Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 14 May 2020

Afternoon | Paper Reference **8FM0/25**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
25: Further Mechanics 1
(Part of options C, E, H and J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P61868A

©2020 Pearson Education Ltd.

1/1/1/1/1/

P 6 1 8 6 8 A 0 1 1 6

Pearson

1. Two particles P and Q have masses m and $4m$ respectively. The particles are at rest on a smooth horizontal plane. Particle P is given a horizontal impulse, of magnitude I, in the direction PQ. Particle P then collides directly with Q. Immediately after this collision, P is at rest and Q has speed w. The coefficient of restitution between the particles is e.

(a) Find I in terms of m and w.

(2)

(b) Show that $e = \frac{1}{4}$

(1)

(c) Find, in terms of m and w, the total kinetic energy lost in the collision between P and Q.

(2)

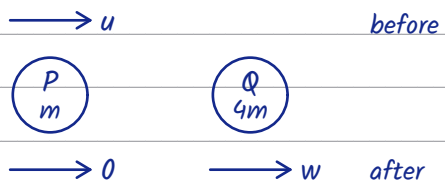
(a) Let P have speed u after the impulse:

$I = mu \Rightarrow u = I/m$

CLM for the collision:

$m \times \frac{I}{m} = m(0) + 4mw$

$\therefore I = 4mw$



(b) From (a), speed of P before collision $u = 4w$

NLR: speed of separation = e × speed of approach

$w - 0 = e(4w - 0)$

$\therefore e = \frac{w}{4w} = \frac{1}{4}$ as required

(c) KE before $= \frac{1}{2}m(4w)^2 = 8mw^2$

KE after $= \frac{1}{2}(4m)w^2 + 0 = 2mw^2$

KE lost $= \frac{1}{2}m(4w)^2 - \frac{1}{2}(4m)w^2$

$= 8mw^2 - 2mw^2$

KE lost $= 6mw^2$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

(Total for Question 1 is 5 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

2. A car of mass 1000 kg moves along a straight horizontal road.

In all circumstances, when the speed of the car is $v\,\text{m s}^{-1}$, the resistance to the motion of the car is modelled as a force of magnitude cv^2 N, where c is a constant.

The maximum power that can be developed by the engine of the car is 50 kW.

At the instant when the speed of the car is $72\,\text{km h}^{-1}$ and the engine is working at its maximum power, the acceleration of the car is $2.25\,\text{m s}^{-2}$

(a) Convert $72\,\text{km h}^{-1}$ into m s^{-1}

(1)

(b) Find the acceleration of the car at the instant when the speed of the car is $144\,\text{km h}^{-1}$ and the engine is working at its maximum power.

(7)

The maximum speed of the car when the engine is working at its maximum power is $V\,\text{km h}^{-1}$.

(c) Find, to the nearest whole number, the value of V.

(4)

(a) $72\text{ km h}^{-1} = 72 \times 1000 \div 3600 = 20\text{ m s}^{-1}$

(b) At 20 m s^{-1}: driving force $F = P/v = 50000/20 = 2500$ N

N2L: $2500 - c(20)^2 = 1000 \times 2.25$

$2500 - 400c = 2250$

$400c = 250 \Rightarrow c = 5/8\ (= 0.625)$

$144\text{ km h}^{-1} = 144 \times 1000 \div 3600 = 40\text{ m s}^{-1}$

Driving force $F = 50000/40 = 1250$ N

N2L: $1250 - (5/8)(40)^2 = 1000a$

$1250 - 1000 = 1000a$

$a = 250/1000 = 0.25\text{ m s}^{-2}$

(c) At maximum speed $W\text{ m s}^{-1}$, acceleration $= 0$:

$$\frac{50000}{W} - \frac{5}{8}W^2 = 0$$

$W^3 = 50000 \times 8/5 = 80000$

$W = \sqrt[3]{80000} = 43.0886\ldots\text{ m s}^{-1}$

$V = 43.0886\ldots \times 3.6 = 155.119\ldots\text{ km h}^{-1}$

$V = 155$ (nearest whole number)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

(Total for Question 2 is 12 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

3. Three particles A, B and C are at rest on a smooth horizontal plane. The particles lie along a straight line with B between A and C.

Particle B has mass $4m$ and particle C has mass km, where k is a positive constant. Particle B is projected with speed u along the plane towards C and they collide directly.

The coefficient of restitution between B and C is $\frac{1}{4}$

(a) Find the range of values of k for which there would be no further collisions.

