Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Friday 16 May 2025

Afternoon

Paper reference **8FM0/22**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
22: Further Pure Mathematics 2
(Part of option A only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. **In this question you must show all stages of your working.**
Solutions based entirely on calculator technology are not acceptable.

(i) (a) Use the Euclidean algorithm to determine the highest common factor h of 105 and 24

(3)

(b) Hence determine integers a and b such that

$$105a + 24b = h$$

(3)

(ii) Determine the remainder when 179^5 is divided by 11

(2)

(i)(a) $105 = 4 \times 24 + 9$

$24 = 2 \times 9 + 6$

$9 = 1 \times 6 + 3$

$6 = 2 \times 3 + 0$

Last non-zero remainder is 3, so $h = 3$

(i)(b) $3 = 9 - 1 \times 6$

$= 9 - 1 \times (24 - 2 \times 9) = 3 \times 9 - 1 \times 24$

$= 3 \times (105 - 4 \times 24) - 1 \times 24$

$= 3 \times 105 - 13 \times 24$

$\therefore a = 3, \ b = -13$

(ii) $179 = 16 \times 11 + 3 \Rightarrow 179 \equiv 3 \pmod{11}$

$179^5 \equiv 3^5 \pmod{11} \equiv 243 \pmod{11}$

$243 = 22 \times 11 + 1 \Rightarrow 243 \equiv 1 \pmod{11}$

$179^5 \equiv 1 \pmod{11}$, so the remainder is 1

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Question 1 continued

Question 1 continued

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Question 1 continued

(Total for Question 1 is 8 marks)

2. (i) Using a suitable algorithm and without performing any division, determine whether 13 306 617 is divisible by 9

(2)

(ii) The group $G = \{1, 3, 7, 9, 11, 13, 17, 19\}$ has multiplication modulo 20 as its operation.

(a) Complete the following Cayley table for G

$\times_{20}$	1	3	7	9	11	13	17	19
1	1	3	7	9	11	13	17	19
3	3	9	1	7	13	19	11	17
7	7	1	9	3	17	11	19	13
9	9	7	3	1	19	17	13	11
11	11	13	17	19	1	3	7	9
13	13	19	11	17	3	9	1	7
17	17	11	19	13	7	1	9	3
19	19	17	13	11	9	7	3	1

A copy of this table is given on page 9 if you need to rewrite your Cayley table.

(3)

(b) State the inverse of the element 7

(1)

(c) Determine the order of the element 13

(1)

(d) Write down a subgroup of G of order 4

(1)

(i) Sum of digits: $1 + 3 + 3 + 0 + 6 + 6 + 1 + 7 = 27$

$27 = 3 \times 9$ is a multiple of 9, so 13 306 617 is divisible by 9

(ii)(b) $7 \times 3 = 21 \equiv 1 \pmod{20}$, so the inverse of 7 is 3

(ii)(c) $13^2 = 169 \equiv 9$, $13^3 \equiv 9 \times 13 = 117 \equiv 17$,

$13^4 \equiv 17 \times 13 = 221 \equiv 1 \pmod{20}$

order of 13 is 4

(ii)(d) $\{1, 3, 7, 9\}$

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Question 2 continued

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Question 2 continued

Only use this grid if you need to rewrite your Cayley table.

$\times_{20}$	1	3	7	9	11	13	17	19
1	1	3	7	9	11	13	17	19
3	3		1			19	11	
7	7	1			17			13
9	9		3	1		17	13	
11	11			19	1			
13	13		11			9	1	7
17	17	11						3
19	19		13		9		3	

(Total for Question 2 is 8 marks)

3. A loan of £180 000 is taken out to buy a house.

The monthly interest rate on the loan is 0.15%

The interest is added to the balance of the loan at the end of each month.

To repay the loan, £900 is repaid at the end of each month, immediately after the interest has been added.

Let B_n thousands of pounds be the balance of the loan at the end of month n after the interest has been added and the £900 repaid.

(a) Explain, in the context of the problem, why the balance of the loan, B_n, can be modelled by the recurrence relation

$$B_n = 1.0015B_{n-1} - 0.9 \qquad B_0 = 180 \qquad n \in \mathbb{Z}^+$$

(2)

(b) State an assumption that must be made for this model to be valid.

(1)

(c) Solve the recurrence relation to determine a closed form for B_n

(5)

(d) Hence determine the time it will take to repay the loan. Give your answer in years and months to the nearest month.

