Pearson Edexcel Level 3 GCE

Friday 19 May 2023

Afternoon

Paper reference **8FM0/27**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
27: Decision Mathematics 1
(Part of options D, F, H and K)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator,
D1 Answer Book (enclosed)

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.
- Do not return the question paper with the D1 Answer Book.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P72813A

©2023 Pearson Education Ltd.
N:1/1/

P 7 2 8 1 3 A

Pearson

Answer ALL questions. Write your answers in the answer book provided.

1.

67 59 46 71 40 48 53 63 45 54 56

The list of eleven numbers shown above is to be sorted into descending order.

Carry out a quick sort to produce the sorted list. You should show the result of each pass and identify the pivots clearly.

(Total for Question 1 is 4 marks)

2.

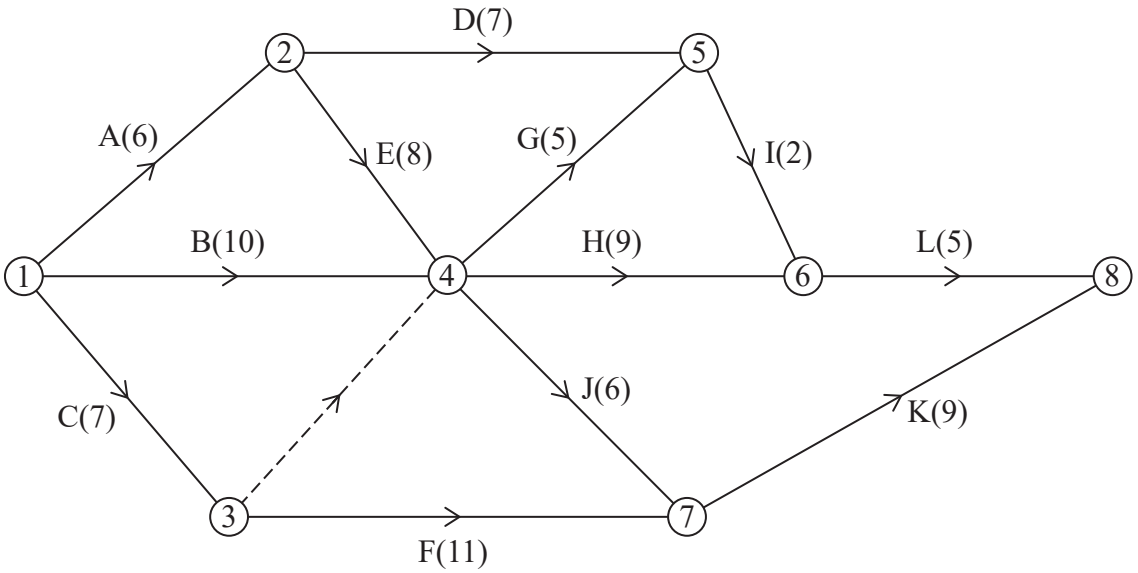


Figure 1

A project is modelled by the activity network shown in Figure 1. The activities are represented by the arcs. The number in brackets on each arc gives the time required, in hours, to complete the corresponding activity. The numbers in circles are the event numbers. Each activity requires one worker, and the project is to be completed in the shortest possible time.

(a) Explain the significance of the dummy activity from event 3 to event 4

(1)

(b) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(3)

(c) State the critical activities.

(1)

(d) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

(1)

(e) Draw a Gantt chart for this project on Grid 1 in the answer book.

(4)

(Total for Question 2 is 10 marks)

3.