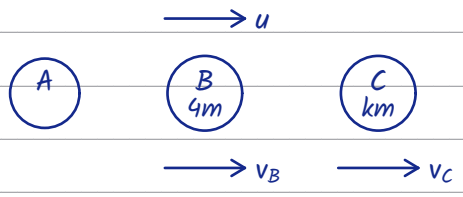
(8)

The magnitude of the impulse on B in the collision between B and C is $3mu$

(b) Find the value of k.

(4)

(a) Velocities after: v_B, v_C (towards C +ve)



CLM: $4mu = 4mv_B + kmv_C$

$\Rightarrow 4u = 4v_B + kv_C$ ...(1)

NLR: $v_C - v_B = \frac{1}{4}u$ ...(2)

From (2): $v_C = v_B + \frac{1}{4}u$

Sub in (1): $4u = 4v_B + kv_B + \frac{1}{4}ku$

$16u = 16v_B + 4kv_B + ku$

$4(k + 4)v_B = u(16 - k)$

$v_B = \frac{u(16 - k)}{4(k + 4)}$ $\quad v_C = \frac{5u}{k + 4}$

$v_C > v_B$, so B and C will not collide again.

A is at rest behind B, so no further collisions

if B does not move back towards A, i.e. $v_B \geq 0$:

$\frac{u(16 - k)}{4(k + 4)} \geq 0$ and $k + 4 > 0 \Rightarrow 16 - k \geq 0$

$\therefore 0 < k \leq 16$

(b) Impulse on B = change in momentum of B:

$-3mu = 4m(v_B - u)$

$\Rightarrow v_B - u = -\frac{3}{4}u \Rightarrow v_B = \frac{1}{4}u$

$\frac{u(16 - k)}{4(k + 4)} = \frac{u}{4} \Rightarrow 16 - k = k + 4$

$2k = 12$

$k = 6$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 12 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

4. A small ball, of mass m, is thrown vertically upwards with speed $\sqrt{8gH}$ from a point O on a smooth horizontal floor. The ball moves towards a smooth horizontal ceiling that is a vertical distance H above O. The coefficient of restitution between the ball and the ceiling is $\frac{1}{2}$

In a model of the motion of the ball, it is assumed that the ball, as it moves up or down, is subject to air resistance of constant magnitude $\frac{1}{2}mg$.

Using this model,

(a) use the work-energy principle to find, in terms of g and H, the speed of the ball immediately before it strikes the ceiling,

(5)

(b) find, in terms of g and H, the speed of the ball immediately before it strikes the floor at O for the first time.

(5)

In a simplified model of the motion of the ball, it is assumed that the ball, as it moves up or down, is subject to no air resistance.

Using this simplified model,

(c) explain, without any detailed calculation, why the speed of the ball, immediately before it strikes the floor at O for the first time, would still be less than $\sqrt{8gH}$

(1)

(a) Let v = speed just before striking the ceiling.

Work done against resistance = $\frac{1}{2}mg \times H = \frac{1}{2}mgH$

Gain in PE = mgH

KE lost = $\frac{1}{2}m(\sqrt{8gH})^2 - \frac{1}{2}mv^2 = \frac{1}{2}m(8gH - v^2)$

Work-energy: KE lost = PE gained + work done

$\frac{1}{2}m(8gH - v^2) = mgH + \frac{1}{2}mgH$

$4gH - \frac{1}{2}v^2 = (3/2)gH$

$v^2 = 8gH - 3gH = 5gH$

$v = \sqrt{5gH}$

(b) NLR at ceiling: rebound speed = $\frac{1}{2}\sqrt{5gH}$

Moving down a distance H, let v_1 = speed at floor.

Work-energy: KE gained = PE lost − work done

$\frac{1}{2}mv_1^2 - \frac{1}{2}m(\frac{1}{2}\sqrt{5gH})^2 = mgH - \frac{1}{2}mgH$

$\frac{1}{2}v_1^2 - (5/8)gH = \frac{1}{2}gH$

$v_1^2 = (5/4)gH + gH = (9/4)gH$

$v_1 = \frac{3}{2}\sqrt{gH}$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

(c) Since $e = \frac{1}{2} < 1$, the impact with the ceiling is not perfectly elastic, so the ball loses kinetic energy in its collision with the ceiling. With no other energy losses, it returns to O with speed less than $\sqrt{8gH}$.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 11 marks)

TOTAL FOR FURTHER MECHANICS 1 IS 40 MARKS

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA