Please check the examination details below before entering your candidate information

Candidate surname	Other names
StemUp Model Solution	

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Friday 19 May 2023

Afternoon

Paper reference **8FM0/26**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. Three particles of masses $4m$, $2m$ and km are placed at the points with coordinates $(-3,-1)$, $(6, 1)$ and $(-1, 5)$ respectively.

Given that the centre of mass of the three particles is at the point with coordinates $(\bar{x}, \bar{y})$

(a) show that $\bar{x} = \dfrac{-k}{k+6}$

(3)

(b) find $\bar{y}$ in terms of k.

(2)

Given that the centre of mass of the three particles lies on the line with equation $y = 2x + 3$

(c) find the value of k.

(2)

A fourth particle is placed at the point with coordinates $(\lambda, 4)$.

Given that the centre of mass of the four particles also lies on the line with equation $y = 2x + 3$

(d) find the value of λ.

(2)

(a) Total mass $= 4m + 2m + km = (k + 6)m$

Moments about the y-axis:

$4m \times (-3) + 2m \times 6 + km \times (-1) = (k + 6)m\,\bar{x}$

$-12 + 12 - k = (k + 6)\,\bar{x}$

$\Rightarrow \quad \bar{x} = \dfrac{-k}{k + 6}$ as required

(b) Moments about the x-axis:

$4m \times (-1) + 2m \times 1 + km \times 5 = (k + 6)m\,\bar{y}$

$-4 + 2 + 5k = (k + 6)\,\bar{y} \quad \Rightarrow \quad \bar{y} = \dfrac{5k - 2}{k + 6}$

(c) $\bar{y} = 2\bar{x} + 3$:

$\dfrac{5k - 2}{k + 6} = \dfrac{-2k}{k + 6} + 3$

$5k - 2 = -2k + 3(k + 6) = k + 18$

$4k = 20 \quad \Rightarrow \quad k = 5$

(d) With $k = 5$, the centre of mass of the three particles is $G\left(\dfrac{-5}{11}, \dfrac{23}{11}\right)$,

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Question 1 continued

which lies on $y = 2x + 3$. The centre of mass of all four particles lies on the line joining G to $(\lambda, 4)$, so for it to be on $y = 2x + 3$ as well, the fourth particle must also lie on $y = 2x + 3$:

$4 = 2\lambda + 3$

$\Rightarrow \quad \lambda = \frac{1}{2}$

Question 1 continued

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Question 1 continued

(Total for Question 1 is 9 marks)

2. A particle P is moving along the x-axis.
At time t seconds, $t \geqslant 0$, P has acceleration $a\,\text{m s}^{-2}$ and velocity $v\,\text{m s}^{-1}$ in the direction of x increasing, where

$$v = e^{2t} + 6e^{t} - kt$$

and k is a positive constant.

When $t = \ln 2$, $a = 0$

(a) Find the value of k.

(4)

When $t = 0$, the particle passes through the fixed point A.
When $t = \ln 2$, the particle is d metres from A.

(b) Showing all stages of your working, find the value of d correct to 2 significant figures.
[*Solutions relying entirely on calculator technology are not acceptable.*]

(4)

(a) $a = \frac{dv}{dt} = 2e^{2t} + 6e^{t} - k$

$t = \ln 2$, $a = 0$: $2e^{2\ln 2} + 6e^{\ln 2} - k = 0$

$2 \times 4 + 6 \times 2 - k = 0$

$k = 20$

(b) $v = \frac{dx}{dt} = e^{2t} + 6e^{t} - 20t$

$x = \int (e^{2t} + 6e^{t} - 20t)\,dt = \frac{1}{2}e^{2t} + 6e^{t} - 10t^2 + C$

$t = 0$, $x = 0$ (at A): $\frac{1}{2} + 6 + C = 0 \quad \Rightarrow \quad C = -\frac{13}{2}$

$t = \ln 2$: $x = \frac{1}{2}e^{2\ln 2} + 6e^{\ln 2} - 10(\ln 2)^2 - \frac{13}{2}$

$x = \frac{1}{2} \times 4 + 6 \times 2 - 10(\ln 2)^2 - \frac{13}{2}$

$x = 2 + 12 - 10(\ln 2)^2 - 6.5 = 7.5 - 10(\ln 2)^2$

$x = 2.6954\ldots$

($v > 0$ for $0 \leqslant t \leqslant \ln 2$, since its least value, at $t = \ln 2$, is $16 - 20\ln 2 > 0$, so P does not turn round and d is this displacement.)

