Pearson Edexcel Level 3 GCE

Friday 16 May 2025

Afternoon

Paper reference **8FM0/28**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
28: Decision Mathematics 2
(Part of option K only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator,
D2 Answer Book (enclosed)

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.
- Do not return the question paper with the D2 Answer Book.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. Five workers, A, B, C, D and E, are each to be assigned to one of five tasks, J, K, L, M and N. Each task must be assigned to exactly one worker and each worker must do exactly one task.

Worker C cannot do task L and worker D cannot do task K.

The profit, in pounds, that each worker will make while assigned to each task is shown in the table below.

	J	K	L	M	N
A	38	33	40	35	32
B	26	24	27	25	23
C	33	29	–	30	27
D	36	–	41	37	33
E	32	27	31	29	25

The Hungarian algorithm is to be used to find the maximum total profit that can be earned by the five workers.

(a) Explain how the contents of the table must be modified to allow the algorithm to be used.

(2)

(b) Reducing rows first, use the Hungarian algorithm to obtain the maximum total profit. You should explain how any initial row and column reductions are made and also how you determine if the table is optimal at each stage.

(7)

(Total for Question 1 is 9 marks)

2.

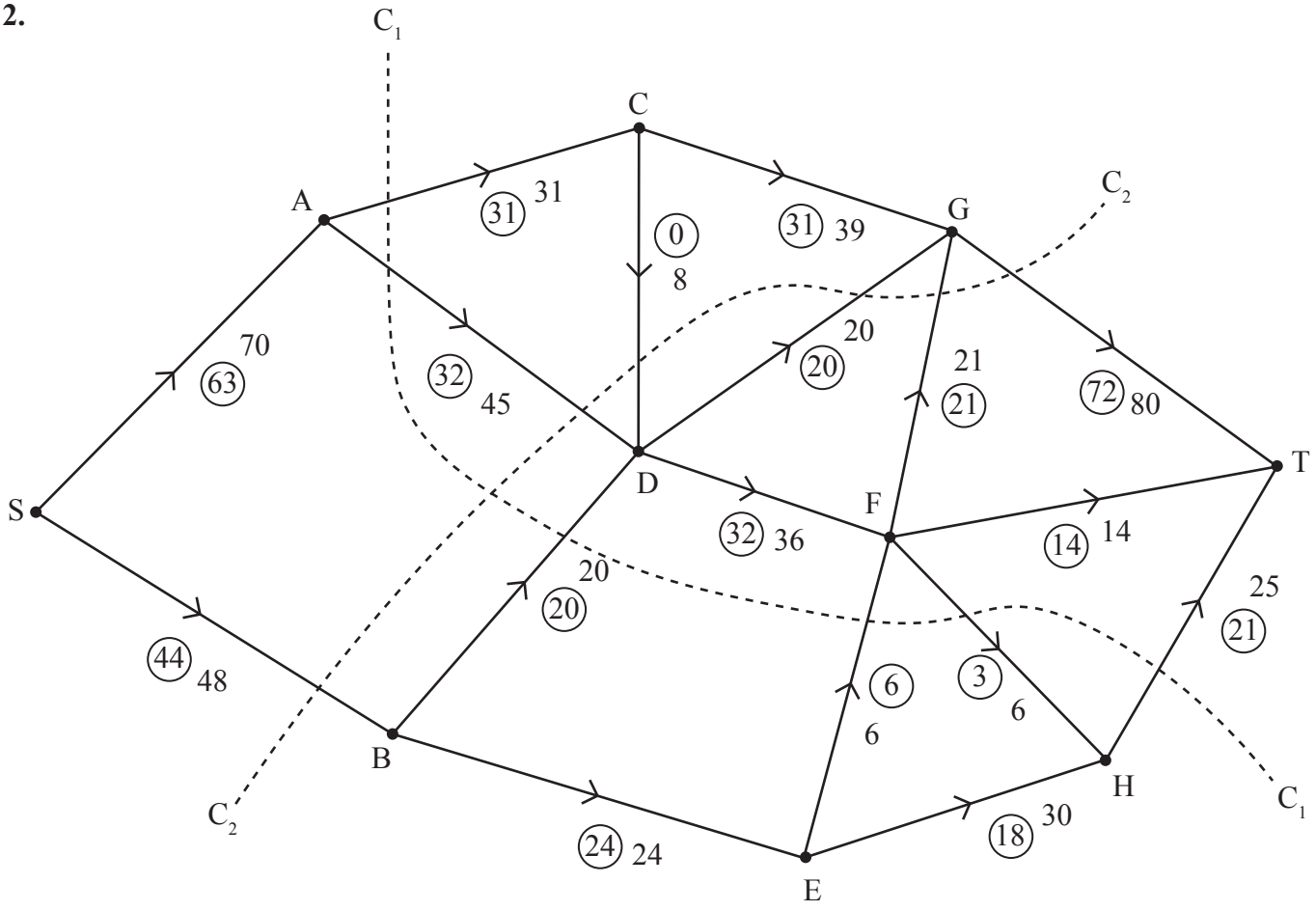


Figure 1

Figure 1 shows a capacitated, directed network of pipes. The number on each arc represents the capacity of the corresponding pipe. The numbers in circles represent a feasible flow from S to T.

(a) State the two conditions satisfied by a feasible flow.

(2)

(b) List the seven saturated arcs in Figure 1.

(1)

(c) Find the capacity of

(i) cut C_1

(ii) cut C_2

(2)

(d) Write down a flow-augmenting route that increases the flow by four units.

(1)

(e) Use the answer to part (d) to draw the resulting flow pattern on Diagram 1 in the answer book.

(2)

(f) Prove that the answer to part (e) is a maximum flow.

(3)

(Total for Question 2 is 11 marks)

