Pearson Edexcel Level 3 GCE

Thursday 16 May 2019

Afternoon | Paper Reference **8FM0-28**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
28: Decision Mathematics 2
(Part of option K only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator,
D2 Answer Book (enclosed)

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the Answer Book provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for each question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. Three workers, A, B and C, are each to be assigned to one of four tasks, P, Q, R and S.

Each worker must be assigned to at most one task, and each task must be done by at most one worker.

The amount, in pounds, that each worker will earn while assigned to each task is shown in the table below.

	P	Q	R	S
A	32	40	37	42
B	29	32	35	41
C	37	33	39	40

The Hungarian algorithm is to be used to find the maximum total amount that can be earned by the three workers.

(a) Explain how the table should be modified.

(2)

(b) (i) Reducing rows first, use the Hungarian algorithm to obtain an allocation which maximises the total earnings.

(ii) Explain how any initial row and column reductions were made and also how you determined if the table was optimal at each stage.

(7)

(Total for Question 1 is 9 marks)

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2. (a) Find the general solution of the recurrence relation

$$u_{n+1} = 3u_n + 2^n \qquad n \geqslant 1$$

(4)

(b) Find the particular solution of this recurrence relation for which $u_1 = u_2$

(2)

(Total for Question 2 is 6 marks)

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3.

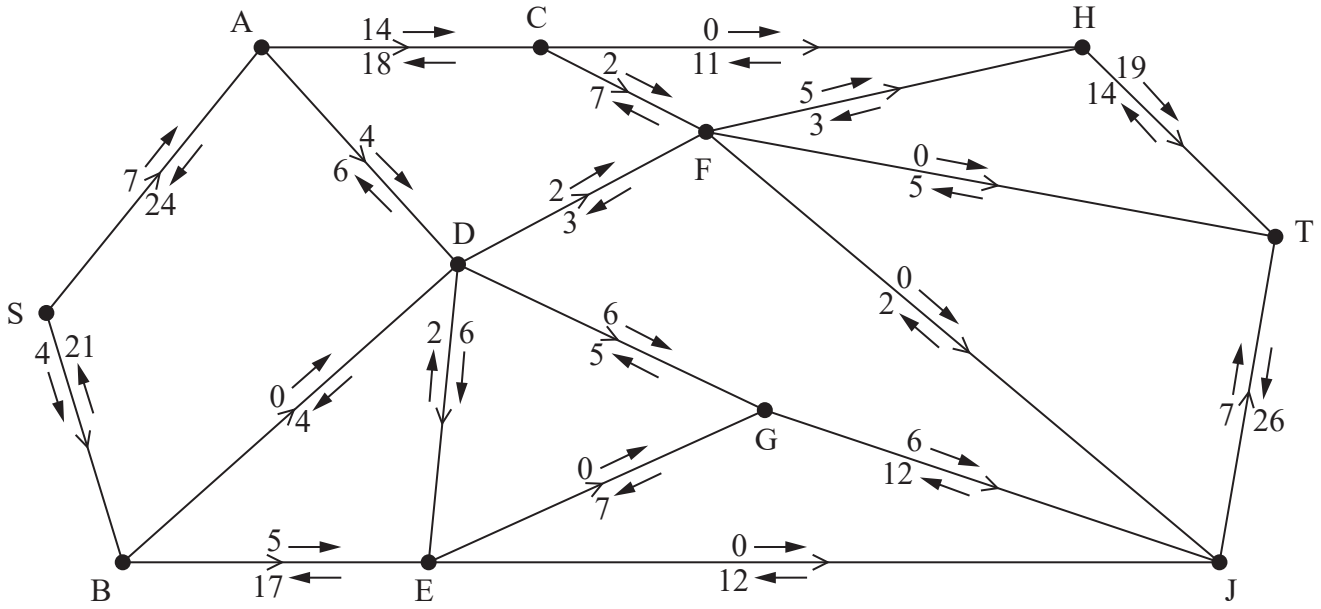


Figure 1

Alexa is monitoring a system of pipes through which fluid can flow from the source, S, to the sink, T. Currently, fluid is flowing through the system from S to T.

Alexa initialises the labelling procedure for this system, and the excess capacities and potential backflows are shown on the arrows either side of each arc, as shown in Figure 1.

(a) State the value of the initial flow.

(1)

(b) Explain why arcs DF and DG can never both be full to capacity.

(1)

(c) Obtain the capacity of the cut that passes through the arcs AC, AD, BD, DE, EG and EJ.