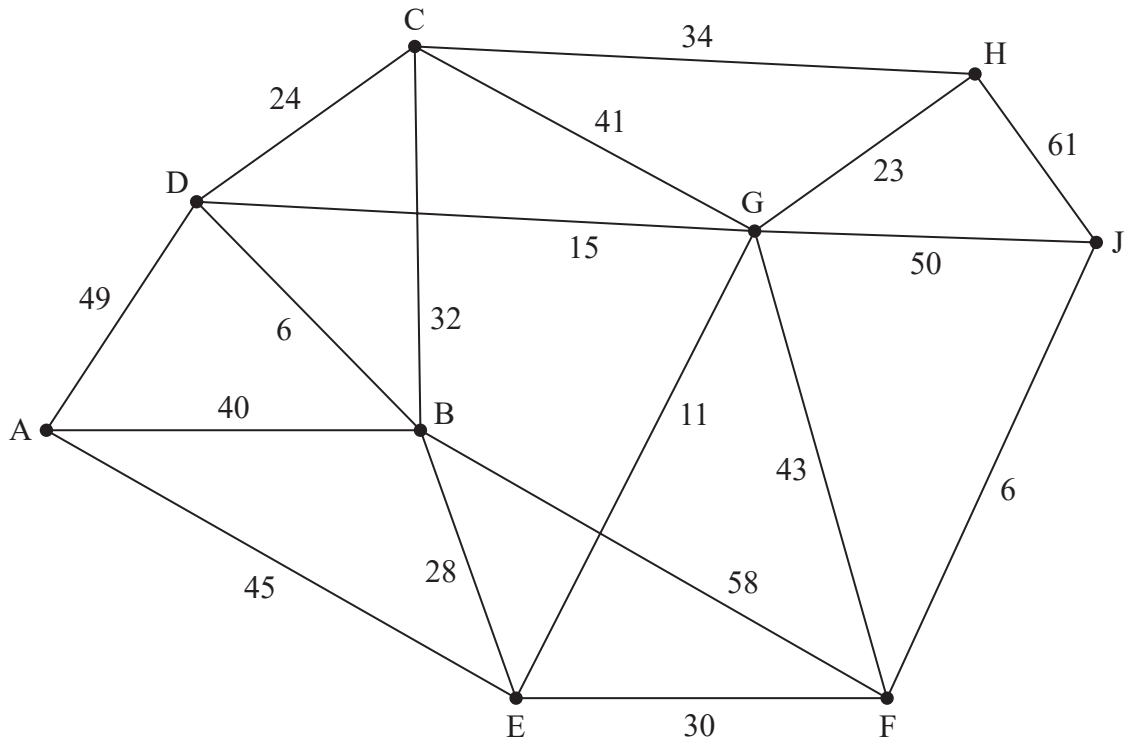


Figure 2

Figure 2 represents a network of train tracks. The number on each edge represents the length, in kilometres, of the corresponding track.
Dyfan wishes to travel from A to J via C. Dyfan wishes to minimise the distance they travel.

Given that Dijkstra's algorithm is to be applied only once to find Dyfan's route,

(a) explain why the algorithm should begin at C.

(1)

(b) Use Dijkstra's algorithm to find the shortest route from A to J via C. State this route and its length.

(6)

(c) Use Prim's algorithm, starting at C, to find a minimum spanning tree for the network. You must clearly state the order in which you select the edges of your tree.

(3)

(d) State the total length, in km, of the minimum spanning tree.

(1)

(Total for Question 3 is 11 marks)

4.

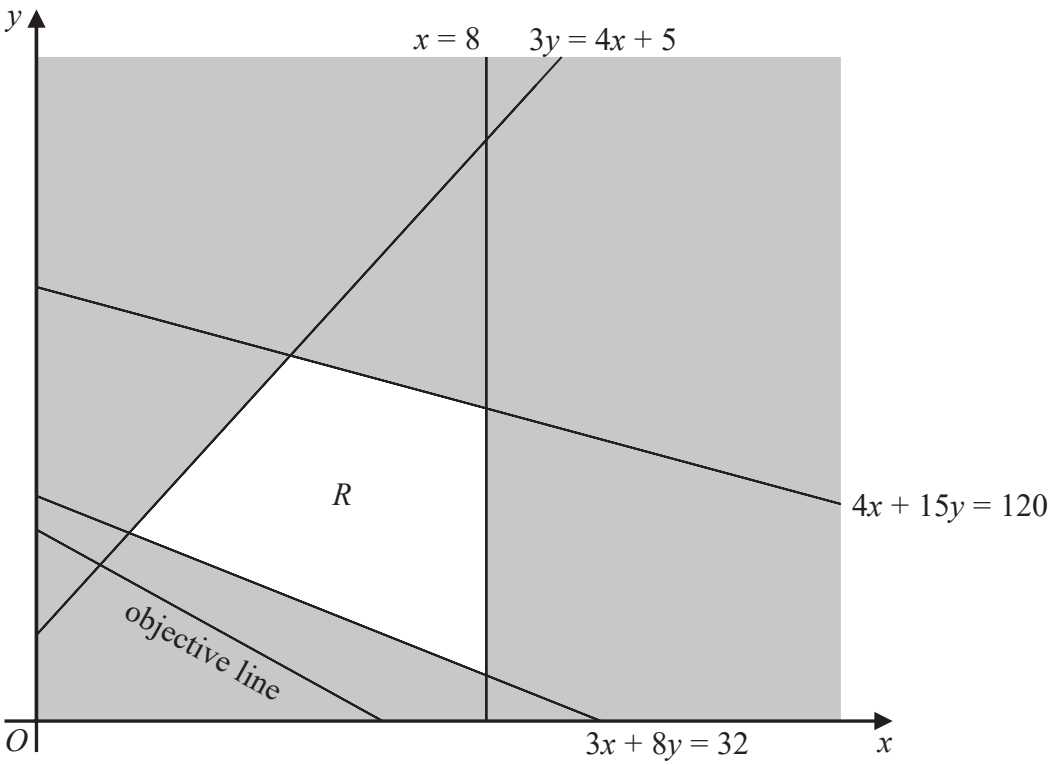


Figure 3

Figure 3 shows the constraints of a linear programming problem in x and y.
The unshaded area, including its boundaries, forms the feasible region, R.
An objective line has been drawn and labelled on the graph.

(a) State the inequalities that define the feasible region.

(2)

The maximum value of the objective function is $\frac{160}{3}$

The minimum value of the objective function is $\frac{883}{41}$

(b) Determine the objective function, showing your working clearly.

(5)

(Total for Question 4 is 7 marks)

5.

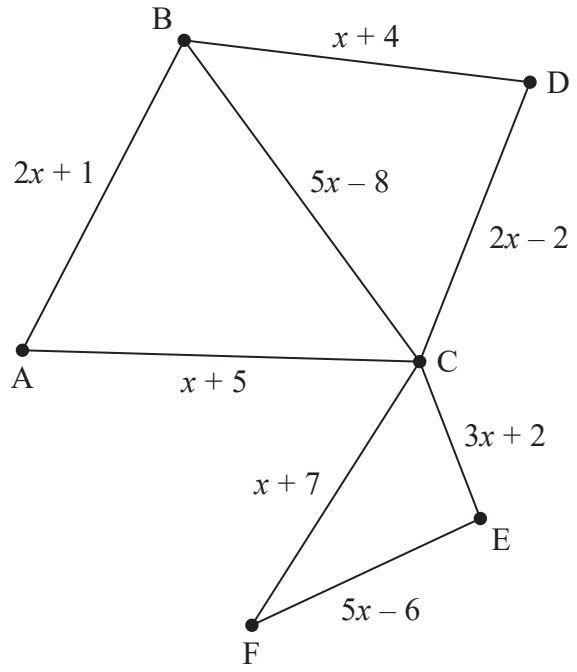


Figure 4

[*The weight of the network is* $20x + 3$]

Figure 4 shows a graph G that contains 8 arcs and 6 vertices.

(a) State the minimum number of arcs that would need to be added to make G into an Eulerian graph.

(1)

(b) Explain whether or not the route A – C – F – E – C – D – B is an example of a path on G.

(1)

Figure 4 represents a network of 8 roads in a city. The expression on each arc gives the time, in minutes, to travel along the corresponding road.

You are given that $x > 1.6$

A route is required that

- starts and finishes at the same vertex
- traverses each road at least once
- minimises the total time taken

The route inspection algorithm is applied to the network in Figure 4 and the time taken for the route is found to be at most 189 minutes.

Given that the inspection route contains two roads that need to be traversed twice,

(c) determine the range of possible values of x, making your reasoning clear.

(6)

(Total for Question 5 is 8 marks)

TOTAL FOR DECISION MATHEMATICS 1 IS 40 MARKS

BLANK PAGE

BLANK PAGE

Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Friday 19 May 2023

Afternoon

Paper reference **8FM0/27**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
27: Decision Mathematics 1
(Part of options D, F, H and K)

D1 Answer Book
Do not return the question paper with the answer book.

Total Marks

Turn over ▶

P72813A

©2023 Pearson Education Ltd.
N:1/1/

P 7 2 8 1 3 A 0 1 1 2

Pearson

1.

67	59	46	71	40	48	53	63	45	54	56	Pivot(s)
67	59	46	71	40	(48)	53	63	45	54	56	48
67	59	71	(53)	63	54	56	48	46	(40)	45	53, 40
67	59	71	(63)	54	56	53	48	46	(45)	40	63, 45
67	(71)	63	59	(54)	56	53	48	(46)	45	40	71, 54, 46
71	(67)	63	59	(56)	54	53	48	46	45	40	67, 56
71	67	63	(59)	56	54	53	48	46	45	40	59
All values fixed, so the list is sorted:											
71	67	63	59	56	54	53	48	46	45	40	

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 1 continued

(Total for Question 1 is 4 marks)

2. (a)

The dummy is needed because activity F depends only on C, but activities G, H and J depend on B, C and E (so C must be complete before G, H and J can start).

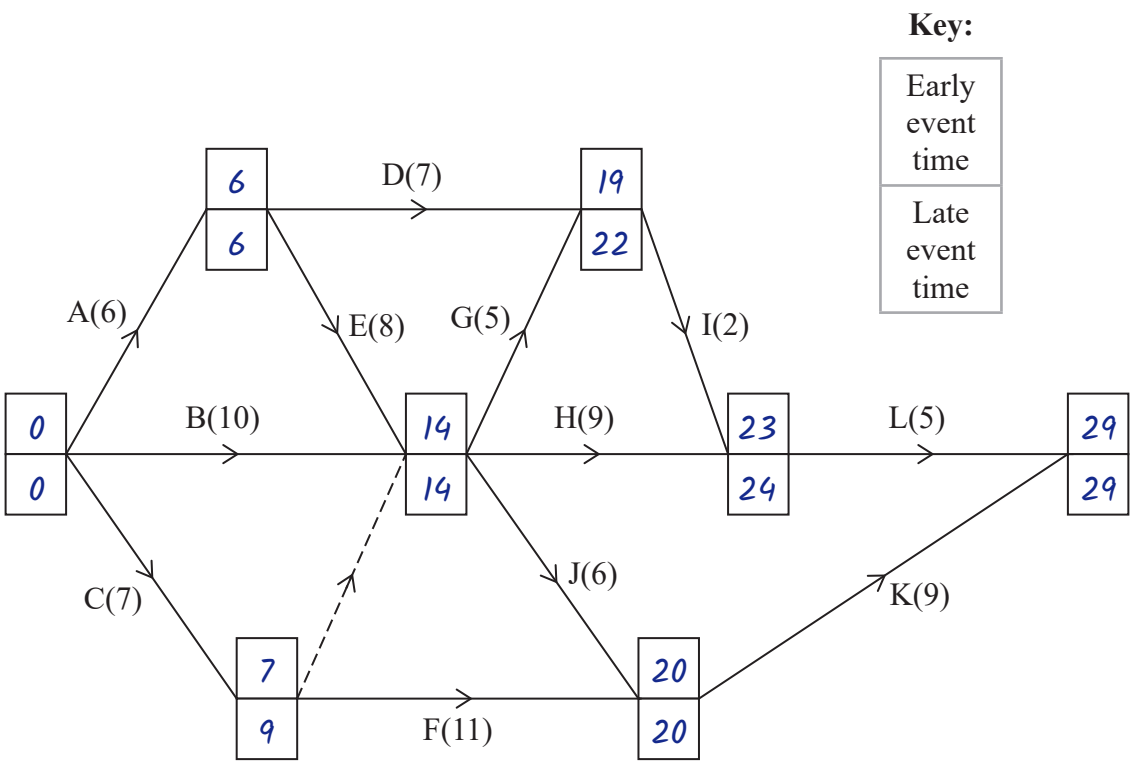


Diagram 1

(c) Critical activities: A, E, J and K

(d) Lower bound = (sum of activity times) ÷ (critical path length)

= (6+10+7+7+8+11+5+9+2+6+9+5) ÷ 29

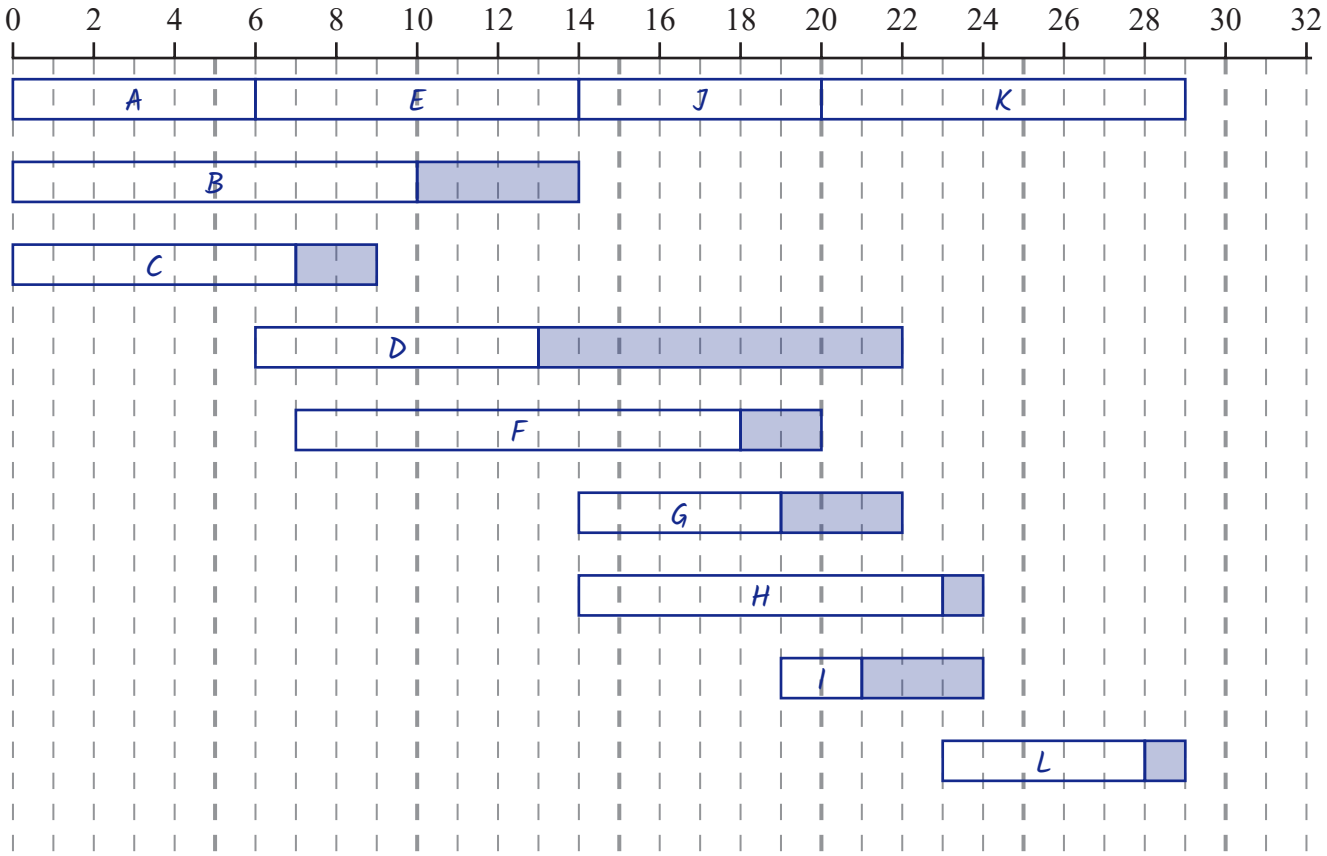
= 85 ÷ 29 = 2.931...

so a lower bound of 3 workers

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued



Grid 1

(e) Shaded sections show the float of each non-critical activity.

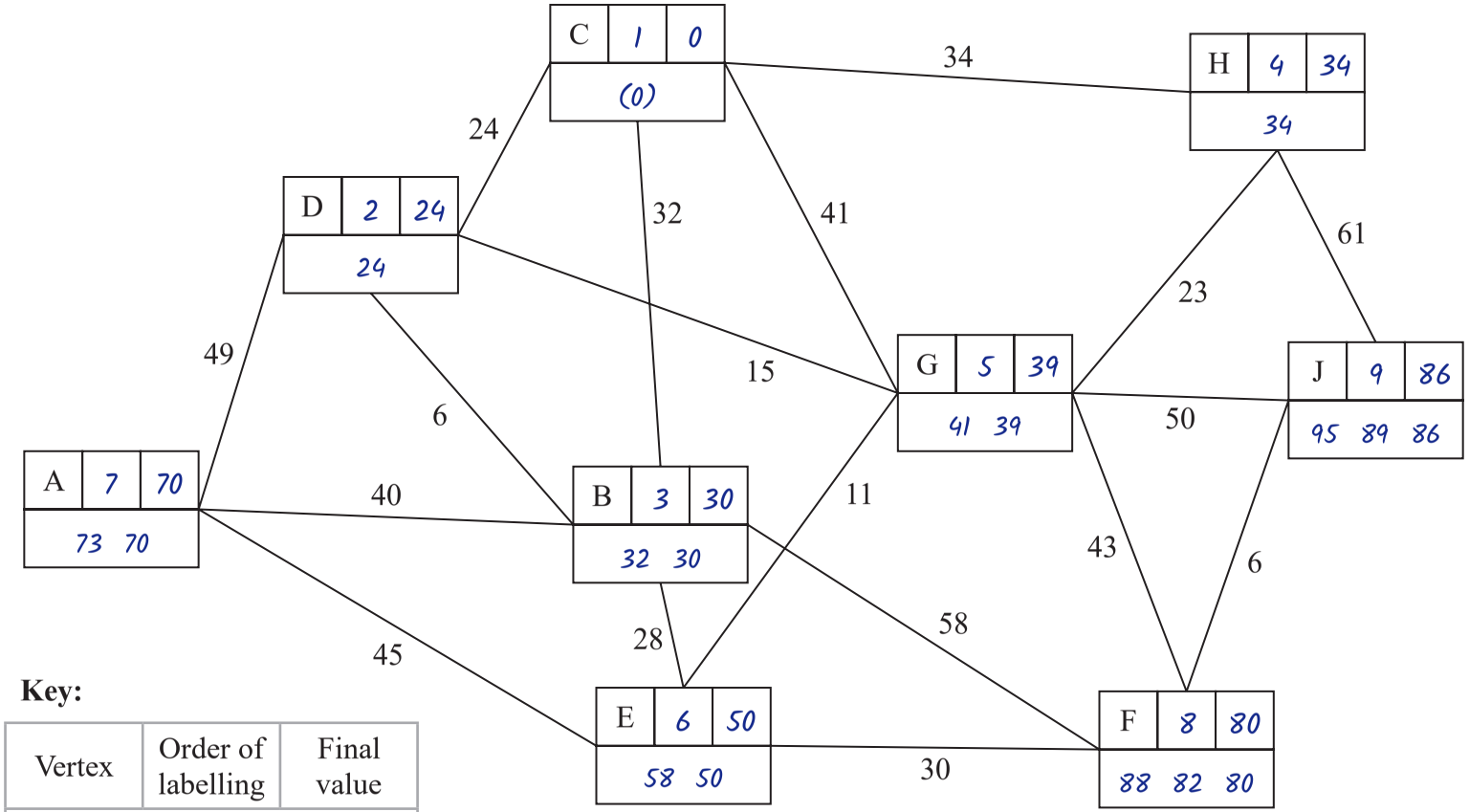
(Total for Question 2 is 10 marks)

3. (a)

Dijkstra's algorithm finds the shortest paths from one start vertex to every other vertex.

The shortest route A to J via C is the shortest path from C to A plus the shortest path from C to J,

so starting at C gives both of these paths from a single application of the algorithm.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest route from A to J via C: A B D C D G E F J

Length of shortest route from A to J via C: 70 + 86 = 156 km

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 3 continued

(b) Tracing back: C to A is C D B A (70), C to J is C D G E F J (86)

(c) Prim's algorithm starting at C, edges selected in order:

CD (24), DB (6), DG (15), GE (11), GH (23), EF (30), FJ (6), AB (40)

(vertices joined in the order C, D, B, G, E, H, F, J, A)

(d) 24 + 6 + 15 + 11 + 23 + 30 + 6 + 40 = 155

Total length of the minimum spanning tree = 155 km

(Total for Question 3 is 11 marks)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

4.

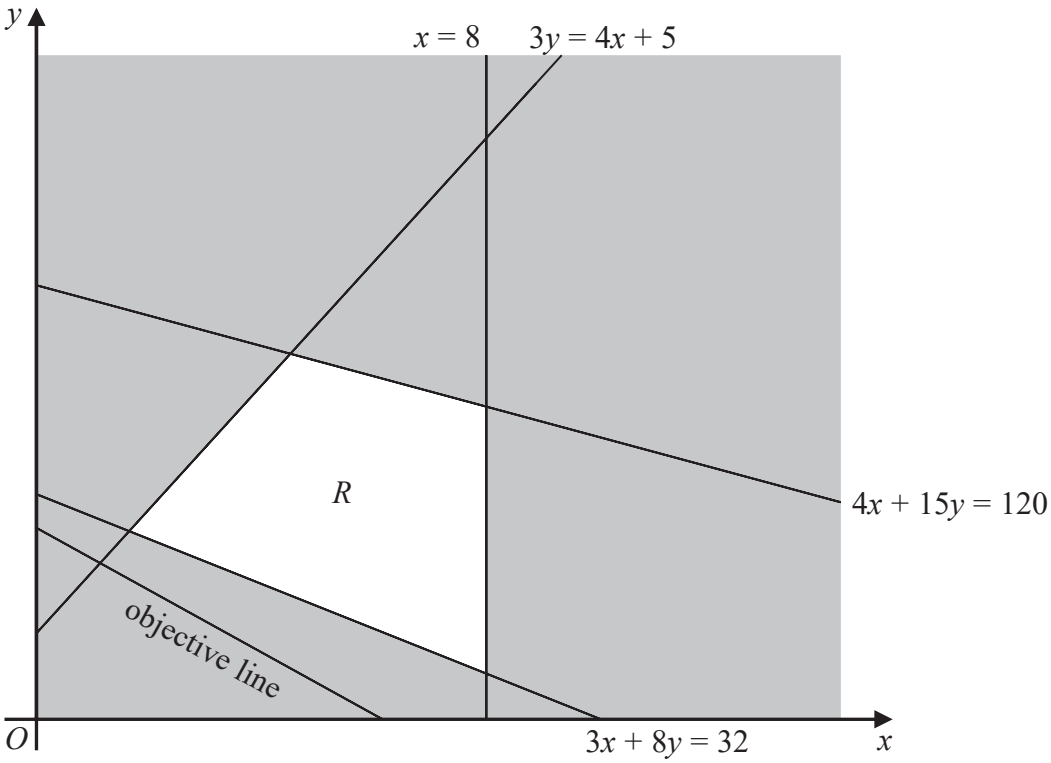


Figure 3

(a) $x \leq 8$, $3x + 8y \geq 32$, $3y \leq 4x + 5$, $4x + 15y \leq 120$

(b) Sliding the objective line parallel to itself across R, the minimum is at the lower left vertex ($3y = 4x + 5$ meets $3x + 8y = 32$) and the maximum is at the upper right vertex ($x = 8$ meets $4x + 15y = 120$).