3. Layla and Mohsin play a zero-sum game represented by the following pay-off matrix for Layla.

	Mohsin plays X	Mohsin plays Y	Mohsin plays Z
Layla plays P	1	−2	2
Layla plays Q	−4	3	−5
Layla plays R	−1	1	−3

(a) (i) Find the play-safe strategies for each player.

(ii) State, giving a reason, whether there is a stable solution to this game.

(3)

(b) Option R is now removed from Layla's choices.

(i) For each of Mohsin's three options, find the expected pay-off to Layla when she plays options P and Q equally often.

(1)

(ii) Use a graphical method to determine Layla's optimal mixed strategy. You should define any variables you use.

(7)

(Total for Question 3 is 11 marks)

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4. A sequence $\{u_n\}$, where $n \geqslant 1$, satisfies the recurrence relation

$$4u_{n+1} + 2u_n = 3n - 5$$

Given that $u_1 = -2$

(a) solve the recurrence relation, giving u_n in terms of n

(6)

(b) hence determine the number of negative terms in the sequence $\{u_n\}$. You must justify your answer.

(3)

(Total for Question 4 is 9 marks)

TOTAL FOR DECISION MATHEMATICS 2 IS 40 MARKS

Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

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Total Marks

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1. (a) Subtract every entry from a constant, e.g. 50, so the maximisation becomes a minimisation. Then replace the two blank cells (CL and DK) by a large number, e.g. 100 (greater than 41).

	J	K	L	M	N
A	38	33	40	35	32
B	26	24	27	25	23
C	33	29	–	30	27
D	36	–	41	37	33
E	32	27	31	29	25

You may not need to use all of these tables.

T1

	J	K	L	M	N
A	12	17	10	15	18
B	24	26	23	25	27
C	17	21	100	20	23
D	14	100	9	13	17
E	18	23	19	21	25

(b) T1: every entry subtracted from 50, 100 in the blank cells CL and DK.
T2: reduce each row by its minimum: A by 10, B by 23, C by 17, D by 9, E by 18.

T2

	J	K	L	M	N
A	2	7	0	5	8
B	1	3	0	2	4
C	0	4	83	3	6
D	5	91	0	4	8
E	0	5	1	3	7

T3: now reduce each column of T2 by its minimum: no reduction in columns J and L, reduce K by 3, M by 2 and N by 4.

T3

	J	K	L	M	N
A	2	4	0	3	4
~~B~~	~~1~~	~~0~~	~~0~~	~~0~~	~~0~~
C	0	1	83	1	2
D	5	88	0	2	4
E	0	2	1	1	3

T3: all zeros covered by a minimum of 3 lines (row B, columns J and L).
3 < 5 so the table is not optimal.
Smallest uncovered element = 1:
subtract 1 from each uncovered element,
add 1 to elements covered twice (BJ, BL),
elements covered once stay the same.

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Question 1 continued

T4

	J	K	L	M	N
A	2	3	0	2	3
~~B~~	~~2~~	~~0~~	~~1~~	~~0~~	~~0~~
~~C~~	~~0~~	~~0~~	~~83~~	~~0~~	~~1~~
D	5	87	0	1	3
~~E~~	~~0~~	~~1~~	~~1~~	~~0~~	~~2~~

T5

	J	K	L	M	N
A	1	2	0	1	2
~~B~~	~~2~~	~~0~~	~~2~~	~~0~~	~~0~~
~~C~~	~~0~~	~~0~~	~~84~~	~~0~~	~~1~~
D	4	86	0	0	2
E	0	1	2	0	2

T6

	J	K	L	M	N
A	1	2	(0)	1	2
B	2	0	2	0	(0)
C	0	(0)	84	0	1
D	4	86	0	(0)	2
E	(0)	1	2	0	2

	J	K	L	M	N
A					
B					
C					
D					
E					

	J	K	L	M	N
A					
B					
C					
D					
E					

T4: the zeros now need a minimum of 4 lines (rows B, C, E and column L).
4 < 5 so the table is not optimal.
Smallest uncovered element = 1:
subtract 1 from each uncovered element,
add 1 to elements covered twice
(BL, CL, EL), others unchanged.

T5: the zeros now need a minimum of 5 lines (rows B, C, columns J, L, M).
5 lines = size of the 5 × 5 table,
so the table is optimal.

T6: allocation from the ringed zeros:
A to L, B to N, C to K, D to M, E to J

Maximum profit
= 40 + 23 + 29 + 37 + 32
= £161

(Total for Question 1 is 9 marks)



2.