(1)

(d) Use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

(3)

(e) Use your answers to part (d) to find a maximum flow pattern for this system of pipes and draw it on Diagram 1 in the answer book.

(1)

(f) Prove that the answer to part (e) is optimal.

(3)

(Total for Question 3 is 10 marks)

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4. The table below gives the pay-off matrix for a zero-sum game between two players, Aljaz and Brendan. The values in the table show the pay-offs for Aljaz.

		Brendan		
		Option X	Option Y	Option Z
Aljaz	Option P	−6	−1	2
	Option Q	5	4	−7
	Option R	5	6	3

(a) (i) Show that this game is stable.

(ii) State the value of this game to Brendan.

(3)

Option R is removed from Aljaz's choices and the reduced game, with option R removed, is no longer stable.

(b) Find the best strategy for Aljaz in this reduced game, defining any variable you use.

(7)

(c) Explain why Brendan should never play option Y

(1)

Let Brendan play option X with probability q

(d) (i) Explain why q satisfies the equation $6q - 2(1 - q) = 1.6$

(ii) Hence find the best strategy for Brendan in this reduced game.

(4)

(Total for Question 4 is 15 marks)

TOTAL FOR DECISION MATHEMATICS 2 IS 40 MARKS

END

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Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

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Answer Book
Do not return the question paper with the answer book.

Total Marks

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1. (a)

Subtract every entry from 42 (the largest value) so that the problem of maximising earnings becomes a minimisation problem (minimise 42 − earnings). Add a dummy row X (a dummy worker) with all entries equal, e.g. 42 (earning £0), so that the table is square (4 × 4).

	P	Q	R	S
A	32	40	37	42
B	29	32	35	41
C	37	33	39	40

You may not need to use all of these tables
You may not need to use all the rows and columns

	P	Q	R	S	
A	10	2	5	0	
B	13	10	7	1	
C	5	9	3	2	
X	42	42	42	42	

(b)(i) 42 − each entry, dummy row X with entries 42, 42, 42, 42

	P	Q	R	S	
A	10	2	5	0	
B	12	9	6	0	
C	3	7	1	0	
X	~~0~~	~~0~~	~~0~~	~~0~~	

(ii) Reduce rows: subtract the least value in each row from every element of that row: row A − 0, row B − 1, row C − 2, row X − 42. Every column already contains a 0 (row X), so no column reduction is needed. Zeros covered by 2 lines (row X, column S). 2 < 4, so not optimal.

	P	Q	R	S	
A	9	1	4	0	
B	11	8	5	0	
C	2	6	0	0	
X	~~0~~	~~0~~	~~0~~	~~1~~	

Least uncovered element = 1: subtract 1 from uncovered elements, add 1 to elements covered twice (XS), others unchanged. Zeros covered by 3 lines (row X, columns R, S). 3 < 4, so not optimal.

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	P	Q	R	S	
A	8	0	4	0	
B	10	7	5	0	
C	1	5	0	0	
X	~~0~~	~~0~~	~~1~~	~~2~~	

Least uncovered element = 1: subtract 1 from uncovered elements, add 1 to elements covered twice (XR, XS), others unchanged.

Zeros now need 4 lines to cover them (row X, columns Q, R, S).

4 = 4 (n × n table, n lines), so optimal.

	P	Q	R	S	
A					
B					
C					

Allocation:

A – Q, B – S, C – R (X – P, dummy)

	P	Q	R	S	
A					
B					
C					

Maximum total earnings

= 40 + 41 + 39 = £120

	P	Q	R	S	
A					
B					
C					

	P	Q	R	S	
A					
B					
C					

(Total for Question 1 is 9 marks)

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2.

(a) Auxiliary equation: $m - 3 = 0 \Rightarrow m = 3$

Complementary function: $u_n = A(3^n)$

Particular solution: try $u_n = k(2^n)$

$k(2^{n+1}) = 3k(2^n) + 2^n$

$2k(2^n) = 3k(2^n) + 2^n$

$2k = 3k + 1 \Rightarrow k = -1$

General solution: $u_n = A(3^n) - 2^n$

(b) $u_1 = 3A - 2, \quad u_2 = 9A - 4$

$u_1 = u_2 \Rightarrow 3A - 2 = 9A - 4 \Rightarrow 6A = 2$

$A = \frac{1}{3}$

$u_n = \frac{1}{3}(3^n) - 2^n$

$u_n = 3^{n-1} - 2^n$

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Question 2 continued

(Total for Question 2 is 6 marks)

3.

(a) Initial flow = flow out of S = 24 + 21 = 45

(b) Maximum flow into D = AD + BD = 10 + 4 = 14

Capacity of DF + DG = (2 + 3) + (6 + 5) = 5 + 11 = 16

14 < 16, so the flow out of D can never reach 16, hence DF and DG can never both be full to capacity.

(c) Capacity of cut = AC + AD + BD + DE + EG + EJ

= 32 + 10 + 4 + 0 + 7 + 12 = 65

(DE goes from the sink side to the source side so contributes 0)

(d) Flow-augmenting routes:

S A C F H T – 2

S A D G J T – 4

S B E D G J T – 2 (using the backflow of 2 on DE)

Total increase = 2 + 4 + 2 = 8

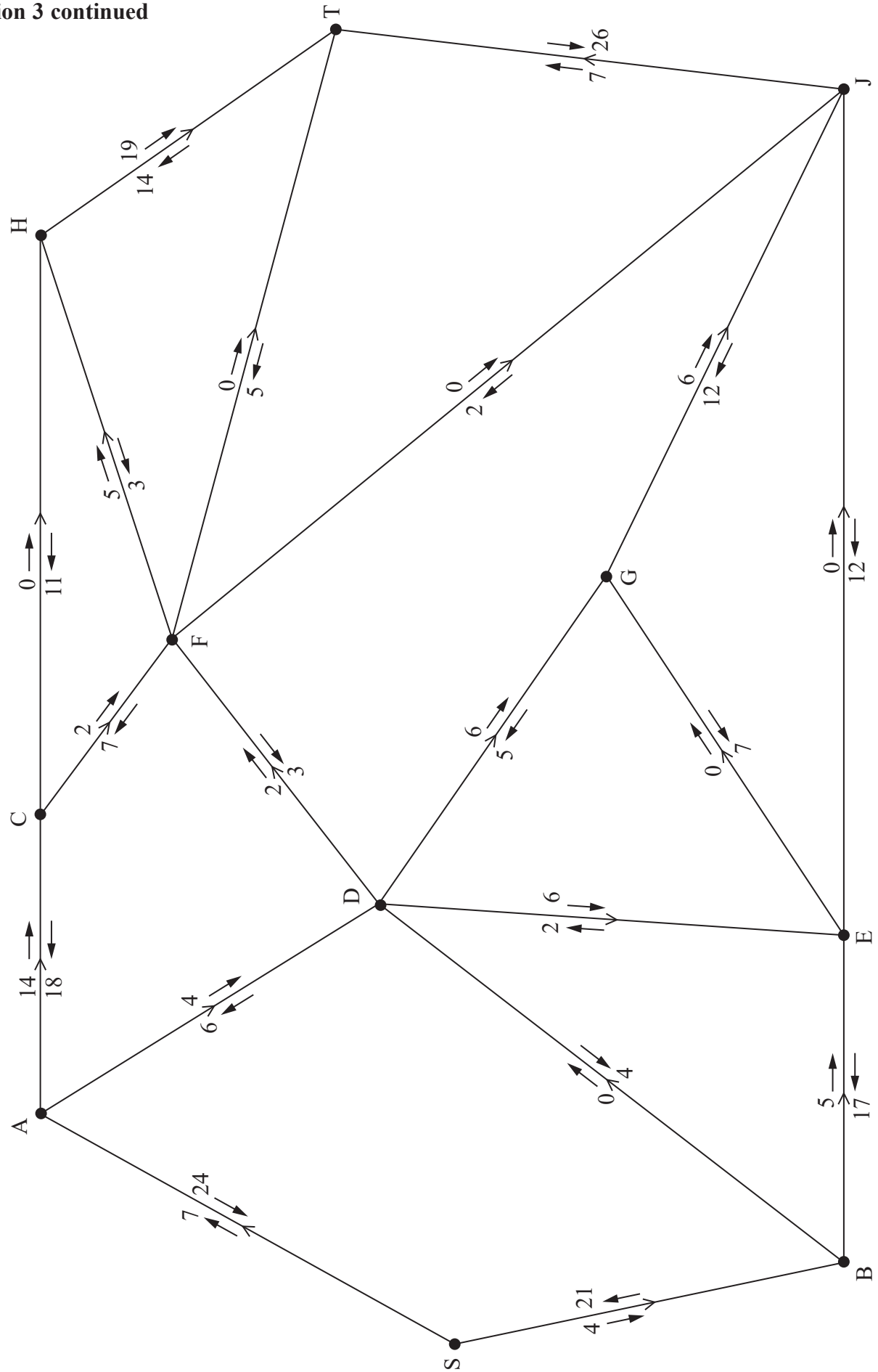
Maximum flow = 45 + 8 = 53

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Question 3 continued



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Question 3 continued