$d = 2.7$ (2 s.f.)

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Question 2 continued

(Total for Question 2 is 8 marks)

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3. A girl is cycling round a circular track.
The girl and her bicycle have a combined mass of 55 kg.
The coefficient of friction between the track surface and the tyres of the bicycle is μ.

The track is banked at an angle of 15° to the horizontal.

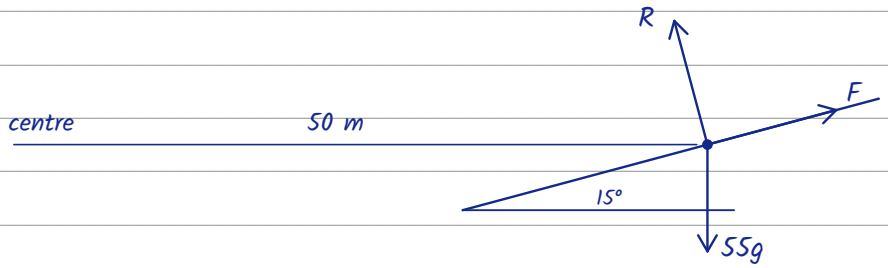
The girl and her bicycle are modelled as a particle moving in a horizontal circle of radius 50 m
The minimum speed at which the girl can cycle round this circle without slipping is 4.5 m s^{-1}

Using the model, find the value of μ.

(9)

Minimum speed, so the girl is on the point of slipping down the slope:

friction F acts up the slope and F = μR.



R(←, towards centre): $R \sin 15° - F \cos 15° = \frac{55 \times 4.5^2}{50}$ $(= 22.275)$

R(↑): $R \cos 15° + F \sin 15° = 55g$ $(= 539)$

Using $F = \mu R$:

$R(\sin 15° - \mu \cos 15°) = 22.275$ ...(1)

$R(\cos 15° + \mu \sin 15°) = 539$ ...(2)

(1) ÷ (2): $\frac{\sin 15° - \mu \cos 15°}{\cos 15° + \mu \sin 15°} = \frac{22.275}{539}$

$539 \sin 15° - 539\mu \cos 15° = 22.275 \cos 15° + 22.275\mu \sin 15°$

$\mu(539 \cos 15° + 22.275 \sin 15°) = 539 \sin 15° - 22.275 \cos 15°$

$\mu = \frac{539 \sin 15° - 22.275 \cos 15°}{539 \cos 15° + 22.275 \sin 15°} = \frac{117.987...}{526.399...}$

$\mu = 0.22414...$

$\mu = 0.224$ (3 s.f.)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 9 marks)

4.

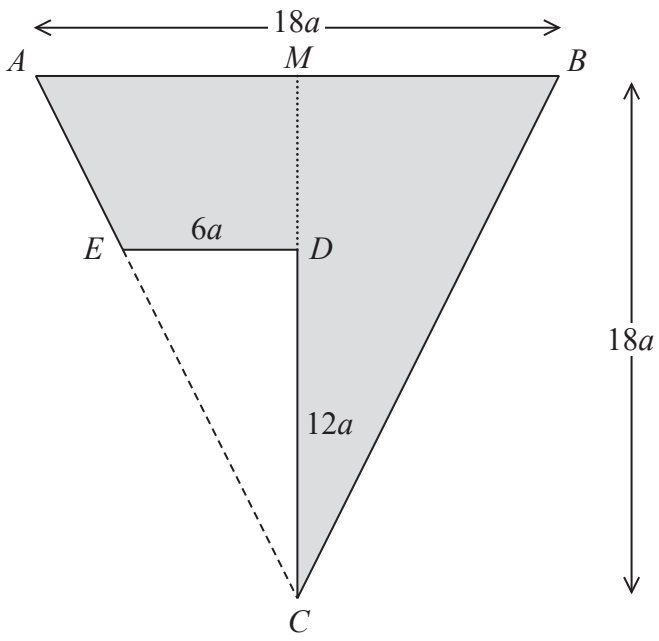


Figure 1

A uniform triangular lamina ABC is isosceles, with $AC = BC$. The midpoint of AB is M. The length of AB is $18a$ and the length of CM is $18a$.

The triangular lamina CDE, with $DE = 6a$ and $CD = 12a$, has ED parallel to AB and MDC is a straight line.

Triangle CDE is removed from triangle ABC to form the lamina L, shown shaded in Figure 1.

The distance of the centre of mass of L from MC is d.

(a) Show that $d = \frac{4}{7}a$

(4)

The lamina L is suspended by two light inextensible strings. One string is attached to L at A and the other string is attached to L at B.
The lamina hangs in equilibrium in a vertical plane with the strings vertical and AB horizontal.
The weight of L is W

(b) Find, in terms of W, the tension in the string attached to L at B

(3)

The string attached to L at B breaks, so that L is now suspended from A.
When L is hanging in equilibrium in a vertical plane, the angle between AB and the downward vertical through A is $\theta°$

(c) Find the value of θ

(7)

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Question 4 continued

(a) Area ABC $= \frac{1}{2} \times 18a \times 18a = 162a^2$, Area CDE $= \frac{1}{2} \times 6a \times 12a = 36a^2$

Area of L $= 162a^2 - 36a^2 = 126a^2$

Centre of mass of CDE is $\frac{1}{3} \times 6a = 2a$ from MC (on the side of E);

centre of mass of ABC lies on MC (symmetry).

Moments about MC (taking the B side as positive):

$162a^2 \times 0 = 36a^2 \times (-2a) + 126a^2 \times d$

$126a^2 d = 72a^3$

$d = \frac{72a}{126} = \frac{4}{7}a$ as required

(b) Horizontal distance of the centre of mass of L from A $= 9a + \frac{4}{7}a = \frac{67}{7}a$

M(A): $T \times 18a = W \times \left(9a + \frac{4}{7}a\right) = \frac{67}{7}aW$

$T = \frac{67}{126}W \quad (= 0.532W)$

(c) Distances below AB (centroid of a triangle is one third of the way up from its base):

ABC: $\frac{1}{3} \times 18a = 6a$ below AB

CDE: D and E are $18a - 12a = 6a$ below AB, so centroid is $6a + \frac{1}{3} \times 12a = 10a$ below AB

Moments about AB:

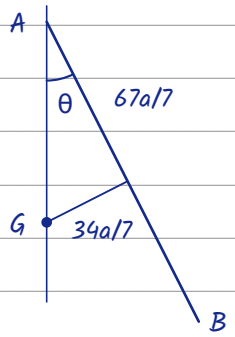
$126a^2 \times \bar{y} = 162a^2 \times 6a - 36a^2 \times (6a + 4a)$

$126a^2 \bar{y} = 972a^3 - 360a^3 = 612a^3$

$\bar{y} = \frac{612a}{126} = \frac{34}{7}a$ below AB

When L hangs freely from A, the centre of mass G is vertically below A.

Question 4 continued



G is $\frac{67}{7}a$ along AB from A and $\frac{34}{7}a$ perpendicular to AB,

and θ is the angle between AB and AG:

$$\tan\theta = \frac{34a \,/\, 7}{67a \,/\, 7} = \frac{34}{67}$$

$\theta = 26.906\ldots$

$\theta = 26.9$

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Question 4 continued

Question 4 continued

(Total for Question 4 is 14 marks)

TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS

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