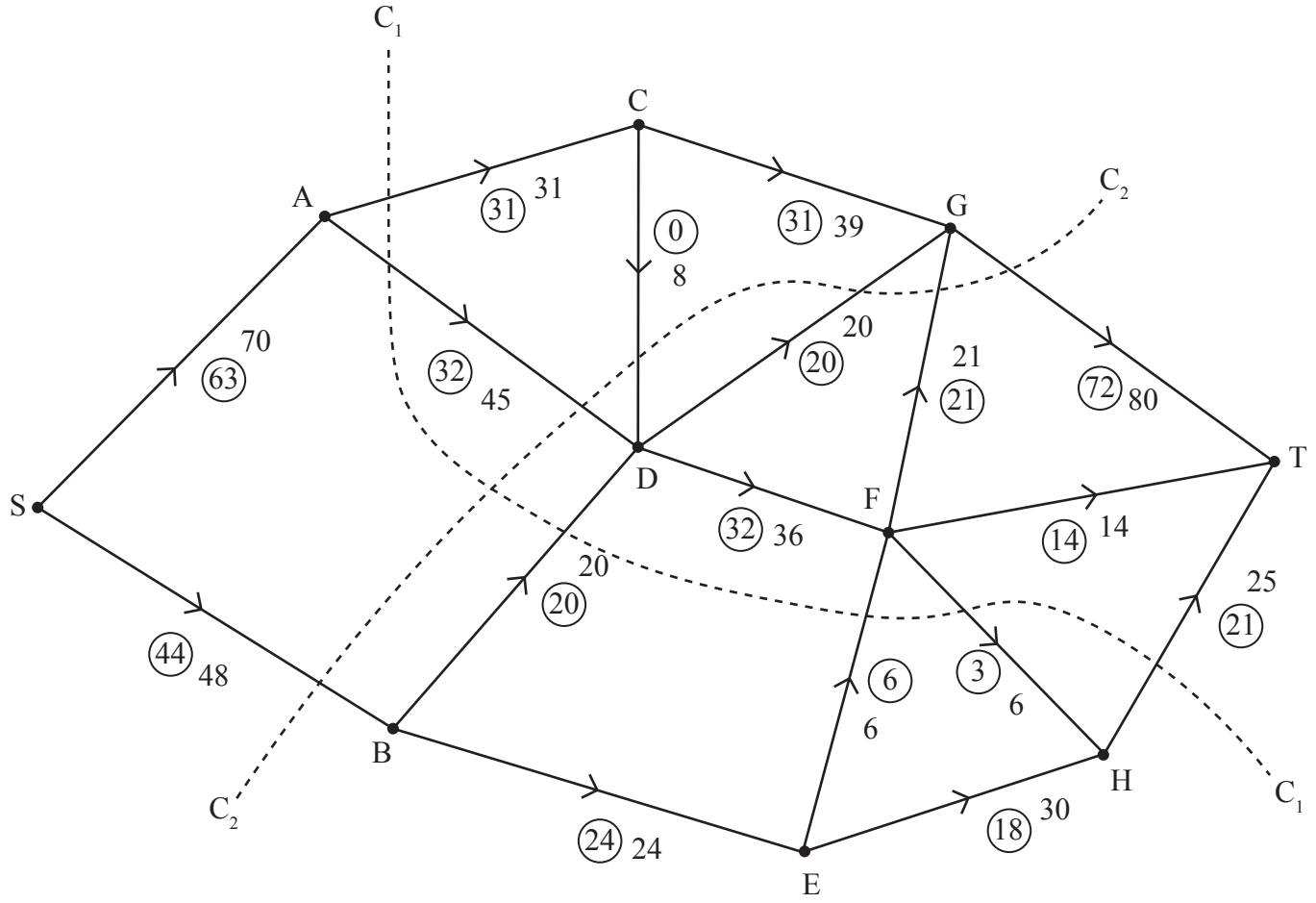


Figure 1

(a) On every arc, the flow along the arc must not exceed the capacity of the arc.

For every vertex except S and T, the total flow in = the total flow out.

(b) AC, BD, BE, DG, EF, FG, FT

(c)(i) Capacity of C_1 = 31 + 45 + 20 + 6 + 25 = 127

(ii) Capacity of C_2 = 80 + 8 + 45 + 48 = 181

(d) S A D F E H T (increases the flow by 4)

(e) See Diagram 1 (flows written on each arc).