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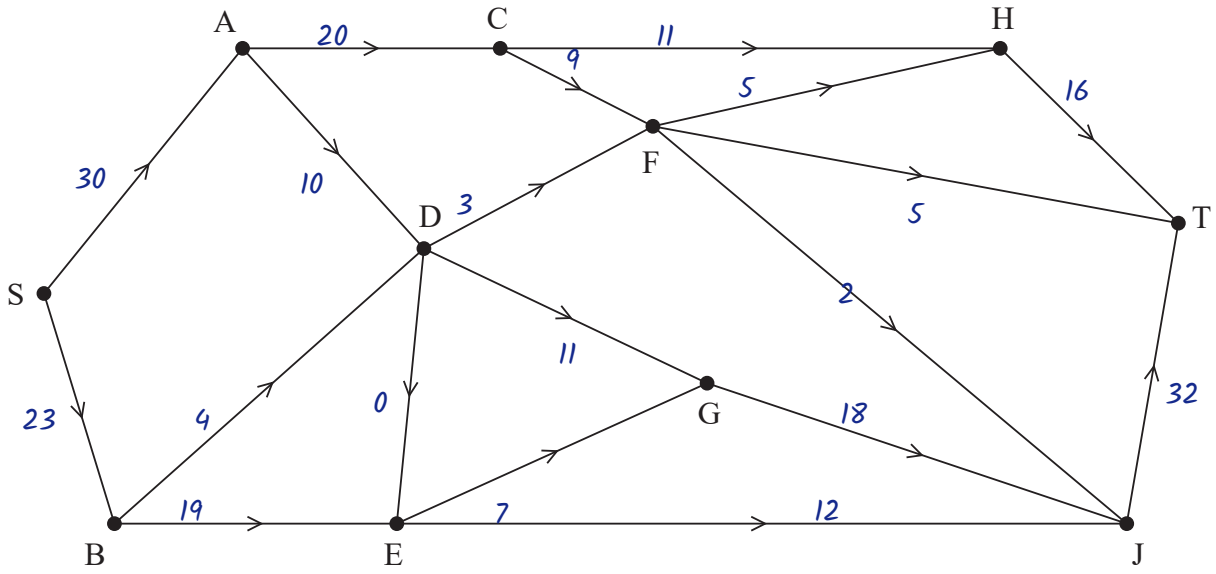


Diagram 1

(e) flows shown on Diagram 1

(f) Consider the cut through CH, CF, AD, BD, DE, EG and EJ

Value = 11 + 9 + 10 + 4 + 0 + 7 + 12 = 53

(DE goes from the sink side to the source side so contributes 0)

Flow found in (e) = 53 = value of this cut

By the maximum flow – minimum cut theorem, maximum flow = minimum cut,

so the flow of 53 is maximal (and the cut is a minimum cut).

(Total for Question 3 is 10 marks)

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4.

		Brendan		
		Option X	Option Y	Option Z
Aljaz	Option P	−6	−1	2
	Option Q	5	4	−7
	Option R	5	6	3

(a)(i) Row minima: P −6, Q −7, R 3 ⇒ row maximin = 3

Column maxima: X 5, Y 6, Z 3 ⇒ column minimax = 3

Row maximin = column minimax (3 = 3), so the game is stable.

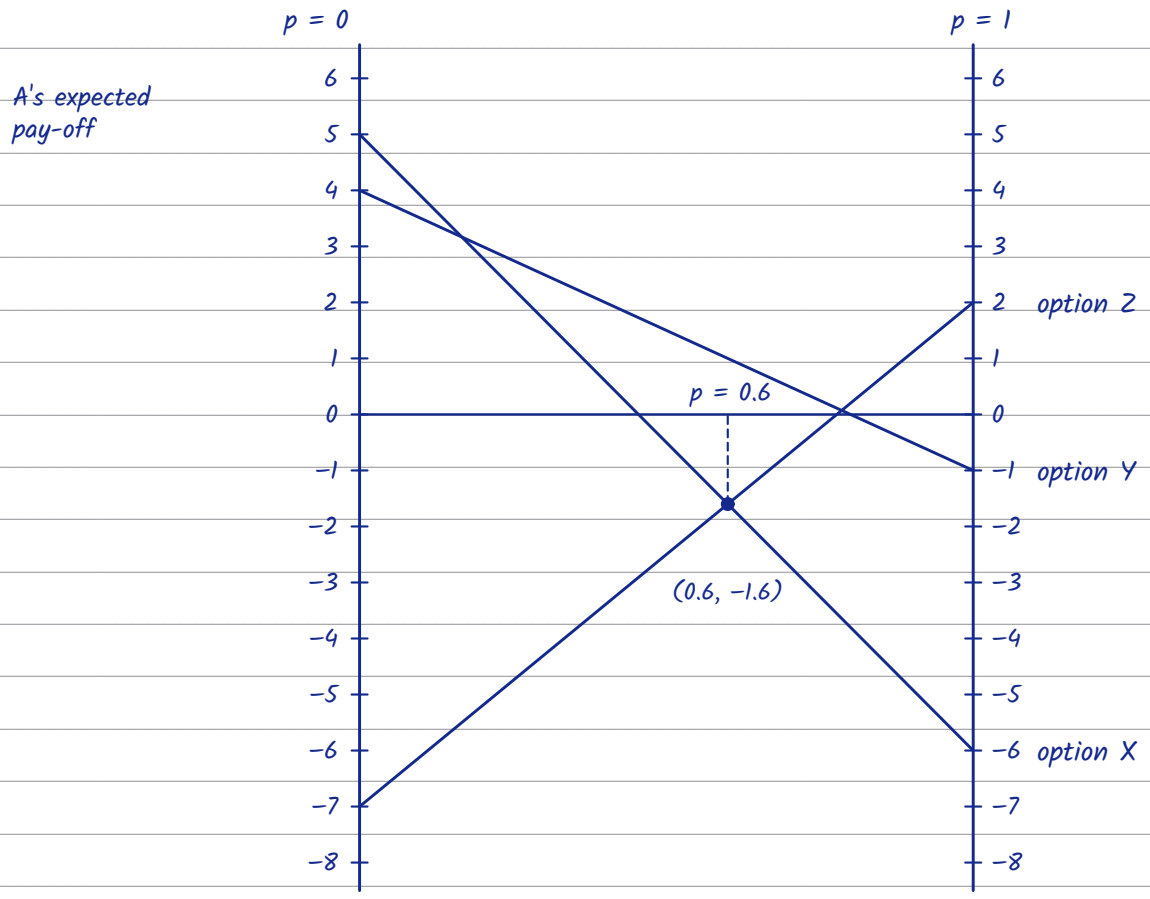
(ii) Value of the game to Brendan = −3

(b) Let Aljaz play option P with probability p and option Q with probability 1 − p

If B plays X, A's expected gain = −6p + 5(1 − p) = 5 − 11p

If B plays Y, A's expected gain = −p + 4(1 − p) = 4 − 5p

If B plays Z, A's expected gain = 2p − 7(1 − p) = 9p − 7



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Question 4 continued

The optimal point is the highest point of the lowest boundary (shown on the graph), where lines X and Z intersect:

$5 - 11p = 9p - 7 \quad \Rightarrow \quad 20p = 12 \quad \Rightarrow \quad p = 0.6$

Aljaz should play option P with probability 0.6 and option Q with probability 0.4

(value of the game to Aljaz = $5 - 11(0.6) = -1.6$)

(c) From the graph, for $0 \leqslant p \leqslant 0.6$ line Z lies below line Y, and for $0.6 < p \leqslant 1$ line X lies below line Y. So for every value of p Brendan does better (Aljaz gains less) by playing X or Z than by playing Y, so he should never play option Y.

(d)(i) Brendan plays X with probability q and Z with probability $1 - q$.

The table shows Aljaz's pay-offs, so Brendan's pay-offs are their negatives.

If Aljaz plays P, Brendan's expected gain = $-(-6q + 2(1 - q)) = 6q - 2(1 - q)$

Value of the game to Brendan = $-$(value to Aljaz) = $-(5 - 11(0.6)) = -(-1.6) = 1.6$

At the optimal strategy Brendan's expected gain equals the value of the game,

so $6q - 2(1 - q) = 1.6$

(ii) $6q - 2 + 2q = 1.6 \quad \Rightarrow \quad 8q = 3.6 \quad \Rightarrow \quad q = 0.45$

Brendan should play option X with probability 0.45 and option Z with probability 0.55 (and never play option Y).

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Question 4 continued

(Total for Question 4 is 15 marks)

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