Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 14 May 2020

Afternoon | Paper Reference **8FM0/21**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
21: Further Pure Mathematics 1
(Part of options A, B, C and D)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} = 2y^2 - x - 1$$

where $\frac{dy}{dx} = 3$ and $y = 0$ at $x = 0$

Use the approximations

$$\left(\frac{d^2y}{dx^2}\right)_n \approx \frac{(y_{n+1} - 2y_n + y_{n-1})}{h^2} \quad \text{and} \quad \left(\frac{dy}{dx}\right)_n \approx \frac{(y_{n+1} - y_{n-1})}{2h}$$

with $h = 0.1$ to find an estimate for the value of y at $x = 0.2$

(7)

$\left(\frac{d^2y}{dx^2}\right)_0 = 2y_0^2 - x_0 - 1 = 2(0)^2 - 0 - 1 = -1$

$\left(\frac{dy}{dx}\right)_0 = 3 \Rightarrow \frac{y_1 - y_{-1}}{2(0.1)} = 3 \Rightarrow y_1 - y_{-1} = 0.6$

$\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{(0.1)^2} \Rightarrow \frac{y_1 - 2(0) + y_{-1}}{0.01} = -1$

$\Rightarrow y_1 + y_{-1} = -0.01$

Adding: $2y_1 = 0.6 - 0.01 = 0.59 \Rightarrow y_1 = 0.295$ (y at x = 0.1)

$\left(\frac{d^2y}{dx^2}\right)_1 = 2y_1^2 - x_1 - 1 = 2(0.295)^2 - 0.1 - 1 = -0.92595$

$\left(\frac{d^2y}{dx^2}\right)_1 \approx \frac{y_2 - 2y_1 + y_0}{(0.1)^2} \Rightarrow \frac{y_2 - 2(0.295) + 0}{0.01} = -0.92595$

$y_2 = 2(0.295) - 0 - 0.01(0.92595) = 0.59 - 0.0092595$

$y_2 = 0.5807405...$

At $x = 0.2$, $y \approx 0.581$ (3 s.f.)

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Question 1 continued

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Question 1 continued

(Total for Question 1 is 7 marks)

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2. Use algebra to determine the values of x for which

$$\frac{x+1}{2x^2+5x-3} > \frac{x}{4x^2-1}$$

(5)

$\frac{x+1}{(2x-1)(x+3)} - \frac{x}{(2x-1)(2x+1)} > 0$

$\frac{(x+1)(2x+1) - x(x+3)}{(2x-1)(2x+1)(x+3)} > 0$

$\frac{2x^2+3x+1-x^2-3x}{(2x-1)(2x+1)(x+3)} > 0 \Rightarrow \frac{x^2+1}{(2x-1)(2x+1)(x+3)} > 0$

Multiply by $(2x-1)^2(2x+1)^2(x+3)^2 > 0$:

$(x^2+1)(2x-1)(2x+1)(x+3) > 0$

$x^2+1 > 0$ for all real x, so it has no critical values.

Critical values: $x = -3$, $x = -\frac{1}{2}$, $x = \frac{1}{2}$

Sign: $x < -3$ (−), $-3 < x < -\frac{1}{2}$ (+), $-\frac{1}{2} < x < \frac{1}{2}$ (−), $x > \frac{1}{2}$ (+)

$\{x \in \mathbb{R} : -3 < x < -\frac{1}{2}\} \cup \{x \in \mathbb{R} : x > \frac{1}{2}\}$

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Question 2 continued

(Total for Question 2 is 5 marks)

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3. (i) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to prove that

$$\cot x + \tan\left(\frac{x}{2}\right) = \operatorname{cosec} x \qquad x \neq n\pi,\ n \in \mathbb{Z}$$

(2)

(ii)

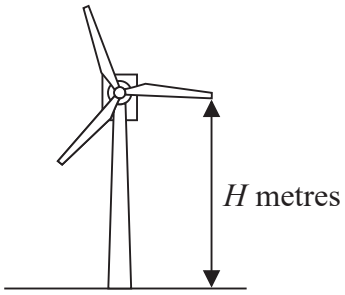


Figure 1

An engineer models the vertical height above the ground of the tip of one blade of a wind turbine, shown in Figure 1. The ground is assumed to be horizontal.

The vertical height of the tip of the blade above the ground, H metres, at time x seconds after the wind turbine has reached its constant operating speed, is modelled by the equation

$$H = 90 - 30\cos(120x)° - 40\sin(120x)° \qquad \text{(I)}$$

(a) Show that $H = 60$ when $x = 0$

(1)

Using the substitution $t = \tan(60x)°$

(b) show that equation (I) can be rewritten as

$$H = \frac{120t^2 - 80t + 60}{1 + t^2}$$

(3)

(c) Hence find, according to the model, the value of x when the tip of the blade is 100 m above the ground for the first time after the wind turbine has reached its constant operating speed.