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Question 2 continued

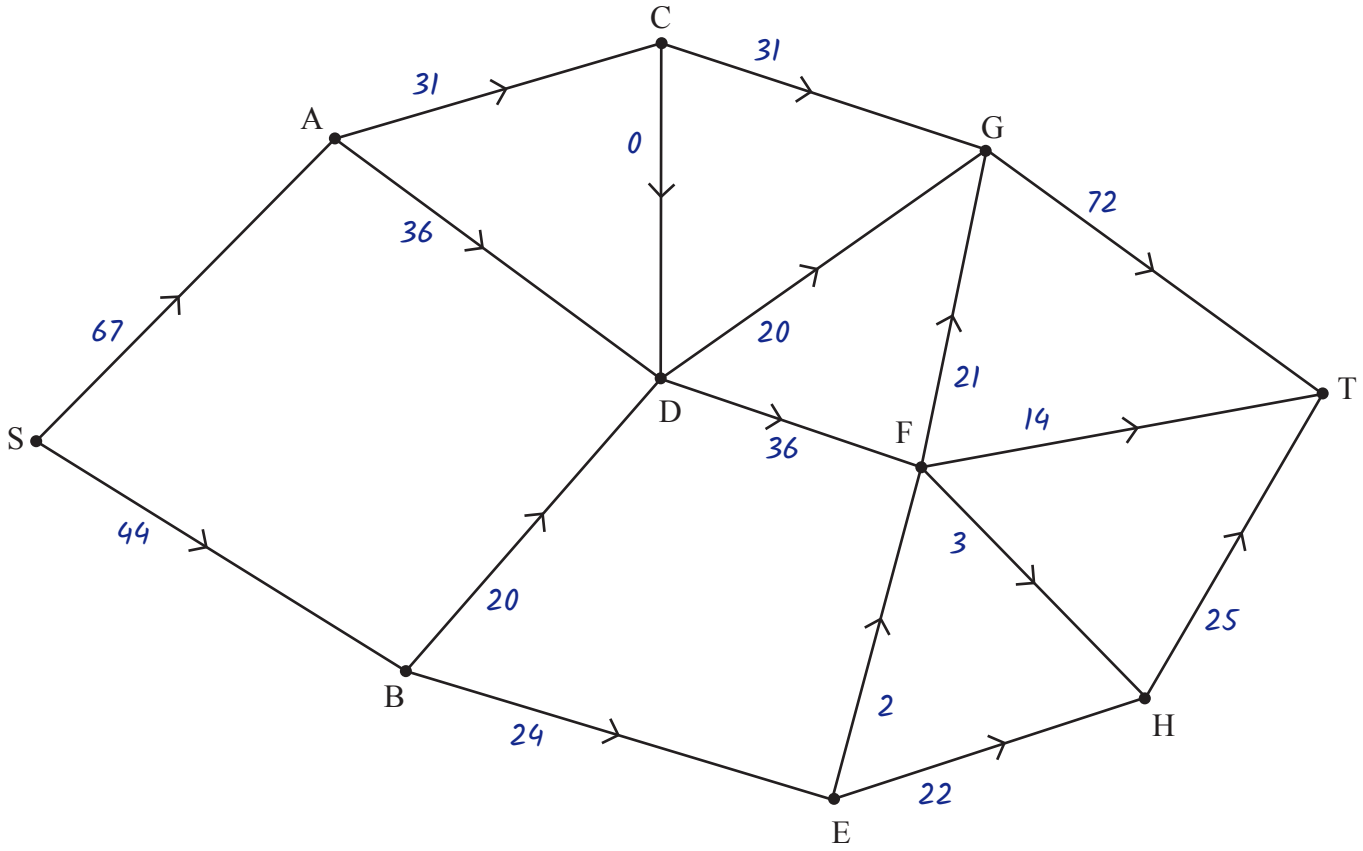


Diagram 1

(f) Consider the cut through arcs AC, CD, DG, DF and BE
(source side {S, A, B, D}, sink side {C, E, F, G, H, T}).
Capacity of cut = 31 + 0 + 20 + 36 + 24 = 111
(CD goes from the sink side to the source side, so it contributes 0)
Value of the flow in Diagram 1 = 67 + 44 = 111
Value of flow = capacity of this cut = 111, so by the
maximum flow minimum cut theorem the flow is a maximum.

(Total for Question 2 is 11 marks)

3.

	Mohsin plays X	Mohsin plays Y	Mohsin plays Z
Layla plays P	1	–2	2
Layla plays Q	–4	3	–5
Layla plays R	–1	1	–3

(a)(i) Row minima: P: −2, Q: −5, R: −3 ⇒ max of row minima = −2

Column maxima: X: 1, Y: 3, Z: 2 ⇒ min of column maxima = 1

Layla's play-safe strategy is P. Mohsin's play-safe strategy is X.

(ii) Row maximin (−2) ≠ column minimax (1), so there is no stable solution.

(b)(i) Mohsin plays X: $0.5 \times 1 + 0.5 \times (-4) = -1.5$

Mohsin plays Y: $0.5 \times (-2) + 0.5 \times 3 = 0.5$

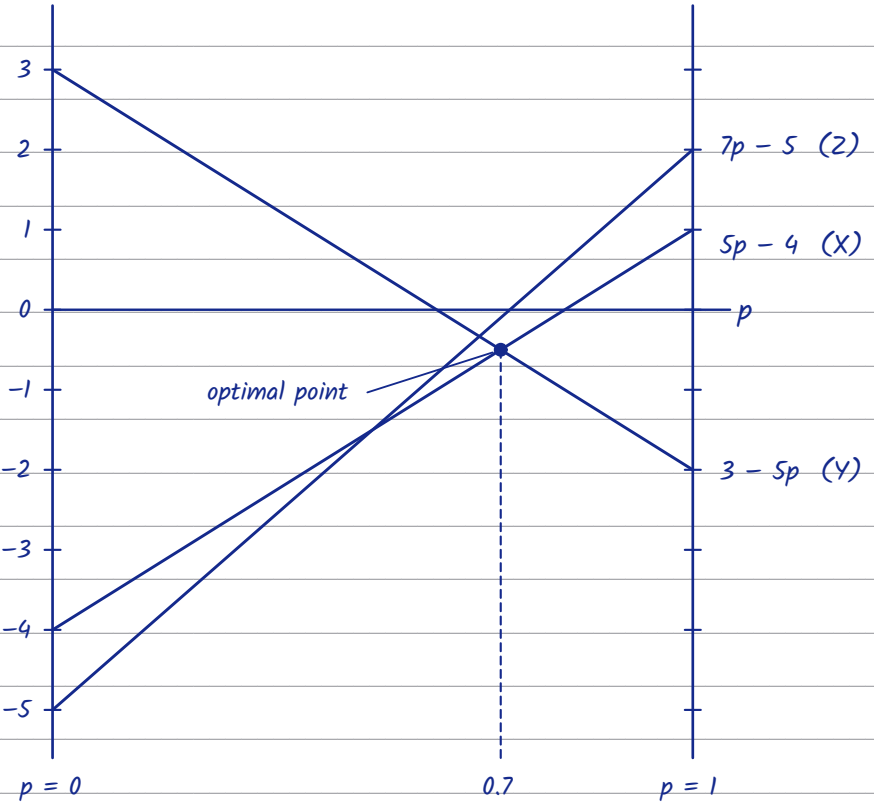
Mohsin plays Z: $0.5 \times 2 + 0.5 \times (-5) = -1.5$

(ii) Let Layla play P with probability p and Q with probability $1 - p$.

If Mohsin plays X, Layla's expected pay-off $= p - 4(1 - p) = 5p - 4$

If Mohsin plays Y, Layla's expected pay-off $= -2p + 3(1 - p) = 3 - 5p$

If Mohsin plays Z, Layla's expected pay-off $= 2p - 5(1 - p) = 7p - 5$



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Question 3 continued

Layla maximises her minimum expected pay-off, so the optimal point is the highest point on the lower boundary of the graph, where $5p - 4$ meets $3 - 5p$.

$5p - 4 = 3 - 5p \Rightarrow 10p = 7 \Rightarrow p = 7/10$

(check: $7p - 5 = -0.1 > -0.5$ at $p = 0.7$, so 2 is not on the lower boundary there)

Layla should play P with probability 7/10 and play Q with probability 3/10.

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 11 marks)

4.

(a) $u_{n+1} = -(1/2)u_n + (3/4)n - 5/4$

Auxiliary equation: $m + 1/2 = 0 \Rightarrow m = -1/2$

Complementary function: $u_n = A(-1/2)^n$

Particular solution: try $u_n = an + b$

$4(a(n + 1) + b) + 2(an + b) = 3n - 5$

Compare n terms: $4a + 2a = 3 \Rightarrow a = 1/2$

Compare constants: $4a + 4b + 2b = -5 \Rightarrow 2 + 6b = -5 \Rightarrow b = -7/6$

General solution: $u_n = A(-1/2)^n + n/2 - 7/6$

$u_1 = -2$: $-2 = A(-1/2) + 1/2 - 7/6$

$-2 = -A/2 - 2/3 \Rightarrow A/2 = 4/3 \Rightarrow A = 8/3$

$u_n = (8/3)(-1/2)^n + n/2 - 7/6$

(b) $u_1 = -4/3 + 1/2 - 7/6 = -2 < 0$

Even n: $(8/3)(-1/2)^n > 0$ and $u_2 = 2/3 + 1 - 7/6 = 1/2 > 0$;

for $n \geq 4$, $n/2 > 7/6$ so both parts are positive. So all even terms are > 0.

Odd n: the first part $-(8/3)(1/2)^n$ is negative but its size keeps decreasing,

$u_3 = -1/3 + 3/2 - 7/6 = 0$ (not negative)

$u_5 = -1/12 + 5/2 - 7/6 = 5/4 > 0$

for odd $n \geq 5$: $(8/3)(1/2)^n \leq 1/12$, while $n/2 - 7/6 \geq 4/3$ and is increasing,

so all odd terms after u_1 are ≥ 0.

Hence there is exactly one negative term in the sequence, u_1.

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Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

TOTAL FOR DECISION MATHEMATICS 2 IS 40 MARKS

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