Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Friday 19 May 2023

Afternoon

Paper reference **8FM0/25**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
25: Further Mechanics 1
(Part of options C, E, H and J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. Two particles, P and Q, of masses $3m$ and $2m$ respectively, are moving on a smooth horizontal plane. They are moving in opposite directions along the same straight line when they collide directly.

Immediately before the collision, P is moving with speed $2u$.

The magnitude of the impulse exerted on P by Q in the collision is $\frac{9mu}{2}$

(a) Find the speed of P immediately after the collision.

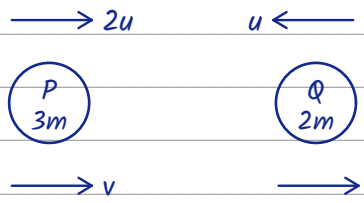
(3)

The coefficient of restitution between P and Q is e.

Given that the speed of Q immediately before the collision is u,

(b) find the value of e.

(5)



Taking the original direction of motion of P as positive.

(a) Impulse-momentum for P: $-\frac{9mu}{2} = 3m(v - 2u)$

$v - 2u = -\frac{3u}{2} \Rightarrow v = \frac{u}{2}$

Speed of P after the collision $= \frac{u}{2}$

(b) Impulse-momentum for Q: $\frac{9mu}{2} = 2m(y - (-u)) = 2m(y + u)$

$y + u = \frac{9u}{4} \Rightarrow y = \frac{5u}{4}$

NEL: $y - v = e(2u + u) \Rightarrow \frac{5u}{4} - \frac{u}{2} = 3eu$

$e = \frac{5u/4 - u/2}{2u + u} = \frac{3u/4}{3u}$

$e = \frac{1}{4}$

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Question 1 continued

(Total for Question 1 is 8 marks)

2. A racing car of mass 750 kg is moving along a straight horizontal road at a constant speed of U **km h**$^{-1}$. The engine of the racing car is working at a constant rate of 60 kW.

The resistance to the motion of the racing car is modelled as a force of magnitude $37.5v$ N, where v m s^{-1} is the speed of the racing car.

Using the model,

(a) find the value of U

(4)

Later on, the racing car is accelerating up a straight road which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{5}{49}$. The engine of the racing car is working at a constant rate of 60 kW.

The total resistance to the motion of the racing car from non-gravitational forces is modelled as a force of magnitude $37.5v$ N, where v m s^{-1} is the speed of the racing car. At the instant when the acceleration of the racing car is 2 m s^{-2}, the speed of the racing car is V m s^{-1}

Using the model,

(b) find the value of V

(4)

(a) Driving force $F = \frac{P}{v} = \frac{60000}{v}$

Constant speed, so $a = 0$. Parallel to the road: $F - 37.5v = 0$

$\frac{60000}{v} - 37.5v = 0 \Rightarrow 37.5v^2 = 60000$

$v^2 = 1600 \Rightarrow v = 40$ m s^{-1}

$U = 40 \times 3600 \div 1000 = 40 \times 3.6$

$U = 144$

(b) Driving force $F = \frac{60000}{V}$

N2L parallel to the slope: $F - 37.5V - 750g \sin\alpha = 750 \times 2$

$\frac{60000}{V} - 37.5V - 750 \times 9.8 \times \frac{5}{49} = 1500$

$\frac{60000}{V} - 37.5V - 750 = 1500$

$60000 - 37.5V^2 = 2250V$

$37.5V^2 + 2250V - 60000 = 0$

$\div 37.5$: $V^2 + 60V - 1600 = 0$

$(V - 20)(V + 80) = 0$

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Question 2 continued

$V = 20$ $(V = -80$ rejected, speed $> 0)$

$V = 20$

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 8 marks)

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3. A stone of mass 0.5 kg is projected vertically upwards with a speed $U\,\text{m s}^{-1}$ from a point A. The point A is 2.5 m above horizontal ground.

The speed of the stone as it hits the ground is $25\,\text{m s}^{-1}$

The motion of the stone from the instant it is projected from A until the instant it hits the ground is modelled as that of a particle moving freely under gravity.

(a) Use the model and the principle of conservation of mechanical energy to find the value of U.

(4)

In reality, the stone will be subject to air resistance as it moves from A to the ground.

(b) State how this would affect your answer to part (a).

(1)

The ground is soft and the stone sinks a vertical distance d cm into the ground. The resistive force exerted on the stone by the ground is modelled as a constant force of magnitude 2000 N and the stone is modelled as a particle.

(c) Use the model and the work-energy principle to find the value of d, giving your answer to 3 significant figures.

(5)

(a) Conservation of mechanical energy, A to the ground (taking m = 0.5):

KE gained = PE lost

$\frac{1}{2}m \times 25^2 - \frac{1}{2}mU^2 = mg \times 2.5$

$\frac{1}{2}(0.5)(625) - \frac{1}{2}(0.5)U^2 = 0.5 \times 9.8 \times 2.5$

$156.25 - 0.25U^2 = 12.25$

$U^2 = 576$

$U = 24$

(b) The value of U would be larger.

(c) d cm = $0.01d$ m

Work done against resistance = $2000 \times 0.01d = 20d$ J

Work-energy principle: WD against resistance = KE lost + PE lost

$2000 \times 0.01d = \frac{1}{2} \times 0.5 \times 25^2 + 0.5g \times 0.01d$

$20d = 156.25 + 0.049d$

$19.951d = 156.25$

$d = 7.83166...$

$d = 7.83$ (3 s.f.)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)

4.



Figure 1

Three particles, P, Q and R, lie at rest on a smooth horizontal plane. The particles are in a straight line with Q between P and R, as shown in Figure 1.

Particle P is projected **towards** Q with speed u. At the same time, R is projected with speed $\frac{1}{2}u$ **away from** Q, in the direction QR.

Particle P has mass m and particle Q has mass $2m$.

The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision between P and Q is $\frac{u(1+e)}{3}$

(6)

It is given that $e > \frac{1}{2}$

(b) Determine whether there is a collision between Q and R.

(2)

(c) Determine the direction of motion of P immediately after the collision between P and Q.

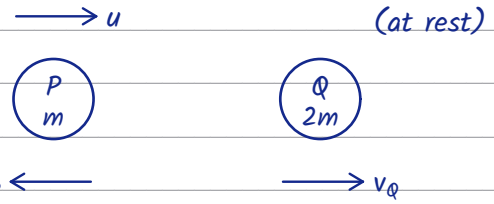
(2)

(d) Find, in terms of m, u and e, the total kinetic energy lost in the collision between P and Q, simplifying your answer.

(3)

(e) Explain how using $e = 1$ could be used to check your answer to part (d).

(1)



(a) Direction of motion of P before impact taken as positive.

CLM: $mu = -mv_P + 2mv_Q \Rightarrow u = -v_P + 2v_Q$

NEL: $v_Q + v_P = e(u - 0) \Rightarrow v_P = eu - v_Q$

Substituting: $u = -(eu - v_Q) + 2v_Q = -eu + 3v_Q$

$3v_Q = u + eu$

$v_Q = \frac{u(1+e)}{3}$ as required

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Question 4 continued

(b) $e > \frac{1}{2} \Rightarrow v_Q = \frac{u(1+e)}{3} > \frac{u(1+\frac{1}{2})}{3} = \frac{1}{2}u$

R moves away from Q at speed $\frac{1}{2}u$, so Q is moving faster than R in the same direction. Hence there is a collision between Q and R.

(c) $v_P = eu - v_Q = eu - \frac{u(1+e)}{3} = \frac{u(2e-1)}{3}$

$e > \frac{1}{2} \Rightarrow 2e - 1 > 0 \Rightarrow v_P > 0$

So the direction of motion of P is reversed: after the collision P moves in the opposite direction to its original direction of motion.

(d) KE lost = KE before − KE after

$= \frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u(2e-1)}{3}\right)^2 - \frac{1}{2}(2m)\left(\frac{u(1+e)}{3}\right)^2$

$= \frac{1}{2}mu^2 - \frac{mu^2}{18}(4e^2 - 4e + 1) - \frac{mu^2}{9}(1 + 2e + e^2)$

$= \frac{1}{2}mu^2 - \frac{mu^2}{18}(6e^2 + 3) = \frac{1}{2}mu^2 - \frac{1}{6}mu^2 - \frac{1}{3}mu^2e^2$

$= \frac{1}{3}mu^2 - \frac{1}{3}mu^2e^2$

Total KE lost $= \frac{1}{3}mu^2(1 - e^2)$

(e) When $e = 1$ the collision is perfectly elastic, so no kinetic energy is lost: substituting $e = 1$ into $\frac{1}{3}mu^2(1 - e^2)$ should give 0, and it does.

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Question 4 continued

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Question 4 continued

Question 4 continued

(Total for Question 4 is 14 marks)

TOTAL FOR FURTHER MECHANICS 1 IS 40 MARKS

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