Minimum point: $3y = 4x + 5 \Rightarrow y = (4x + 5)/3$

$3x + 8(4x + 5)/3 = 32 \Rightarrow 9x + 32x + 40 = 96 \Rightarrow 41x = 56$

$x = 56/41$, $y = (4 \times 56/41 + 5)/3 = 143/41$ so $(56/41, 143/41)$

Maximum point: $x = 8 \Rightarrow 32 + 15y = 120 \Rightarrow 15y = 88$

$y = 88/15$ so $(8, 88/15)$

Let the objective function be $P = ax + by$

$(56/41)a + (143/41)b = 883/41 \Rightarrow 56a + 143b = 883$ (1)

$8a + (88/15)b = 160/3 \Rightarrow 120a + 88b = 800 \Rightarrow 15a + 11b = 100$ (2)

(2) × 13: $195a + 143b = 1300$

subtract (1): $139a = 417 \Rightarrow a = 3$

$15(3) + 11b = 100 \Rightarrow 11b = 55 \Rightarrow b = 5$

Check: $3(56/41) + 5(143/41) = 883/41$, $3(8) + 5(88/15) = 160/3$

Objective function is $P = 3x + 5y$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 7 marks)

5.

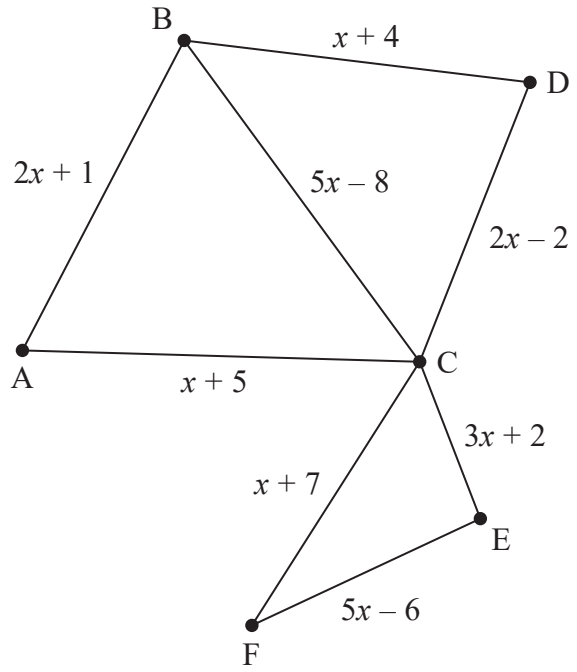


Figure 4

[*The weight of the network is* $20x + 3$]

(a) 1

(b) It is not a path, because vertex C appears twice in the route

(the route contains the cycle C, F, E, C).

(c) Orders: A 2, B 3, C 5, D 2, E 2, F 2 so the odd vertices are B and C

Ways to join B and C:

BC = 5x – 8 (one road)

BA + AC = (2x + 1) + (x + 5) = 3x + 6 (two roads)

BD + DC = (x + 4) + (2x – 2) = 3x + 2 (two roads)

Two roads are repeated, so either BA, AC or BD, DC is repeated.

3x + 6 > 3x + 2 for all x, so BD and DC are the roads repeated.

This must be shorter than repeating BC:

5x – 8 > 3x + 2 ⇒ 2x > 10 ⇒ x > 5

Length of route = (20x + 3) + (3x + 2) ≤ 189

23x + 5 ≤ 189 ⇒ 23x ≤ 184 ⇒ x ≤ 8

∴ 5 < x ≤ 8

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)

TOTAL FOR DECISION MATHEMATICS 1 IS 40 MARKS

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA