Pearson Edexcel Level 3 GCE

Friday 19 May 2023

Afternoon

Paper reference **8FM0/28**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
28: Decision Mathematics 2
(Part of option K only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator,
D2 Answer Book (enclosed)

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.
- Do not return the question paper with the D2 Answer Book.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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P 7 2 8 1 4 A

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1. Five workers, A, B, C, D and E, are available to complete four tasks, P, Q, R and S.

Each worker can only be assigned to at most one task, and each task must be done by at most one worker.

Worker B cannot be assigned to task Q and worker E cannot be assigned to task S.

The time, in minutes, that each worker takes to complete each task is shown in the table below.

	P	Q	R	S
A	38	39	37	37
B	39	–	39	40
C	41	44	40	42
D	40	41	39	38
E	36	39	41	–

The Hungarian algorithm is to be used to find the least total time to complete all four tasks.

(a) Explain how the table should be modified so that the Hungarian algorithm can be applied.

(2)

(b) (i) Use the Hungarian algorithm to obtain an allocation that minimises the total time.

(ii) Explain how you determined if the table was optimal at each stage.

(6)

(c) Calculate the least total time to complete all four tasks.

(1)

(Total for Question 1 is 9 marks)

2.

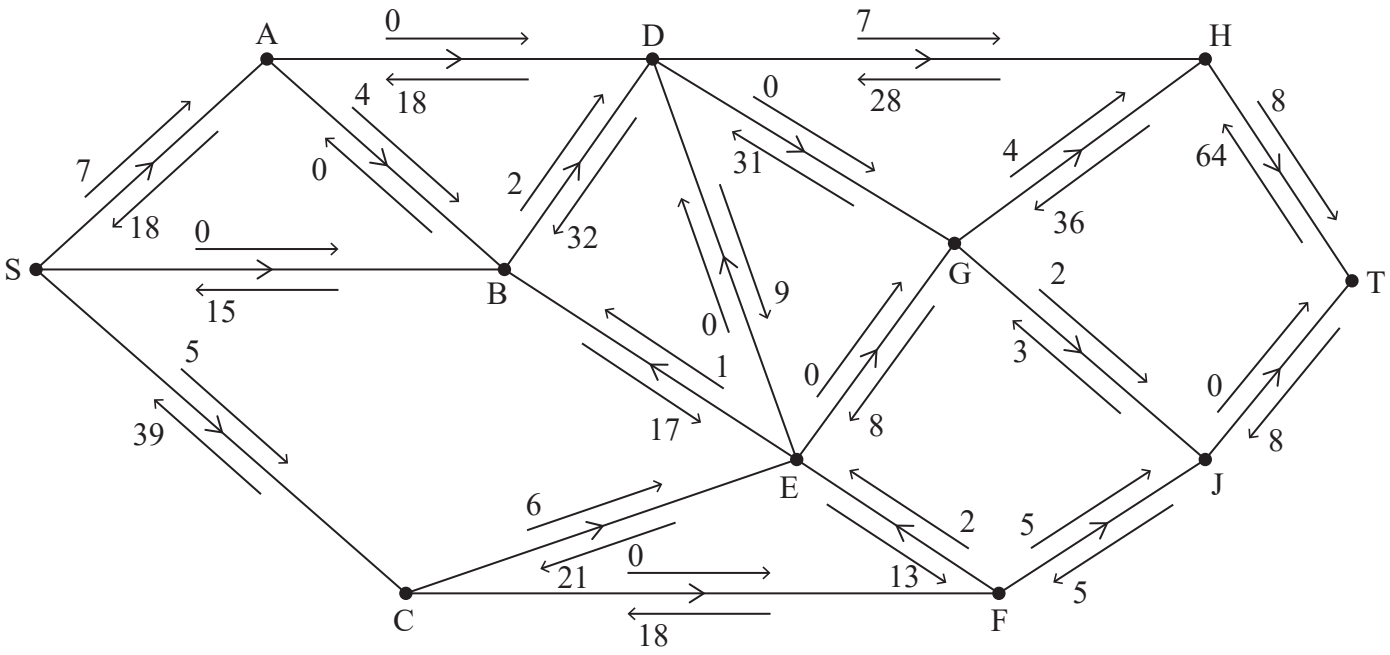


Figure 1

An engineer monitors a system of pipes through which a fluid flows from the source, S, to the sink, T.

The engineer initialises the labelling procedure for this system, and the excess capacities and potential backflows are shown on the arrows either side of each arc, as shown in Figure 1.

(a) State the value of the initial flow.

(1)

(b) Obtain the capacity of the cut that passes through the arcs SA, SB, CE, FE and FJ.

(1)

(c) Use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

(3)

(d) Use your answer to (c) to draw a maximum flow pattern on Diagram 1 in the answer book.

(1)

(e) Prove that the answer to (d) is optimal.

(3)

(Total for Question 2 is 9 marks)

3. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays X	B plays Y
A plays Q	2	–2
A plays R	–1	5
A plays S	3	4
A plays T	0	2

(a) (i) Show that this game is stable.

(ii) State the value of this game to player B.

(3)

Option S is removed from player A's choices and the reduced game, with option S removed, is no longer stable.

(b) Write down the reduced pay-off matrix for player B.

(1)

Let B play option X with probability p and option Y with probability $1 - p$.

(c) Use a graphical method to find the optimal value of p and hence find the best strategy for player B in the reduced game.

(6)

(d) (i) Find the value of the reduced game to player A.

(ii) State which option player A should never play in the reduced game.

(iii) Hence find the best strategy for player A in the reduced game.

(4)

(Total for Question 3 is 14 marks)

4. A sequence $\{u_n\}$, where $n \geqslant 0$, satisfies the recurrence relation

$$u_{n+1} = \frac{3}{2}u_n - 2n^2 - 4 \qquad u_0 = k$$

where k is an integer.

(a) Determine an expression for u_n in terms of n and k.

(6)

Given that $u_{10} > 5000$

(b) determine the minimum possible value of k.

(2)

(Total for Question 4 is 8 marks)

TOTAL FOR DECISION MATHEMATICS 2 IS 40 MARKS

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Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

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Total Marks

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1. (a)

(a) Add a dummy column X (e.g. all values 0) so that the array is square (5 × 5), since there are 5 workers but only 4 tasks.
Replace the dashes in cells BQ and ES by a large number, e.g. 100 (> 44), so that B is never given Q and E is never given S.

	P	Q	R	S
A	38	39	37	37
B	39	–	39	40
C	41	44	40	42
D	40	41	39	38
E	36	39	41	–

You may not need to use all of these tables or all the rows/columns in each table.

	P	Q	R	S	X
A	38	39	37	37	0
B	39	100	39	40	0
C	41	44	40	42	0
D	40	41	39	38	0
E	36	39	41	100	0
min	36	39	37	37	0

(b)(i) Modified table (dummy X, 100 in BQ, ES). Each row already contains a 0 (column X), so reducing rows leaves the table unchanged. Reduce columns: subtract the column minima 36, 39, 37, 37, 0.

	P	Q	R	S	X
A	~~2~~	~~0~~	~~0~~	~~0~~	~~0~~
B	3	61	2	3	0
C	5	5	3	5	0
D	4	2	2	1	0
E	~~0~~	~~0~~	~~4~~	~~63~~	~~0~~

Zeros covered by 3 lines (rows A, E and column X). 3 < 5 so not optimal. Smallest uncovered element = 1: subtract 1 from uncovered elements, add 1 to elements covered twice (AX, EX).

	P	Q	R	S	X
A	~~2~~	~~0~~	~~0~~	~~0~~	~~1~~
B	2	60	1	2	0
C	4	4	2	4	0
D	3	1	1	0	0
E	~~0~~	~~0~~	~~4~~	~~63~~	~~1~~

Now 4 lines are needed to cover the zeros (rows A, E and columns S, X). 4 < 5 so not optimal. Smallest uncovered element = 1: subtract 1 from uncovered elements, add 1 to elements covered twice (AS, AX, ES, EX).

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Question 1 continued

	P	Q	R	S	X
A	~~2~~	~~0~~	~~0~~	~~1~~	~~2~~
B	1	59	0	2	0
C	3	3	1	4	0
D	~~2~~	~~0~~	~~0~~	~~0~~	~~0~~
E	~~0~~	~~0~~	~~4~~	~~64~~	~~2~~

5 lines are now needed to cover all the zeros (rows A, D, E and columns R, X), so the table is optimal.

	P	Q	R	S	X
A	2	(0)	0	1	2
B	1	59	(0)	2	0
C	3	3	1	4	(0)
D	2	0	0	(0)	0
E	(0)	0	4	64	2

Allocation (ringed zeros):
A to Q, B to R, D to S, E to P
(C does no task: C is given dummy X)

	P	Q	R	S	
A					
B					
C					
D					
E					

(ii) At each stage I found the minimum number of lines needed to cover all the zeros. If fewer than 5 lines (the size of the 5 × 5 table) were needed the table was not optimal; when 5 lines were needed the table was optimal.

	P	Q	R	S	
A					
B					
C					
D					
E					

(c) Least total time = 39 + 39 + 38 + 36
= 152 minutes

	P	Q	R	S	
A					
B					
C					
D					
E					

(Total for Question 1 is 9 marks)

2.

(a) Initial flow = 18 + 15 + 39 = 72

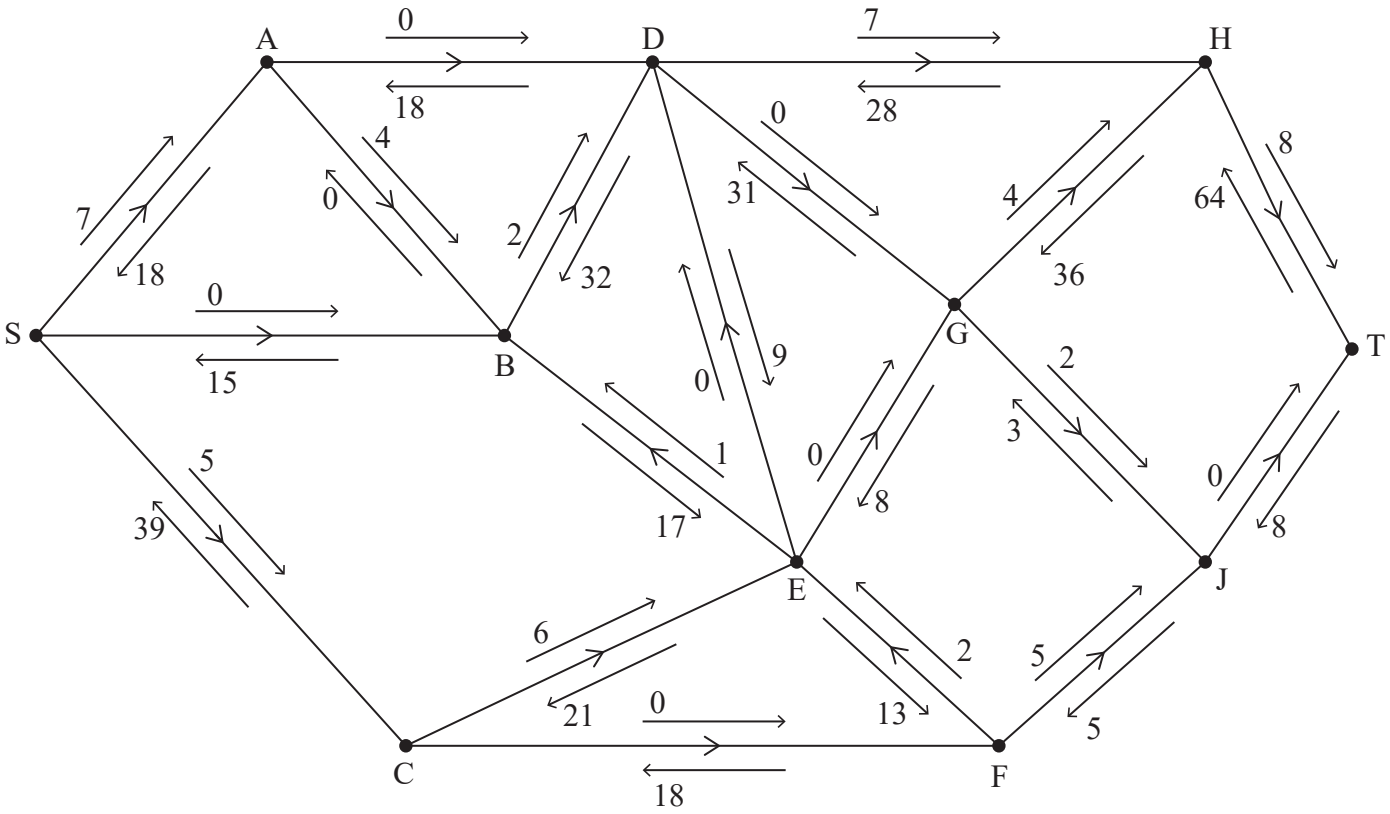
(b) Capacity of cut = 25 + 15 + 27 + 15 + 10 = 92

(c) Flow-augmenting routes:

S C E F J G H T flow 3

S A B D H T flow 2

Maximum flow = 72 + 3 + 2 = 77



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Question 2 continued

(d) Maximum flow pattern shown on Diagram 1 below (total flow 77).

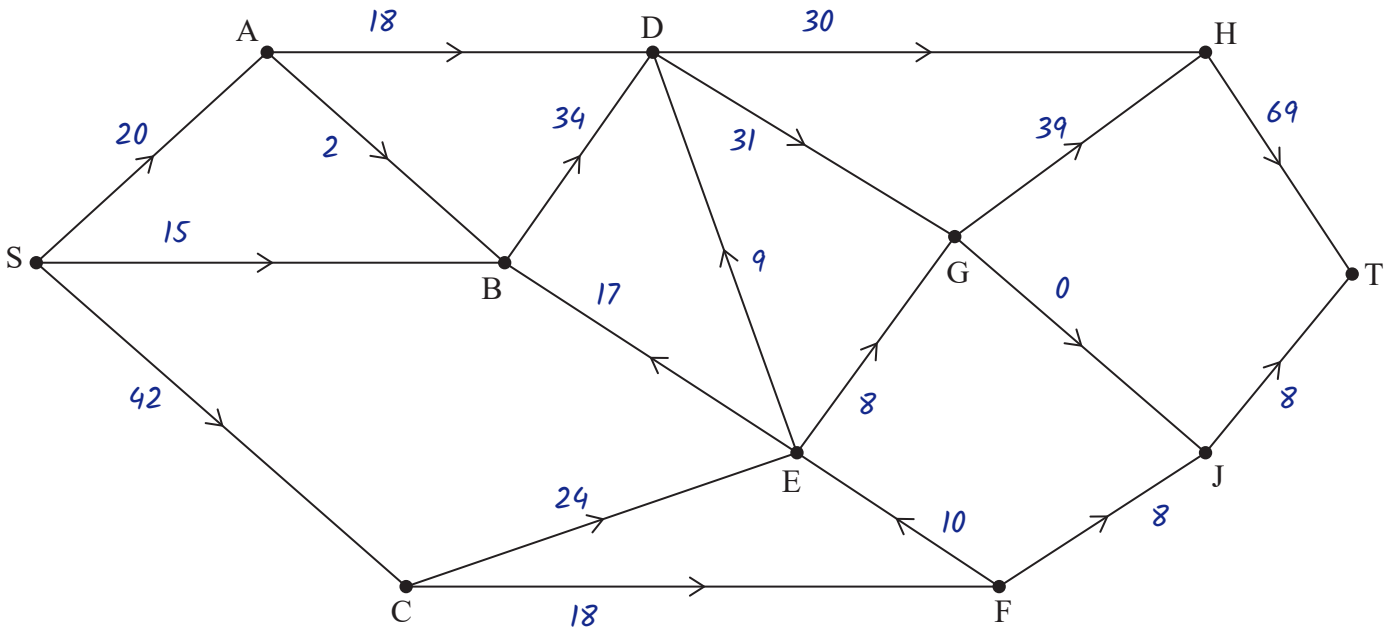


Diagram 1

(e) Cut through arcs AD, BD, ED, EG, GJ and JT:

value = 18 + 34 + 9 + 8 + 0 + 8 = 77

(GJ goes from the sink side to the source side so it contributes 0)

Flow through the network = 77 = value of this cut, so by the maximum flow minimum cut theorem the flow of 77 is a maximum, hence the flow pattern in (d) is optimal.

(Total for Question 2 is 9 marks)

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3.

	B plays X	*B* plays Y
A plays Q	2	–2
A plays R	–1	5
A plays S	3	4
A plays T	0	2

(a)(i) Row minima: Q: −2, R: −1, S: 3, T: 0 ⇒ max(row min) = 3

Column maxima: X: 3, Y: 5 ⇒ min(col max) = 3

Row maximin = column minimax = 3, so the game is stable.

(ii) Value of the game to player B = −3

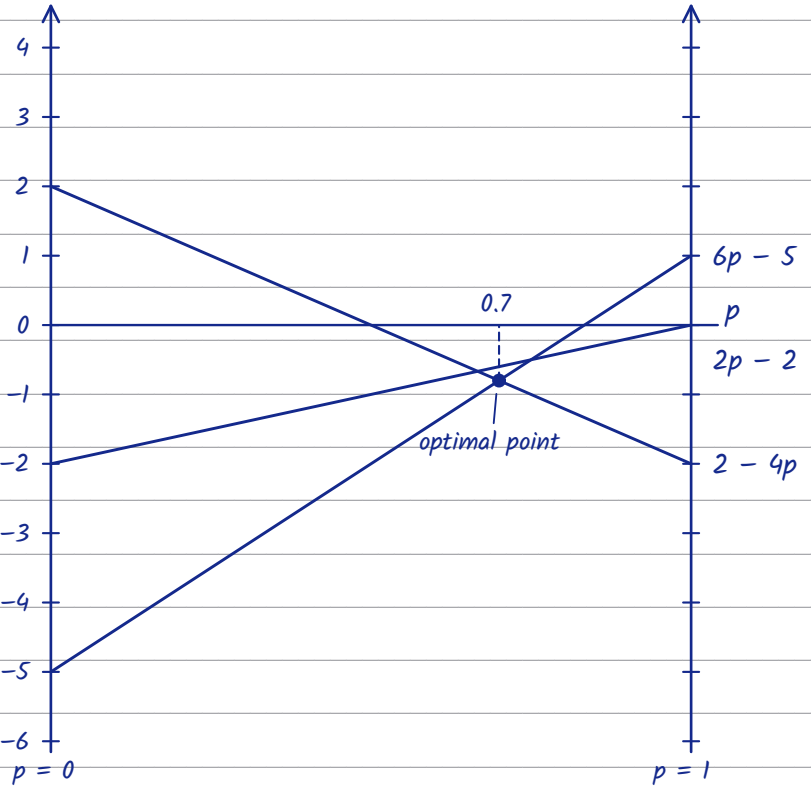
(b) Reduced pay-off matrix for B:

	A plays Q	A plays R	A plays T
B plays X	−2	1	0
B plays Y	2	−5	−2

(c) If A plays Q, B's expected gain = −2p + 2(1 − p) = 2 − 4p

If A plays R, B's expected gain = p − 5(1 − p) = 6p − 5

If A plays T, B's expected gain = 0p − 2(1 − p) = 2p − 2



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Question 3 continued

B maximises the minimum gain, so the optimal point is the highest point of the lower boundary: where $2 - 4p$ meets $6p - 5$.

$2 - 4p = 6p - 5 \Rightarrow 10p = 7 \Rightarrow p = 7/10$

B should play X with probability 7/10 and Y with probability 3/10.

(d)(i) Value to B $= 2 - 4(0.7) = -0.8$

so the value of the reduced game to player A $= 4/5$ $(= 0.8)$

(ii) Player A should never play option T

(at $p = 0.7$, $2p - 2 = -0.6 \neq -0.8$, so T is not at the optimal point)

(iii) Let A play Q with probability q and R with probability $1 - q$.

If B plays X: $2q + (-1)(1 - q) = 4/5$

$3q - 1 = 4/5 \Rightarrow 3q = 9/5 \Rightarrow q = 3/5$

A should play Q with probability 3/5 and R with probability 2/5

(and never play S or T).

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 14 marks)

4.

(a) Auxiliary equation: $m - 3/2 = 0 \Rightarrow m = 3/2$

Complementary function: $u_n = A(1.5)^n$

Particular solution: try $u_n = \alpha n^2 + \beta n + \gamma$

$\alpha(n + 1)^2 + \beta(n + 1) + \gamma = (3/2)(\alpha n^2 + \beta n + \gamma) - 2n^2 - 4$

$\times 2$: $2\alpha n^2 + (4\alpha + 2\beta)n + (2\alpha + 2\beta + 2\gamma) = (3\alpha - 4)n^2 + 3\beta n + (3\gamma - 8)$

Compare coefficients:

n^2: $2\alpha = 3\alpha - 4 \Rightarrow \alpha = 4$

n: $4\alpha + 2\beta = 3\beta \Rightarrow \beta = 4\alpha = 16$

const: $2\alpha + 2\beta + 2\gamma = 3\gamma - 8 \Rightarrow 8 + 32 + 2\gamma = 3\gamma - 8 \Rightarrow \gamma = 48$

General solution: $u_n = A(1.5)^n + 4n^2 + 16n + 48$

$u_0 = k \Rightarrow A + 48 = k \Rightarrow A = k - 48$

$u_n = (k - 48)(1.5)^n + 4n^2 + 16n + 48$

(b) $u_{10} = (k - 48)(1.5)^{10} + 4(10)^2 + 16(10) + 48 > 5000$

$(k - 48)(57.6650...) + 608 > 5000$

$k - 48 > 4392 \div 57.6650... = 76.1636...$

$k > 124.1636...$

Minimum possible value of $k = 125$

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Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

TOTAL FOR DECISION MATHEMATICS 2 IS 40 MARKS

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