Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Friday 16 May 2025

Afternoon

Paper reference **8FM0/21**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
21: Further Pure Mathematics 1
(Part of options A, B, C and D)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P74070A

©2025 Pearson Education Ltd.
Y:1/1/1/1/1/1/1/

P 7 4 0 7 0 A 0 1 2 0

Pearson

1. **In this question you must show all stages of your working.**
Solutions based entirely on calculator technology are not acceptable.

The surface temperature of the water in a lake during a particular year is modelled by the equation

$$S = 12 - \frac{15}{2}\cos x° - \frac{27}{10}\sin x° \qquad (\mathrm{I})$$

where S is the temperature in degrees Celsius and x is the number of days after the start of the year.

(a) Use the model to write down the surface temperature of the water in the lake at the start of the year.

(1)

Using the substitution $t = \tan\left(\frac{x}{2}\right)$

(b) show that equation (I) can be rewritten as

$$S = \frac{At^2 + Bt + C}{10(1 + t^2)}$$

where A, B and C are integers to be determined.

(3)

(c) Hence determine, according to the model, the number of days after the start of the year when the surface temperature of the water in the lake is 10 °C for the **second** time that year. Give your answer to the nearest day.

(5)

(a) $x = 0$: $S = 12 - 7.5\cos 0° - 2.7\sin 0° = 12 - 7.5 = 4.5$ °C

(b) $\cos x = \frac{1 - t^2}{1 + t^2}$, $\sin x = \frac{2t}{1 + t^2}$

$S = 12 - \frac{15}{2}\left(\frac{1 - t^2}{1 + t^2}\right) - \frac{27}{10}\left(\frac{2t}{1 + t^2}\right)$

$S = \frac{120(1 + t^2) - 75(1 - t^2) - 54t}{10(1 + t^2)}$

$S = \frac{195t^2 - 54t + 45}{10(1 + t^2)}$ $\quad A = 195,\ B = -54,\ C = 45$

(c) $\frac{195t^2 - 54t + 45}{10(1 + t^2)} = 10 \Rightarrow 195t^2 - 54t + 45 = 100 + 100t^2$

$95t^2 - 54t - 55 = 0$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 1 continued

$t = \dfrac{54 \pm \sqrt{(54^2 + 4 \times 95 \times 55)}}{2 \times 95} = \dfrac{54 \pm \sqrt{23816}}{190} \Rightarrow t = 1.09644\ldots \text{ or } t = -0.52802\ldots$

$t = 1.09644\ldots$: $\dfrac{x}{2} = \tan^{-1}(1.09644\ldots) = 47.63\ldots° \Rightarrow x = 95.27\ldots$ (first time)

$t = -0.52802\ldots$: $\tan^{-1}(-0.52802\ldots) = -27.835\ldots°$, but $0° < \dfrac{x}{2} < 182.5°$

so $\dfrac{x}{2} = 180° - 27.835\ldots° = 152.164\ldots° \Rightarrow x = 304.329\ldots$

The temperature is 10 °C for the second time after 304 days.

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 1 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 1 continued

(Total for Question 1 is 9 marks)

2. Water is leaking from a hole in the base of a large spherical tank. The depth of water, H metres, in the tank is modelled by the differential equation

$$3(5H - H^2)\frac{dH}{dt} = -4\sqrt{H} \qquad 0 < H < 5$$

where t is the time in hours after the leak started.

The depth of water in the tank 10 minutes after the leak started was 2 m.

Use two applications of the approximation formula

$$\left(\frac{dy}{dx}\right)_n \approx \frac{(y_{n+1} - y_n)}{h}$$

to estimate the depth of water in the tank, one hour after the leak started.

(7)

10 minutes = $\frac{1}{6}$ hour. Steps of 25 minutes from $t = \frac{1}{6}$ to $t = 1$, so $h = \frac{25}{60} = \frac{5}{12}$

$\left(\frac{dH}{dt}\right)_0 = \frac{-4\sqrt{H_0}}{3(5H_0 - H_0^2)} = \frac{-4\sqrt{2}}{3(5 \times 2 - 2^2)} = -\frac{2\sqrt{2}}{9} = -0.314269...$

$H_1 \approx H_0 + h\left(\frac{dH}{dt}\right)_0 = 2 + \frac{5}{12} \times (-0.314269...)$

$= 1.869054...$ $\left(= \frac{108 - 5\sqrt{2}}{54}\right)$

$\left(\frac{dH}{dt}\right)_1 = \frac{-4\sqrt{1.869054...}}{3(5 \times 1.869054... - 1.869054...^2)} = \frac{-5.468534...}{17.555722...} = -0.311495...$

$H_2 \approx H_1 + h\left(\frac{dH}{dt}\right)_1 = 1.869054... + \frac{5}{12} \times (-0.311495...)$

$= 1.739264...$

Depth of water one hour after the leak started ≈ 1.74 m

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

Question 2 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

(Total for Question 2 is 7 marks)

3.

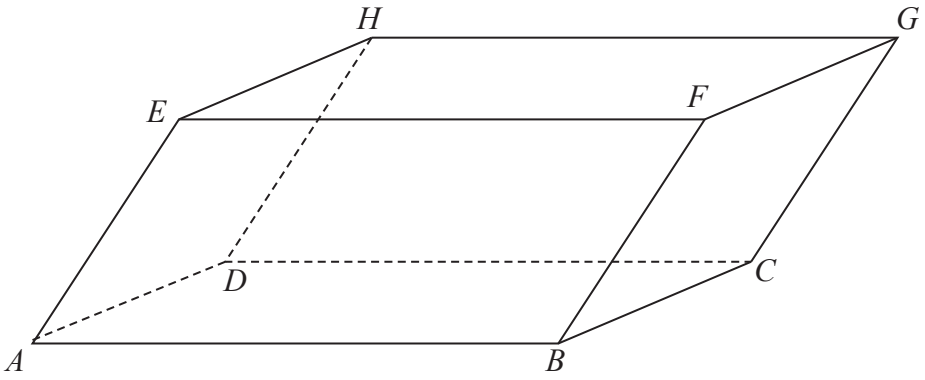


Figure 1

Figure 1 shows the parallelepiped $ABCDHEFG$.

