Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

# Pearson Edexcel Level 3 GCE

## Friday 19 May 2023

Afternoon

Paper reference **8FM0/22**

# Further Mathematics

**Advanced Subsidiary**
**Further Mathematics options**
**22: Further Pure Mathematics 2**
**(Part of option A only)**

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  *– there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets
  *– use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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**1.** The operation * is defined on the set $G = \{0, 1, 2, 3\}$ by

$$x * y \equiv x + y - 2xy \pmod 4$$

(a) Complete the Cayley table below.

**(2)**

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 0 | 3 | 2 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 2 | 1 | 0 |

(b) Show that $G$ is a group under the operation *

(You may assume the associative law is satisfied.)

**(3)**

(c) State the order of each element of $G$.

**(2)**

(d) State whether $G$ is a cyclic group, giving a reason for your answer.

**(1)**

(a) see table above

(b) Closure: every entry in the table is in $G = \{0, 1, 2, 3\}$,
so $G$ is closed under *
Identity: $0 * x = x * 0 = x$ for all $x \in G$, so 0 is the identity
Inverses: $0 * 0 = 0$, $1 * 1 = 0$, $2 * 2 = 0$, $3 * 3 = 0$
so 0 is self-inverse and 1, 2 and 3 are each self-inverse,
i.e. every element has an inverse in $G$
Associativity: given (the associative law is satisfied)
$\therefore$ $G$ is a group under *

(c) 0 has order 1
1, 2 and 3 each have order 2

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**Question 1 continued**

(d) G is not cyclic: there is no element of order 4 (the order of G),
so no element generates G

**(Total for Question 1 is 8 marks)**

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**2.** A linear transformation $T:\mathbb{R}^2 \to \mathbb{R}^2$ is represented by the matrix

$$\mathbf{M} = \begin{pmatrix} 5 & 1 \\ k & -3 \end{pmatrix}$$

where $k$ is a constant.

Given that matrix **M** has a repeated eigenvalue,

(a) determine

(i) the value of $k$

(ii) the eigenvalue.

**(6)**

(b) Hence determine a Cartesian equation of the invariant line under $T$.

**(2)**

(a)(i)

$\det(M - \lambda I) = \begin{vmatrix} 5-\lambda & 1 \\ k & -3-\lambda \end{vmatrix} = (5-\lambda)(-3-\lambda) - k = 0$

$\Rightarrow -15 - 5\lambda + 3\lambda + \lambda^2 - k = 0$

$\Rightarrow \lambda^2 - 2\lambda - 15 - k = 0$

Repeated eigenvalue $\Rightarrow$ discriminant $b^2 - 4ac = 0$

$(-2)^2 - 4(1)(-15-k) = 0$

$4 + 60 + 4k = 0 \Rightarrow 4k = -64$

$k = -16$

(a)(ii)

$\lambda^2 - 2\lambda - 15 + 16 = 0 \Rightarrow \lambda^2 - 2\lambda + 1 = 0$

$\Rightarrow (\lambda - 1)^2 = 0$

eigenvalue $\lambda = 1$

(b) Invariant line through O in the direction of the eigenvector:

$\begin{pmatrix} 5 & 1 \\ -16 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 1\begin{pmatrix} x \\ y \end{pmatrix}$

$5x + y = x \Rightarrow y = -4x$

(check: $-16x - 3y = -16x + 12x = -4x = y$ ✓)

Invariant line: $y = -4x$

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**Question 2 continued**

**Question 2 continued**

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**Question 2 continued**

**(Total for Question 2 is 8 marks)**

**3.** A complex number $z$ is represented by the point $P$ on an Argand diagram.

Given that

$$\arg\left(\frac{z-4-\mathrm{i}}{z-2-7\mathrm{i}}\right)=\frac{\pi}{2}$$

(a) sketch the locus of $P$ as $z$ varies,

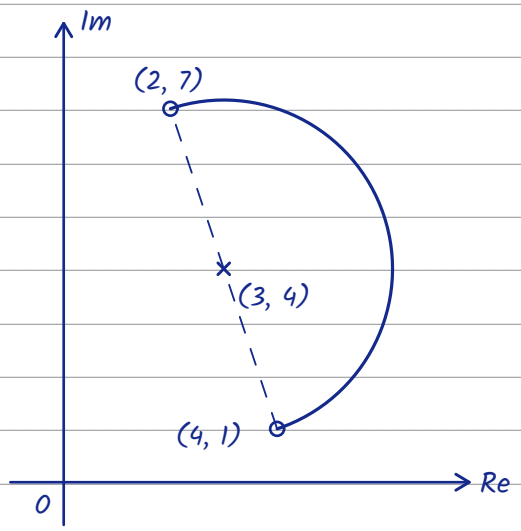
**(2)**

(b) determine the exact maximum possible value of $|z|$

**(5)**

(a)

locus of P:
semicircle to the right
of the line joining
4 + i and 2 + 7i
(end points excluded)




(b) Angle at P is 90°, so the arc is part of a circle with diameter joining (4, 1) and (2, 7)

Centre = midpoint = $\left(\frac{4+2}{2}, \frac{1+7}{2}\right) = (3, 4)$

Radius $r = \frac{1}{2}\sqrt{((4-2)^2 + (1-7)^2)} = \frac{1}{2}\sqrt{40} = \sqrt{10}$

Distance from O to centre $d = \sqrt{(3^2 + 4^2)} = 5$

Maximum $|z|$ where the line through O and (3, 4) meets the arc (this point is on the right-hand semicircle), so

$|z|_{max} = d + r$

$|z|_{max} = 5 + \sqrt{10}$

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**Question 3 continued**

**(Total for Question 3 is 7 marks)**

**4.** A student takes out a loan for £1000 from a bank.

The bank charges 0.5% monthly interest on the amount of the loan yet to be repaid.

At the end of each month

- the interest is added to the loan
- the student then repays £50

Let $U_n$ be the amount of money owed $n$ months after the loan was taken out.

The amount of money owed by the student is modelled by the recurrence relation

$$U_n = 1.005U_{n-1} - A \qquad U_0 = 1000 \qquad n \in \mathbb{Z}^+$$

where $A$ is a constant.

(a) (i) State the value of the constant $A$.

(ii) Explain, in the context of the problem, the value 1.005

**(2)**

Using the value of $A$ found in part (a)(i),

(b) solve the recurrence relation

$$U_n = 1.005U_{n-1} - A \qquad U_0 = 1000 \qquad n \in \mathbb{Z}^+$$

**(5)**

(c) Hence determine, according to the model, the number of months it will take to completely repay the loan.

**(2)**

(a)(i) $A = 50$

(a)(ii) 0.5% interest is added each month to the amount owed, so the amount owed is multiplied by $1 + 0.005 = 1.005$ each month

(b) $U_n = CF + PS$

CF: $U_n = c(1.005)^n$

PS: try $U_n = \lambda$: $\lambda = 1.005\lambda - 50$

$\Rightarrow -0.005\lambda = -50 \Rightarrow \lambda = 10\,000$

$\therefore U_n = c(1.005)^n + 10\,000$

$U_0 = 1000$: $1000 = c(1.005)^0 + 10\,000 \Rightarrow c = -9000$

$\therefore U_n = 10\,000 - 9000(1.005)^n$

(c) Loan repaid when $U_n = 0$: $10\,000 - 9000(1.005)^n = 0$

$(1.005)^n = \frac{10\,000}{9000} = \frac{10}{9} \Rightarrow n = \frac{\log(10/9)}{\log 1.005} = 21.1247\ldots$

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**Question 4 continued**

$(U_{21} = 6.22 > 0, \quad U_{22} = -43.75 < 0)$

$\therefore$ it takes 22 months to completely repay the loan

**Question 4 continued**

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**Question 4 continued**

**(Total for Question 4 is 9 marks)**

**5.** (i) Making your reasoning clear and using modulo arithmetic, show that

$$214^6 \text{ is divisible by } 8$$

**(3)**

(ii) The following 7-digit number has four unknown digits

$$\boxed{a}\,5\,\boxed{b}\,8\,\boxed{a}\,\boxed{b}\,0$$

Given that the number is divisible by 11

(a) determine the value of the digit $a$.

**(2)**

Given that the number is also divisible by 3

(b) determine the possible values of the digit $b$.

**(3)**

(i) $214 = 26 \times 8 + 6 \Rightarrow 214 \equiv 6 \pmod 8$

$214^2 \equiv 6^2 = 36 \equiv 4 \pmod 8$

$214^6 = (214^2)^3 \equiv 4^3 = 64 \equiv 0 \pmod 8$

$214^6 \equiv 0 \pmod 8$, therefore $214^6$ is divisible by 8

(ii)(a) Divisible by 11 $\Rightarrow$ alternating sum of digits $\equiv 0 \pmod{11}$

$a - 5 + b - 8 + a - b + 0 = 2a - 13 \equiv 0 \pmod{11}$

$0 \leq a \leq 9 \Rightarrow -13 \leq 2a - 13 \leq 5$, so $2a - 13 = -11$

$a = 1$

(ii)(b) Divisible by 3 $\Rightarrow$ sum of digits is a multiple of 3

$1 + 5 + b + 8 + 1 + b + 0 = 15 + 2b = 3n$

15 is a multiple of 3, so $2b \equiv 0 \pmod 3 \Rightarrow b \equiv 0 \pmod 3$

$b = 0, 3, 6$ or 9

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**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 8 marks)**

**TOTAL FOR FURTHER PURE MATHEMATICS 2 IS 40 MARKS**

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