a - x)^2 = x^2$

$3a^2 + 9a^2 - 6ax + x^2 = x^2$

$6ax = 12a^2 \Rightarrow x = 2a \quad \therefore AP = 2a$ as required

(b) $BP = 3a - 2a = a$, $\cos \angle APB = \frac{a}{2a} = \frac{1}{2} \Rightarrow \angle APB = 60°$

Same string, smooth bead, so tension T in both portions.

Horizontally, towards B (radius a):

$T + T\cos 60° = \frac{mU^2}{a}$

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Question 2 continued

$T + \frac{1}{2}T = \frac{mU^2}{a} \Rightarrow \frac{3}{2}T = \frac{mU^2}{a}$

$T = \frac{2mU^2}{3a}$

(c) Resolving vertically (R = normal reaction from the table):

$R + T \sin 60° = mg \Rightarrow R + \frac{\sqrt{3}}{2}T = mg$

$R = mg - \frac{2mU^2}{3a} \times \frac{\sqrt{3}}{2} = mg - \frac{\sqrt{3}mU^2}{3a}$

Bead stays on the table, so $R > 0$:

$mg - \frac{2mU^2\sqrt{3}}{3a \times 2} > 0 \Rightarrow \frac{\sqrt{3}U^2}{3a} < g$

$U^2 < \frac{3ag}{\sqrt{3}} = ag\sqrt{3}$ $\therefore U^2 < ag\sqrt{3}$ as required

(d) R would be negative, so the bead would lift off (leave) the table.

(e) The tension would vary along the string (it would not be the same throughout the string).

Question 2 continued

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Question 2 continued

(Total for Question 2 is 13 marks)

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3. At time $t = 0$, a toy electric car is at rest at a fixed point O. The car then moves in a horizontal straight line so that at time t seconds ($t > 0$) after leaving O, the velocity of the car is $v\,\text{m s}^{-1}$ and the acceleration of the car is modelled as $(p + qv)\,\text{m s}^{-2}$, where p and q are constants.

When $t = 0$, the acceleration of the car is $3\,\text{m s}^{-2}$

When $t = T$, the acceleration of the car is $\frac{1}{2}\,\text{m s}^{-2}$ and $v = 4$

(a) Show that

$$8\frac{\mathrm{d}v}{\mathrm{d}t} = (24 - 5v)$$

(6)

(b) Find the exact value of T, simplifying your answer.

(6)

(a) Acceleration $= p + qv$

$t = 0$: $v = 0$ and acceleration $= 3$ $\Rightarrow$ $p + 0 = 3$ $\Rightarrow$ $p = 3$

$t = T$: $v = 4$ and acceleration $= \frac{1}{2}$ $\Rightarrow$ $3 + 4q = \frac{1}{2}$

$4q = -\frac{5}{2}$ $\Rightarrow$ $q = -\frac{5}{8}$

Acceleration $= \frac{dv}{dt} = 3 - \frac{5}{8}v$

$\times 8$: $8\frac{dv}{dt} = (24 - 5v)$ as required

(b) Separating the variables:

$$\int \frac{8}{24 - 5v}\,dv = \int dt$$

$$-\frac{8}{5}\ln(24 - 5v) = t + C$$

$t = 0$, $v = 0$: $C = -\frac{8}{5}\ln 24$

$$t = \frac{8}{5}\ln 24 - \frac{8}{5}\ln(24 - 5v) = \frac{8}{5}\ln\left(\frac{24}{24 - 5v}\right)$$

$v = 4$, $t = T$: $T = \frac{8}{5}\ln\left(\frac{24}{24 - 20}\right) = \frac{8}{5}\ln\left(\frac{24}{4}\right)$

$$T = \frac{8}{5}\ln 6$$

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Question 3 continued

Question 3 continued

(Total for Question 3 is 12 marks)

TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS

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