(2)

(a) $B_0 = 180$ because the loan is £180 000 before any interest is added or any payment made, and B_n is measured in £1000s

Interest of 0.15% is added each month, so the balance is multiplied by 100.15% = 1.0015, giving $1.0015B_{n-1}$

£900 = 0.9 thousand pounds is then repaid, so subtract 0.9:

$B_n = 1.0015B_{n-1} - 0.9$

(b) The interest rate stays the same (0.15% per month) and the monthly repayment stays at £900 throughout the loan

(c) $B_n = CF + PS$

CF: $B_n = a(1.0015)^n$

PS: try $B_n = b$: $b = 1.0015b - 0.9$

$\Rightarrow 0.0015b = 0.9 \Rightarrow b = 600$

$\therefore B_n = a(1.0015)^n + 600$

$B_0 = 180$: $180 = a(1.0015)^0 + 600 \Rightarrow a = -420$

$\therefore B_n = 600 - 420(1.0015)^n \qquad (n \geq 0)$

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Question 3 continued

(d) Repaid when $600 - 420(1.0015)^n = 0$

$(1.0015)^n = \frac{600}{420} = \frac{10}{7} \Rightarrow n = \frac{\ln(10/7)}{\ln 1.0015} = 237.96...$

so 238 months $(238 = 19 \times 12 + 10)$

Time to repay the loan: 19 years and 10 months

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

$$\mathbf{A} = \begin{pmatrix} 6 & -1 \\ 2 & 3 \end{pmatrix}$$

Determine a matrix **P** and a diagonal matrix **D** such that

$$\mathbf{P}^{-1}\mathbf{AP} = \mathbf{D}$$

(7)

$|A - \lambda I| = \begin{vmatrix} 6-\lambda & -1 \\ 2 & 3-\lambda \end{vmatrix} = (6-\lambda)(3-\lambda) + 2 = 0$

$\Rightarrow 18 - 9\lambda + \lambda^2 + 2 = 0 \Rightarrow \lambda^2 - 9\lambda + 20 = 0$

$\Rightarrow (\lambda - 5)(\lambda - 4) = 0 \Rightarrow \lambda = 5, \lambda = 4$

$\lambda = 5$: $\begin{pmatrix} 6 & -1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 5\begin{pmatrix} x \\ y \end{pmatrix}$

$6x - y = 5x \Rightarrow y = x$ $(2x + 3y = 5y \Rightarrow y = x$ ✓)

eigenvector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda = 4$: $\begin{pmatrix} 6 & -1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 4\begin{pmatrix} x \\ y \end{pmatrix}$

$6x - y = 4x \Rightarrow y = 2x$ $(2x + 3y = 4y \Rightarrow y = 2x$ ✓)

eigenvector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$P = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ $D = \begin{pmatrix} 5 & 0 \\ 0 & 4 \end{pmatrix}$

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 7 marks)

5. In an Argand diagram, the curve C with equation

$$\arg\left(\frac{z+4}{z-2\mathrm{i}}\right)=\frac{\pi}{4}$$

represents an arc of a circle.

Given that $z=x+\mathrm{i}y$, where x and y are real numbers,

(a) show that this circle has equation

$$x^2+y^2+ax+by+c=0$$

where a, b and c are constants to be determined.

(4)

(b) For the curve C, determine the exact minimum value of $|z|$

(3)

(a) $z = x + iy$:

$$\frac{z+4}{z-2i}=\frac{x+4+iy}{x+i(y-2)}=\frac{(x+4+iy)(x-i(y-2))}{(x+i(y-2))(x-i(y-2))}$$

$$=\frac{x^2+4x+y^2-2y}{x^2+(y-2)^2}+\frac{2x-4y+8}{x^2+(y-2)^2}i$$

$\arg\left(\frac{z+4}{z-2i}\right)=\frac{\pi}{4}$ $\Rightarrow$ real part = imaginary part (both positive)

$x^2+y^2+4x-2y=2x-4y+8$

$\Rightarrow x^2+y^2+2x+2y-8=0$

$a=2,\ b=2,\ c=-8$

(b) $(x+1)^2+(y+1)^2=8+1+1=10$

centre $(-1,-1)$, radius $r=\sqrt{10}$

distance from O to centre $=\sqrt{(1^2+1^2)}=\sqrt{2}<\sqrt{10}$, so O is inside the circle; the nearest point of C lies on the line through O and the centre, beyond the centre (on the arc C)

$|z|_{min}=\sqrt{10}-\sqrt{(1^2+1^2)}$

$|z|_{min}=\sqrt{10}-\sqrt{2}$

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Question 5 continued

Question 5 continued

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TOTAL FOR FURTHER PURE MATHEMATICS 2 IS 40 MARKS

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