Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Friday 17 May 2024

Afternoon

Paper reference **8FM0/23**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
23: Further Statistics 1
(Part of options B, E, F and G)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from statistical tables should be quoted in full. If a calculator is used instead of tables the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. Sharma believes that each computer game he sells appeals equally to all age ranges.
To investigate this, he takes a random sample of 100 people who play these games and asks them which of the games A, B or C they prefer.
The results are summarised in the table below.

Computer game		A	B	C
Age range	< 20	8	15	6
	20 – 30	21	12	9
	> 30	6	10	13

(a) Write down hypotheses for a suitable test to assess Sharma's belief.

(1)

(b) For the test, calculate the expected frequency for

(i) those players aged under 20 who prefer game C

(ii) those players aged between 20 and 30 who prefer game A

(2)

(c) State the degrees of freedom of the test statistic for this test.

(1)

Sharma correctly calculates the test statistic for this test to be 11.542 (to 3 decimal places).

(d) Using a 5% significance level, and stating your critical value, comment on Sharma's belief.

(2)

(a) H_0: There is no association between age range and preferred game

H_1: There is an association between age range and preferred game

(b) (i) $E = \frac{29 \times 28}{100} = 8.12$

(ii) $E = \frac{42 \times 35}{100} = 14.7$

(c) All $E_i > 5$ (smallest is 8.12), so no pooling is needed

$\nu = (3 - 1)(3 - 1) = 4$

(d) Critical value: $\chi^2_4(0.05) = 9.488$

$11.542 > 9.488$, so the result is significant: reject H_0

Sharma's belief is not supported: there is significant evidence of an association between age range and computer game preference.

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Question 1 continued

Question 1 continued

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Question 1 continued

(Total for Question 1 is 6 marks)

2. A manager keeps a record of accidents in a canteen.

Accidents occur randomly with an average of 2.7 per month. The manager decides to model the number of accidents with a Poisson distribution.

(a) Give a reason why a Poisson distribution could be a suitable model in this situation.

(1)

(b) Assuming that a Poisson model is suitable, find the probability of

(i) at least 3 accidents in the next month,

(1)

(ii) no more than 10 accidents in a 3-month period,

(2)

(iii) at least 2 months with no accidents in an 8-month period.

(4)

One day, two members of staff bump into each other in the canteen and each report the accident to the manager. The canteen manager is unsure whether to record this as one or two accidents.

Given that the manager still wants to model the number of accidents per month with a Poisson distribution,

(c) state

- a property of the Poisson distribution that the manager should consider when deciding how to record this situation
- whether the manager should record this as one or two accidents

(1)

The manager introduces some new procedures to try and reduce the average number of accidents per month.

During the following 12 months the total number of accidents is 22
The manager claims that the accident rate has been reduced.

(d) Use a 5% level of significance to carry out a suitable test to assess the manager's claim.
You should state your hypotheses clearly and the p-value used in your test.

(4)

(a) Accidents occur randomly (independently) and at a constant average rate of 2.7 per month.

(b) A = number of accidents in a month, $A \sim Po(2.7)$

(i) $P(A \geqslant 3) = 1 - P(A \leqslant 2) = 1 - 0.49362...$

$= 0.50637... = 0.506$

(ii) T = number of accidents in 3 months, $T \sim Po(3 \times 2.7) = Po(8.1)$

$P(T \leqslant 10) = 0.805837... = 0.806$

(iii) $P(A = 0) = e^{-2.7} = 0.067205...$

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Question 2 continued

M = number of months with no accidents, $M \sim B(8, 0.067205...)$

$P(M \geqslant 2) = 1 - P(M \leqslant 1)$

$= 1 - [(0.932794...)^8 + 8 \times 0.067205... \times (0.932794...)^7]$

$= 1 - 0.903542... = 0.096457...$

$= 0.0965$

(c) In a Poisson model events (accidents) must occur singly (independently), so the manager should record this as one accident.

(d) λ = mean number of accidents per month

H_0: $\lambda = 2.7$ H_1: $\lambda < 2.7$

Y = number of accidents in 12 months. Under H_0: $Y \sim Po(12 \times 2.7) = Po(32.4)$

p-value = $P(Y \leqslant 22) = 0.03512...$

$0.0351 < 0.05$, so the result is significant: reject H_0

There is evidence to support the manager's claim that the mean number of accidents per month has decreased.

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 13 marks)

3. The discrete random variable X has probability distribution,

x	–1	0	1	3	7
$P(X = x)$	p	r	p	0.3	r

where p and r are probabilities.

Given that $E(X) = 1.95$

find the exact value of $E\left(\sqrt{X+1}\right)$ giving your answer in the form $a+b\sqrt{2}$ where a and b are rational.

(6)

$E(X) = -1 \times p + 0 \times r + 1 \times p + 3 \times 0.3 + 7r = 0.9 + 7r$

$0.9 + 7r = 1.95 \Rightarrow 7r = 1.05 \Rightarrow r = 0.15$

$\Sigma P(X = x) = 1: \quad 2p + 0.3 + 2r = 1$

$2p = 1 - 0.3 - 0.3 = 0.4 \Rightarrow p = 0.2$

x	-1	0	1	3	7
$\sqrt{(x+1)}$	0	1	$\sqrt{2}$	2	$2\sqrt{2}$
$P(X = x)$	0.2	0.15	0.2	0.3	0.15

$E(\sqrt{(X+1)}) = 0 \times 0.2 + 1 \times 0.15 + \sqrt{2} \times 0.2 + 2 \times 0.3 + 2\sqrt{2} \times 0.15$

$= 0.15 + 0.2\sqrt{2} + 0.6 + 0.3\sqrt{2}$

$= 0.75 + 0.5\sqrt{2}$

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Question 3 continued

(Total for Question 3 is 6 marks)

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4. Robin shoots 8 arrows at a target each day for 100 days.

The number of times he hits the target each day is summarised in the table below.

Number of hits	0	1	2	3	4	5	6	7	8
Frequency	1	10	30	34	17	4	2	0	2

Misha believes that these data can be modelled by a binomial distribution.

(a) State, in context, two assumptions that are implied by the use of this model.

(2)

(b) Find an estimate for the proportion of arrows Robin shoots that hit the target.

(2)

Misha calculates expected frequencies, to 2 decimal places, as follows.

Number of hits	0	1	2	3	4	5	6	7	8
Expected frequency	2.81	12.67	r	28.05	19.73	s	2.50	0.40	0.03

(c) Find the value of r and the value of s

(3)

Misha correctly used a suitable test to assess her belief.

(d) (i) Explain why she used a test with 3 degrees of freedom.

(2)

(ii) Complete the test using a 5% level of significance.
You should clearly state your hypotheses, test statistic, critical value and conclusion.

(6)

(a) *The probability of an arrow hitting the target is constant.*

The arrows are shot independently of each other.

(b) $\hat{p} = \dfrac{0\times1 + 1\times10 + 2\times30 + 3\times34 + 4\times17 + 5\times4 + 6\times2 + 7\times0 + 8\times2}{100 \times 8}$

$= \dfrac{288}{800} = \dfrac{2.88}{8} = 0.36$

(c) $X \sim B(8, 0.36)$

$r = 100 \times P(X = 2) = 100 \times 0.24936... = 24.94$

$s = 100 - (2.81 + 12.67 + 24.94 + 28.05 + 19.73 + 2.50 + 0.40 + 0.03)$

$s = 8.87$

(d)(i) *Some expected frequencies are less than 5, so classes must be pooled*

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Question 4 continued

to make all $E_i > 5$: hits 0 and 1 together, and 5 or more together, leaving 5 classes.

There are 2 constraints (the total of 100 and p was estimated from the data), so $\nu = 5 - 2 = 3$

(d)(ii) H_0: A binomial distribution is a suitable model for these data

H_1: A binomial distribution is a suitable model for these data

Hits	0 or 1	2	3	4	5 or more
O_i	11	30	34	17	8
E_i	15.48	24.94	28.05	19.73	11.80
$\frac{(O_i - E_i)^2}{E_i}$	1.297	1.027	1.262	0.378	1.224

Test statistic $= \Sigma\frac{(O_i - E_i)^2}{E_i} = 5.1867... = 5.19$

Critical value: $\chi^2_3(0.05) = 7.815$

$5.19 < 7.815$, so the result is not significant: do not reject H_0

The data are consistent with Misha's belief: the binomial distribution B(8, 0.36) is a suitable model for the number of hits per day.

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Question 4 continued

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Question 4 continued

Question 4 continued

(Total for Question 4 is 15 marks)

TOTAL FOR FURTHER STATISTICS 1 IS 40 MARKS

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