(5)

(i) $t = \tan\left(\frac{x}{2}\right)$: $\cot x = \frac{1}{\tan x} = \frac{1 - t^2}{2t}$, $\sin x = \frac{2t}{1 + t^2}$

$LHS = \cot x + \tan\left(\frac{x}{2}\right) = \frac{1 - t^2}{2t} + t = \frac{1 - t^2 + 2t^2}{2t} = \frac{1 + t^2}{2t}$

$= \frac{1}{\sin x} = \operatorname{cosec} x = RHS$ as required

(ii)(a) $x = 0$: $H = 90 - 30\cos 0° - 40\sin 0° = 90 - 30(1) - 0$

$H = 60$ as required

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Question 3 continued

(b) $t = \tan(60x)°$: $\cos(120x)° = \dfrac{1 - t^2}{1 + t^2}$, $\sin(120x)° = \dfrac{2t}{1 + t^2}$

$$H = 90 - 30\left(\frac{1 - t^2}{1 + t^2}\right) - 40\left(\frac{2t}{1 + t^2}\right)$$

$$H = \frac{90(1 + t^2) - 30(1 - t^2) - 80t}{1 + t^2} = \frac{90 + 90t^2 - 30 + 30t^2 - 80t}{1 + t^2}$$

$$H = \frac{120t^2 - 80t + 60}{1 + t^2}$$ as required

(c) $\dfrac{120t^2 - 80t + 60}{1 + t^2} = 100 \Rightarrow 120t^2 - 80t + 60 = 100 + 100t^2$

$20t^2 - 80t - 40 = 0 \Rightarrow t^2 - 4t - 2 = 0$

$$t = \frac{4 \pm \sqrt{(16 + 8)}}{2} = 2 \pm \sqrt{6}$$

$t = 2 + \sqrt{6}$: $60x = \tan^{-1}(2 + \sqrt{6}) = 77.3196...°$

$t = 2 - \sqrt{6}$: $60x = \tan^{-1}(2 - \sqrt{6}) + 180° = 155.79...°$ (later time)

First time: $60x = 77.3196... \Rightarrow x = 1.28866...$

$x = 1.29$ (seconds)

Question 3 continued

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Question 3 continued

(Total for Question 3 is 11 marks)

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4.

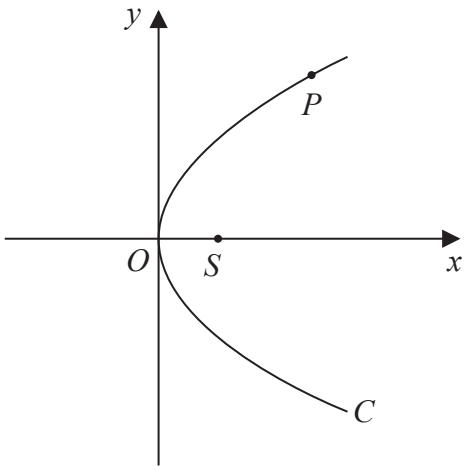


Figure 2

Figure 2 shows a sketch of the parabola C with equation $y^2 = 4ax$, where a is a positive constant. The point S is the focus of C and the point $P(ap^2, 2ap)$ lies on C where $p > 0$

(a) Write down the coordinates of S.

(1)

(b) Write down the length of SP in terms of a and p.

(1)

The point $Q(aq^2, 2aq)$, where $p \neq q$, also lies on C.
The point M is the midpoint of PQ.

Given that $pq = -1$

(c) prove that, as P varies, the locus of M has equation

$$y^2 = 2a(x - a)$$

(5)

(a) $S(a, 0)$

(b) SP = distance from P to directrix $x = -a$

$SP = ap^2 + a$

(c) $M\left(\frac{ap^2 + aq^2}{2}, \frac{2ap + 2aq}{2}\right) = \left(\frac{a(p^2 + q^2)}{2}, a(p + q)\right)$

$y^2 = a^2(p + q)^2 = a^2(p^2 + 2pq + q^2)$

$pq = -1$: $y^2 = a^2(p^2 + q^2 - 2)$

$2a(x - a) = 2a\left(\frac{1}{2}ap^2 + \frac{1}{2}aq^2 - a\right) = a^2(p^2 + q^2) - 2a^2$

$= a^2(p^2 + q^2 - 2)$

$\therefore$ $y^2 = 2a(x - a)$, so the locus of M has equation $y^2 = 2a(x - a)$

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Question 4 continued

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Question 4 continued

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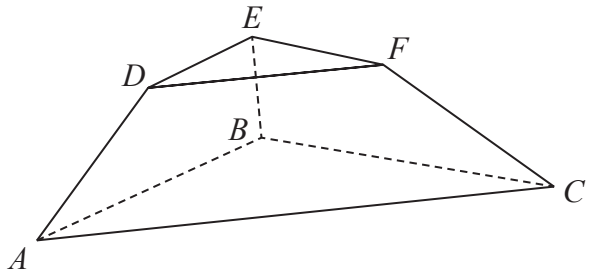


Figure 3

Figure 3 shows a solid display stand with parallel triangular faces *ABC* and *DEF*.
Triangle *DEF* is similar to triangle *ABC*.

With respect to a fixed origin *O*,
the points *A*, *B* and *C* have coordinates (3, –3, 1), (–5, 3, 3) and (1, 7, 5) respectively and
the points *D*, *E* and *F* have coordinates (2, –1, 8), (–2, 2, 9) and (1, 4, 10) respectively.
The units are in centimetres.

(a) Show that the area of the triangular face *DEF* is $\frac{1}{2}\sqrt{339}$ cm^2

(3)

(b) Find, in cm^3, the exact volume of the display stand.

(7)

(a) $\overrightarrow{DE} = \begin{pmatrix} -2-2 \\ 2-(-1) \\ 9-8 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}$, $\overrightarrow{DF} = \begin{pmatrix} 1-2 \\ 4-(-1) \\ 10-8 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix}$

$\overrightarrow{DE} \times \overrightarrow{DF} = \begin{pmatrix} (3)(2)-(1)(5) \\ (1)(-1)-(-4)(2) \\ (-4)(5)-(3)(-1) \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ -17 \end{pmatrix}$

Area $DEF = \frac{1}{2}|\overrightarrow{DE} \times \overrightarrow{DF}| = \frac{1}{2}\sqrt{(1^2 + 7^2 + (-17)^2)} = \frac{1}{2}\sqrt{339}$ cm^2

(b) Extend AD and CF to meet at the apex T of a tetrahedron $TABC$.

$\overrightarrow{AD} = \begin{pmatrix} -1 \\ 2 \\ 7 \end{pmatrix}$, $\overrightarrow{CF} = \begin{pmatrix} 0 \\ -3 \\ 5 \end{pmatrix}$: $\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 5 \end{pmatrix} + \mu\begin{pmatrix} 0 \\ -3 \\ 5 \end{pmatrix}$

x: $3 - \lambda = 1 \Rightarrow \lambda = 2$; y: $-3 + 2\lambda = 7 - 3\mu \Rightarrow \mu = 2$; z: $1 + 14 = 5 + 10$ ✓

$T(1, 1, 15)$ (so $AT = 2AD$: linear scale factor 2)

$\overrightarrow{AT} = \begin{pmatrix} -2 \\ 4 \\ 14 \end{pmatrix}$, $\overrightarrow{BT} = \begin{pmatrix} 6 \\ -2 \\ 12 \end{pmatrix}$, $\overrightarrow{CT} = \begin{pmatrix} 0 \\ -6 \\ 10 \end{pmatrix}$, $\overrightarrow{BT} \times \overrightarrow{CT} = \begin{pmatrix} (-2)(10)-(12)(-6) \\ (12)(0)-(6)(10) \\ (6)(-6)-(-2)(0) \end{pmatrix} = \begin{pmatrix} 52 \\ -60 \\ -36 \end{pmatrix}$

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Question 5 continued

$$\text{Vol } TABC = \frac{1}{6}|AT \cdot (BT \times CT)| = \frac{1}{6}|-2(52) + 4(-60) + 14(-36)| = \frac{848}{6} = \frac{424}{3}$$

$$DT = \begin{vmatrix} -1 \\ 2 \\ 7 \end{vmatrix}, \quad ET = \begin{vmatrix} 3 \\ -1 \\ 6 \end{vmatrix}, \quad FT = \begin{vmatrix} 0 \\ -3 \\ 5 \end{vmatrix}, \quad ET \times FT = \begin{vmatrix} (-1)(5) - (6)(-3) \\ (6)(0) - (3)(5) \\ (3)(-3) - (-1)(0) \end{vmatrix} = \begin{vmatrix} 13 \\ -15 \\ -9 \end{vmatrix}$$

$$\text{Vol } TDEF = \frac{1}{6}|DT \cdot (ET \times FT)| = \frac{1}{6}|-1(13) + 2(-15) + 7(-9)| = \frac{106}{6} = \frac{53}{3}$$

$$\text{Volume of stand} = \frac{424}{3} - \frac{53}{3} = \frac{371}{3} \text{ cm}^3$$

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Question 5 continued

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Question 5 continued

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TOTAL FOR FURTHER PURE MATHEMATICS 1 IS 40 MARKS

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