Relative to a fixed origin O, the points A, B, D and E have coordinates (3, 2, 7), (4, 4, 3), (4, 1, t) and (t, –1, 10) respectively, where t is a constant.

(a) Determine, in simplest form in terms of t, $\overrightarrow{AB} \times \overrightarrow{AD}$

(3)

Given that the volume of the parallelepiped is 9

(b) determine the possible values of t.

(6)

(a) $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$ $\overrightarrow{AD} = \begin{pmatrix} 1 \\ -1 \\ t-7 \end{pmatrix}$

$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 1 & -1 & t-7 \end{vmatrix} = \begin{pmatrix} 2(t-7)-4 \\ -[(t-7)+4] \\ -1-2 \end{pmatrix} = \begin{pmatrix} 2t-18 \\ 3-t \\ -3 \end{pmatrix}$

i.e. $(2t-18)i + (3-t)j - 3k$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

(b) $\overrightarrow{AE} = \begin{pmatrix} t-3 \\ -3 \\ 3 \end{pmatrix}$ Volume $V = |\overrightarrow{AE} \cdot (\overrightarrow{AB} \times \overrightarrow{AD})|$

$\begin{pmatrix} t-3 \\ -3 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2t-18 \\ 3-t \\ -3 \end{pmatrix} = (t-3)(2t-18) - 3(3-t) - 9$

$= 2t^2 - 24t + 54 - 9 + 3t - 9 = 2t^2 - 21t + 36$

$|2t^2 - 21t + 36| = 9$

$2t^2 - 21t + 36 = 9 \Rightarrow 2t^2 - 21t + 27 = 0 \Rightarrow (2t-3)(t-9) = 0 \Rightarrow t = \frac{3}{2}, 9$

$2t^2 - 21t + 36 = -9 \Rightarrow 2t^2 - 21t + 45 = 0 \Rightarrow (2t-15)(t-3) = 0 \Rightarrow t = \frac{15}{2}, 3$

$t = \frac{3}{2}$, $t = 3$, $t = \frac{15}{2}$ or $t = 9$

Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 9 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

4. **In this question you must show all stages of your working.**
Solutions based on calculator technology are not acceptable.

Determine the values of x for which

$$\frac{x-8}{x} \leqslant \frac{7}{x(x-2)}$$

giving your answer in set notation.

(6)

$\frac{x-8}{x} - \frac{7}{x(x-2)} \leq 0$

$\frac{(x-8)(x-2)-7}{x(x-2)} \leq 0$

$\frac{x^2-10x+9}{x(x-2)} \leq 0 \quad \Rightarrow \quad \frac{(x-1)(x-9)}{x(x-2)} \leq 0$

Critical values: $x = 0$, $x = 2$ (denominator zero, excluded), $x = 1$, $x = 9$ (included)

Signs: $x < 0$: + $\quad 0 < x < 1$: − $\quad 1 < x < 2$: + $\quad 2 < x < 9$: − $\quad x > 9$: +

So $0 < x \leq 1$ or $2 < x \leq 9$

$\{x \in \mathbb{R} : 0 < x \leq 1\} \cup \{x \in \mathbb{R} : 2 < x \leq 9\}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 6 marks)

5. The parabola C has equation $y^2 = 16x$

The point $P(4t^2, 8t)$ lies on C.

(a) Show that an equation for the tangent to C at P is

$$yt - x = 4t^2$$

(3)

The line l passes through the origin and is perpendicular to the tangent to C at P.
The line l and the tangent to C at P intersect at the point Q.

(b) Determine, in simplest form in terms of t, the coordinates of Q.

(3)

(c) Hence show that, as t varies, the point Q lies on the curve with equation

$$y^2 = \frac{Ax^3}{x + B}$$

where A and B are integers to be determined.

(3)

(a) $x = 4t^2$, $y = 8t$: $\frac{dx}{dt} = 8t$, $\frac{dy}{dt} = 8$ $\Rightarrow$ $\frac{dy}{dx} = \frac{8}{8t} = \frac{1}{t}$

Tangent: $y - 8t = \frac{1}{t}(x - 4t^2)$

$ty - 8t^2 = x - 4t^2$ $\Rightarrow$ $yt - x = 4t^2$ as required

(b) Gradient of tangent $= \frac{1}{t}$ $\Rightarrow$ gradient of $l = -t$, so l: $y = -tx$

$(-tx)t - x = 4t^2$ $\Rightarrow$ $-x(t^2 + 1) = 4t^2$

$x = \frac{-4t^2}{t^2 + 1}$, $y = -tx = \frac{4t^3}{t^2 + 1}$

$Q\left(\frac{-4t^2}{t^2 + 1}, \frac{4t^3}{t^2 + 1}\right)$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 5 continued

(c) $y = -tx \Rightarrow t = -\frac{y}{x} \Rightarrow y^2 = t^2x^2$

$x(t^2 + 1) = -4t^2 \Rightarrow xt^2 + 4t^2 = -x \Rightarrow t^2(x + 4) = -x$

$t^2 = \frac{-x}{x + 4}$

$y^2 = \frac{-x}{x + 4} \times x^2 \Rightarrow y^2 = \frac{-x^3}{x + 4}$

$A = -1, \ B = 4$

Question 5 continued

(Total for Question 5 is 9 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 1 IS 40 MARKS

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA