Pearson Edexcel Level 3 GCE

Friday 17 May 2024

Afternoon

Paper reference **8FM0/28**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
28: Decision Mathematics 2
(Part of option K only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator, D2 Answer Book (enclosed)

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.
- Do not return the question paper with the D2 Answer Book.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P75678A

©2024 Pearson Education Ltd.
F:1/1/1/

P 7 5 6 7 8 A

Pearson

stemup.co.uk

1.

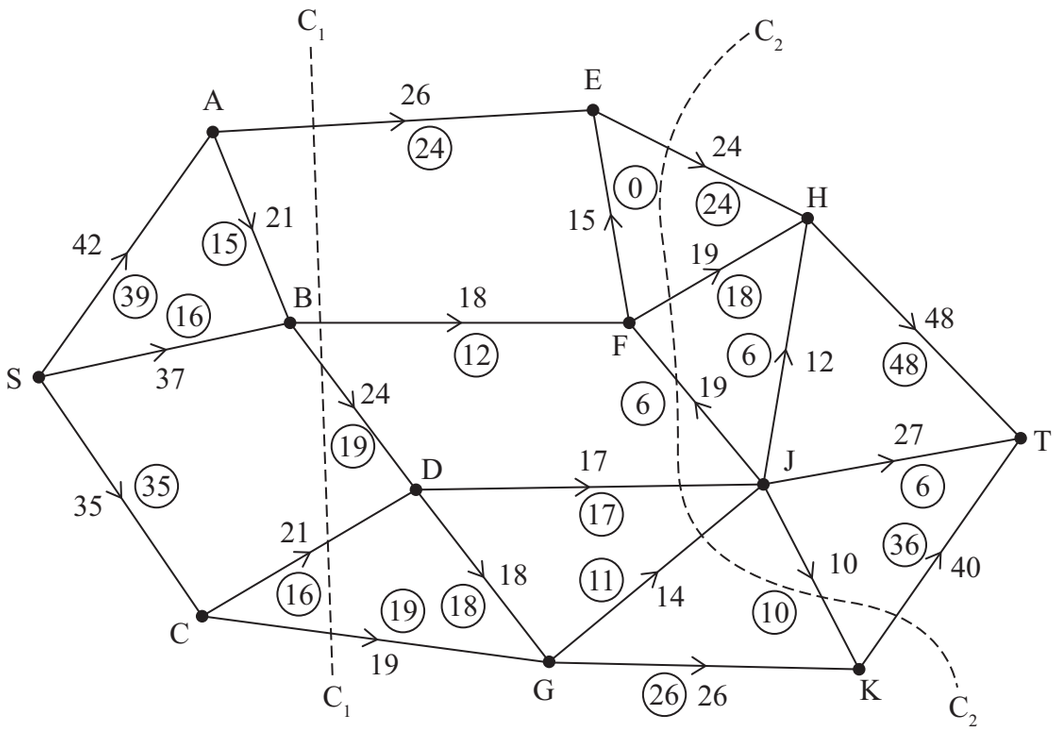


Figure 1

Figure 1 shows a capacitated, directed network of pipes. The number on each arc represents the capacity of the corresponding pipe. The numbers in circles represent a feasible flow from S to T.

(a) State the value of this flow.

(1)

(b) Explain why arcs CD and CG cannot both be saturated.

(1)

(c) Find the capacity of

(i) cut C_1

(ii) cut C_2

(2)

(d) Write down a flow augmenting route of weight 6 which saturates BF.

(1)

The flow augmenting route in part (d) is applied to give an increased flow.

(e) Prove that this increased flow is maximal.

(3)

(Total for Question 1 is 8 marks)

stemup.co.uk

2. A team of 5 players, A, B, C, D and E, competes in a quiz. Each player must answer one of 5 rounds, P, Q, R, S and T.

Each player must be assigned to exactly one round, and each round must be answered by exactly one player.

Player B cannot answer round Q, player D cannot answer round T, and player E cannot answer round R.

The number of points that each player is expected to earn in each round is shown in the table.

	P	**Q**	**R**	**S**	**T**
A	32	40	35	41	37
B	38	–	40	27	33
C	41	28	37	36	35
D	35	33	38	36	–
E	40	38	–	39	34

The team wants to maximise its total expected score.

The Hungarian algorithm is to be used to find the maximum total expected score that can be earned by the 5 players.

(a) Explain how the table should be modified.

(2)

(b) (i) Reducing rows first, use the Hungarian algorithm to obtain an allocation which maximises the total expected score.

(ii) Calculate the maximum total expected score.

(6)

(Total for Question 2 is 8 marks)



stemup.co.uk

3. Haruki and Meera play a zero-sum game. The game is represented by the following pay-off matrix for Haruki.

		Meera		
		Option X	Option Y	Option Z
Haruki	Option A	4	–2	–5
	Option B	1	4	–3
	Option C	–1	6	1
	Option D	–4	5	3

(a) Determine whether the game has a stable solution.

(2)

Option Y for Meera is now removed.

(b) Write down the reduced pay-off matrix for Meera.

(1)

(c) (i) Given that Meera plays Option X with probability p, determine her best strategy.

(ii) State the value of the game to Haruki.

(iii) State which option(s) Haruki should never play.

(7)

The number of points scored by Haruki when he plays Option C and Meera plays Option X changes from –1 to k

Given that the value of the game is now the same for both players,

(d) determine the value of k. You must make your method and working clear.

(4)

(Total for Question 3 is 14 marks)

stemup.co.uk

4. Peter sets up a savings plan. He makes an initial deposit of £D and then pays in £M at the end of each month.

The value of the savings plan, in pounds, is modelled by

$$u_{n+1} = 1.025\, u_n + 1800$$

where $n \geqslant 0$ is an integer and u_n is the total value of the savings plan, in pounds, after n years.

(a) Calculate the value of M

(1)

Given that the value of the savings plan after 1 year is £6 925

(b) solve the recurrence relation for u_n

(5)

(c) Determine the value of D

(1)

(d) Hence determine, using algebra, the number of years it will take for the value of the savings plan to exceed £20 000

(3)

(Total for Question 4 is 10 marks)

TOTAL FOR DECISION MATHEMATICS 2 IS 40 MARKS

stemup.co.uk

BLANK PAGE

stemup.co.uk

BLANK PAGE

stemup.co.uk

BLANK PAGE

stemup.co.uk

Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Friday 17 May 2024

Afternoon

Paper reference **8FM0/28**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
28: Decision Mathematics 2
(Part of option K only)

D2 Answer Book
Do not return the question paper with the answer book.

Total Marks

Turn over ▶

P75678A

©2024 Pearson Education Ltd.
F:1/1/1/

P 7 5 6 7 8 A 0 1 1 2

Pearson

1.

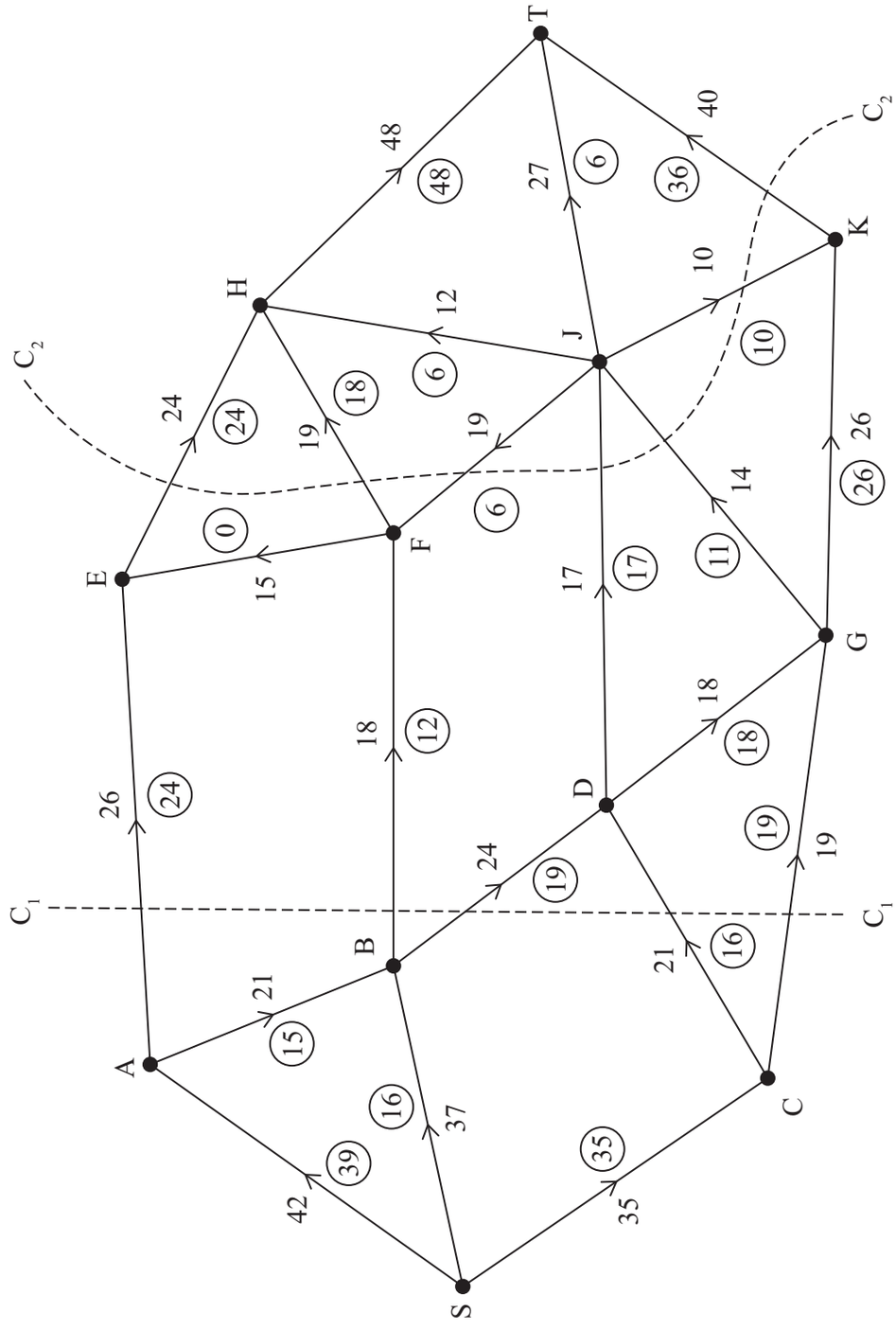


Figure 1

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

Question 1 continued

(a) Value of the flow = 39 + 16 + 35 = 90

(b) SC is the only arc into C, so the maximum flow into C is 35.
Capacity of CD + capacity of CG = 21 + 19 = 40
35 < 40, so the flow out of C can never be 40, hence CD and CG cannot both be saturated.

(c)(i) C_1 = AE + BF + BD + CD + CG = 26 + 18 + 24 + 21 + 19 = 108
(ii) C_2 = EH + FH + DJ + GJ + KT = 24 + 19 + 17 + 14 + 40 = 114
(JF, JH and JK go from the sink side to the source side, so contribute 0)

(d) S B F J T (flow 6, using the backflow of 6 in JF)

(e) New flow = 90 + 6 = 96
Consider the cut through EH, EF, BF, DJ, DG and CG:
capacity = 24 + 0 + 18 + 17 + 18 + 19 = 96 (FE goes from sink side to source side)
The flow of 96 equals the capacity of this cut.
By the maximum flow – minimum cut theorem, maximum flow = minimum cut,
so the increased flow of 96 is maximal.

(Total for Question 1 is 8 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

2. (a)

Subtract every entry from a constant, e.g. 50 (larger than the largest value, 41), to convert the maximisation problem into a minimisation problem. Then put a large value, e.g. 100, in the three missing cells BQ, DT and ER, so that these cells cannot be selected in the allocation.

	P	Q	R	S	T
A	32	40	35	41	37
B	38	–	40	27	33
C	41	28	37	36	35
D	35	33	38	36	–
E	40	38	–	39	34

You may not need to use all of these tables

	P	Q	R	S	T
A	18	10	15	9	13
B	12	100	10	23	17
C	9	22	13	14	15
D	15	17	12	14	100
E	10	12	100	11	16

(b)(i) 50 – each entry, with 100 in BQ, DT, ER

	P	Q	R	S	T
A	9	1	6	0	4
B	2	90	0	13	7
C	0	13	4	5	6
D	3	5	0	2	88
E	0	2	90	1	6

Reduce rows: subtract the row minima 9, 10, 9, 12, 10

	P	Q	R	S	T
A	~~9~~	~~0~~	~~6~~	~~0~~	~~0~~
B	2	89	0	13	3
C	0	12	4	5	2
D	3	4	0	2	84
E	0	1	90	1	2

Reduce columns: subtract the column minima 0, 1, 0, 0, 4

Zeros covered by 3 lines (row A, columns P and R)

3 < 5, so not optimal

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 7 5 6 7 8 A 0 4 1 2

stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

	P	Q	R	S	T
A	~~10~~	~~0~~	~~7~~	~~0~~	~~0~~
B	2	88	0	12	2
C	0	11	4	4	1
D	3	3	0	1	83
E	~~0~~	~~0~~	~~90~~	~~0~~	~~1~~

Least uncovered element = 1:
subtract 1 from uncovered cells,
add 1 to cells covered twice
(AP, AR), others unchanged.
Now 4 lines (rows A and E,
columns P and R): 4 < 5

	P	Q	R	S	T
A	11	0	8	0	0
B	2	87	0	11	1
C	0	10	4	3	0
D	3	2	0	0	82
E	1	0	91	0	1

Least uncovered element = 1:
subtract 1 from uncovered cells,
add 1 to AP, AR, EP and ER.
Now 5 lines are needed to cover
the zeros, 5 = 5, so optimal.

	P	Q	R	S	T
A					
B					
C					
D					
E					

Allocation:
A – T, B – R, C – P,
D – S, E – Q

(ii) Maximum total expected
score = 37 + 40 + 41 + 36 + 38
= 192

	P	Q	R	S	T
A					
B					
C					
D					
E					

	P	Q	R	S	T
A					
B					
C					
D					
E					

(Total for Question 2 is 8 marks)

3.

		Meera		
		Option X	Option Y	Option Z
Haruki	Option A	4	–2	–5
	Option B	1	4	–3
	Option C	–1	6	1
	Option D	–4	5	3

(a) Row minima: A −5, B −3, C −1, D −4 ⇒ row maximin = −1

Column maxima: X 4, Y 6, Z 3 ⇒ column minimax = 3

−1 ≠ 3, row maximin ≠ column minimax, so the game does not have a stable solution.

(b) Reduced pay-off matrix for Meera:

	Haruki A	Haruki B	Haruki C	Haruki D
Meera X	−4	−1	1	4
Meera Z	5	3	−1	−3

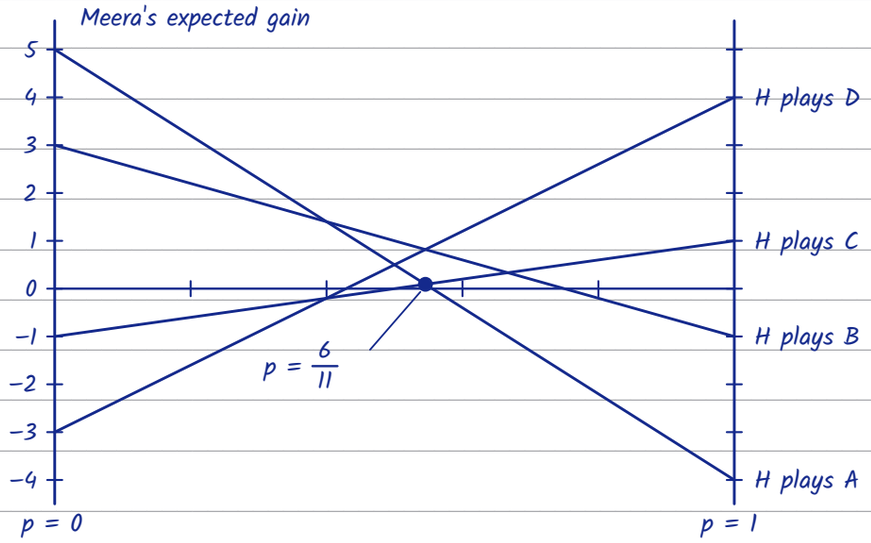
(c)(i) Let Meera play X with probability p and Z with probability 1 − p.

If H plays A, Meera's expected gain = −4p + 5(1 − p) = 5 − 9p

If H plays B, Meera's expected gain = −p + 3(1 − p) = 3 − 4p

If H plays C, Meera's expected gain = p − (1 − p) = 2p − 1

If H plays D, Meera's expected gain = 4p − 3(1 − p) = 7p − 3



DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

Question 3 continued

Meera maximises her minimum expected gain: the optimal point is the highest point on the lower boundary of the graph, where the lines for A and C intersect.

$5 - 9p = 2p - 1 \quad \Rightarrow \quad 11p = 6 \quad \Rightarrow \quad p = \frac{6}{11}$

Meera should play option X with probability $\frac{6}{11}$ and option Z with probability $\frac{5}{11}$

(ii) Value of the game to Meera $= 2 \times \frac{6}{11} - 1 = \frac{1}{11}$

Value of the game to Haruki $= -\frac{1}{11}$

(iii) Haruki should never play option B or option D

(d) The game is zero-sum, so value to Haruki = −(value to Meera).
The values are the same, so $v = -v \Rightarrow$ value of the game $= 0$
If H plays C, Meera's expected gain is now $-kp - (1 - p)$
The optimal point is still where the lines for A and C meet, with value 0:

H plays A: $5 - 9p = 0 \quad \Rightarrow \quad p = \frac{5}{9}$

H plays C: $-k \times \frac{5}{9} - (1 - \frac{5}{9}) = 0$

$-\frac{5k}{9} - \frac{4}{9} = 0 \quad \Rightarrow \quad -5k = 4$

$k = -\frac{4}{5}$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

Question 3 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 14 marks)

stemup.co.uk

4.

(a) $M = 1800 \div 12 = 150$

(b) Auxiliary equation: $m - 1.025 = 0 \Rightarrow m = 1.025$

Complementary function: $u_n = A(1.025)^n$

Particular solution: try $u_n = \lambda$

$\lambda = 1.025\lambda + 1800 \Rightarrow -0.025\lambda = 1800 \Rightarrow \lambda = -72000$

General solution: $u_n = A(1.025)^n - 72000$

$u_1 = 6925$: $1.025A - 72000 = 6925$

$1.025A = 78925 \Rightarrow A = 77000$

$u_n = 77000(1.025)^n - 72000$

(c) $D = u_0 = 77000(1.025)^0 - 72000 = 77000 - 72000$

$D = 5000$

(d) $77000(1.025)^n - 72000 > 20000$

$77000(1.025)^n > 92000$

$(1.025)^n > \frac{92}{77}$

$n \log(1.025) > \log \frac{92}{77}$

$n > \frac{\log(92/77)}{\log(1.025)} = 7.20795...$

$n = 8$, so it takes 8 years for the value to exceed £20 000

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

stemup.co.uk

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

Question 4 continued

(Total for Question 4 is 10 marks)

TOTAL FOR DECISION MATHEMATICS 2 IS 40 MARKS

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

stemup.co.uk