Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 16 May 2019

Afternoon | Paper Reference **8FM0-26**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\ \text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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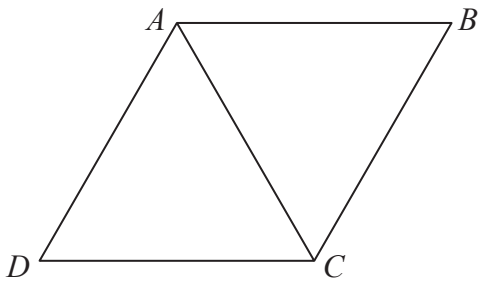


Figure 1

Five identical uniform rods are joined together to form the rigid framework $ABCD$ shown in Figure 1. Each rod has weight W and length $4a$. The points A, B, C and D all lie in the same plane.

The centre of mass of the framework is at the point G.

(a) Explain why G is the midpoint of AC.

(1)

The framework is suspended from the ceiling by two vertical light inextensible strings. One string is attached to the framework at A and the other string is attached to the framework at B. The framework hangs freely in equilibrium with AB horizontal.

(b) Find

(i) the tension in the string attached at A,

(ii) the tension in the string attached at B.

(4)

A particle of weight kW is now attached to the framework at D and a particle of weight $2kW$ is now attached to the framework at C. The framework remains in equilibrium with AB horizontal and the strings vertical.

Either string will break if the tension in it exceeds $6W$.

(c) Find the greatest possible value of k.

(4)

(a) The rods are identical and uniform, so the rhombus ABCD is symmetrical about AC and about BD. The axes of symmetry meet at the midpoint of AC, so G is the midpoint of AC.

(b) Taking A as origin, horizontal distances to the right of A:

G is at distance a (midpoint of AC), B at 4a

M(A): $T_B \times 4a = 5W \times a \Rightarrow T_B = \frac{5W}{4}$

R(↑): $T_A + T_B = 5W \Rightarrow T_A = 5W - \frac{5W}{4} = \frac{15W}{4}$

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Question 1 continued

(i) $T_A = \frac{15W}{4}$ (ii) $T_B = \frac{5W}{4}$

(c) Distances from B: G is 3a, D is 6a, C is 2a

T_A is the larger tension, so the string at A breaks first: $T_A = 6W$

M(B): $6W \times 4a = 5W \times 3a + kW \times 6a + 2kW \times 2a$

$24aW = 15aW + 10kaW$

$10k = 9$

Greatest value: $k = 0.9$

Question 1 continued

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Question 1 continued

(Total for Question 1 is 9 marks)

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2. A car moves in a straight line along a horizontal road. The car is modelled as a particle. At time t seconds, where $t \geqslant 0$, the speed of the car is v m s^{-1}

At the instant when $t = 0$, the car passes through the point A with speed 2 m s^{-1}

The acceleration, a m s^{-2}, of the car is modelled by

$$a = \frac{4}{2+v}$$

in the direction of motion of the car.

(a) Use algebraic integration to show that $v = \sqrt{8t+16} - 2$

(6)

At the instant when the car passes through the point B, the speed of the car is 4 m s^{-1}

(b) Use algebraic integration to find the distance AB.

(6)

(a) $\frac{dv}{dt} = \frac{4}{2+v} \Rightarrow \int (2+v)\,dv = \int 4\,dt$

$\frac{(2+v)^2}{2} = 4t + C$

$t = 0, v = 2$: $\frac{(2+2)^2}{2} = 0 + C \Rightarrow C = 8$

$\frac{(2+v)^2}{2} = 4t + 8$

$(2+v)^2 = 8t + 16$

$2 + v = \sqrt{(8t+16)}$ (positive root since $v > 0$)

$\therefore \quad v = \sqrt{(8t+16)} - 2$ as required

(b) At B: $v = 4 \Rightarrow \sqrt{(8t+16)} = 6 \Rightarrow 8t + 16 = 36 \Rightarrow t = 2.5$

$\frac{dx}{dt} = \sqrt{(8t+16)} - 2$

$x = \int ((8t+16)^{1/2} - 2)\,dt = \frac{1}{12}(8t+16)^{3/2} - 2t + C$

$t = 0, x = 0$ (at A): $\frac{1}{12}(16)^{3/2} + C = 0 \Rightarrow \frac{64}{12} + C = 0 \Rightarrow C = -\frac{16}{3}$

$t = 2.5$: $AB = \frac{1}{12}(36)^{3/2} - 2(2.5) - \frac{16}{3}$

$AB = \frac{216}{12} - 5 - \frac{16}{3} = 18 - 5 - \frac{16}{3}$

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Question 2 continued

$AB = \frac{23}{3}$ m $(= 7.67\text{ m})$

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 12 marks)

3. A light inextensible string has length $8a$. One end of the string is attached to a fixed point A and the other end of the string is attached to a fixed point B, with A vertically above B and $AB = 4a$. A small ball of mass m is attached to a point P on the string, where $AP = 5a$.

The ball moves in a horizontal circle with constant speed v, with both AP and BP taut.

The string will break if the tension in it exceeds $\frac{3mg}{2}$

By modelling the ball as a particle and assuming the string does not break,

(a) show that $\frac{9ag}{4} < v^2 \leqslant \frac{27ag}{4}$ **(7)**

(b) find the least possible time needed for the ball to make one complete revolution. **(2)**

(a) $AP = 5a$, $BP = 8a - 5a = 3a$, $AB = 4a$ $\Rightarrow$ (3, 4, 5) triangle, angle $ABP = 90°$

So BP is horizontal, radius $r = 3a$. Angle $PAB = \theta$: $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$

Tensions T_A in AP and T_B in BP.

$R(\uparrow)$: $T_A \cos\theta = mg \Rightarrow \frac{4}{5}T_A = mg \Rightarrow T_A = \frac{5mg}{4}$

$R(\rightarrow$, towards B$)$: $T_B + T_A \sin\theta = \frac{mv^2}{r}$

$T_B + \frac{3}{5} \times \frac{5mg}{4} = \frac{mv^2}{3a} \Rightarrow T_B = \frac{mv^2}{3a} - \frac{3mg}{4}$

BP taut: $T_B > 0 \Rightarrow \frac{mv^2}{3a} > \frac{3mg}{4} \Rightarrow v^2 > \frac{9ag}{4}$

String does not break: $T_A = \frac{5mg}{4} < \frac{3mg}{2}$ always, and

$T_B \leqslant \frac{3mg}{2} \Rightarrow \frac{mv^2}{3a} - \frac{3mg}{4} \leqslant \frac{3mg}{2} \Rightarrow \frac{mv^2}{3a} \leqslant \frac{9mg}{4} \Rightarrow v^2 \leqslant \frac{27ag}{4}$

$\therefore \frac{9ag}{4} < v^2 \leqslant \frac{27ag}{4}$ as required

(b) Least time when v is greatest: $v^2 = \frac{27ag}{4} \Rightarrow v = \frac{3\sqrt{(3ag)}}{2}$

$T = \frac{2\pi r}{v} = \frac{2\pi(3a)}{3\sqrt{(3ag)}\,/\,2} = \frac{4\pi a}{\sqrt{(3ag)}}$

Least time $= 4\pi\sqrt{\left(\frac{a}{3g}\right)}$ $(= 2.32\sqrt{a}$ s$)$

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Question 3 continued

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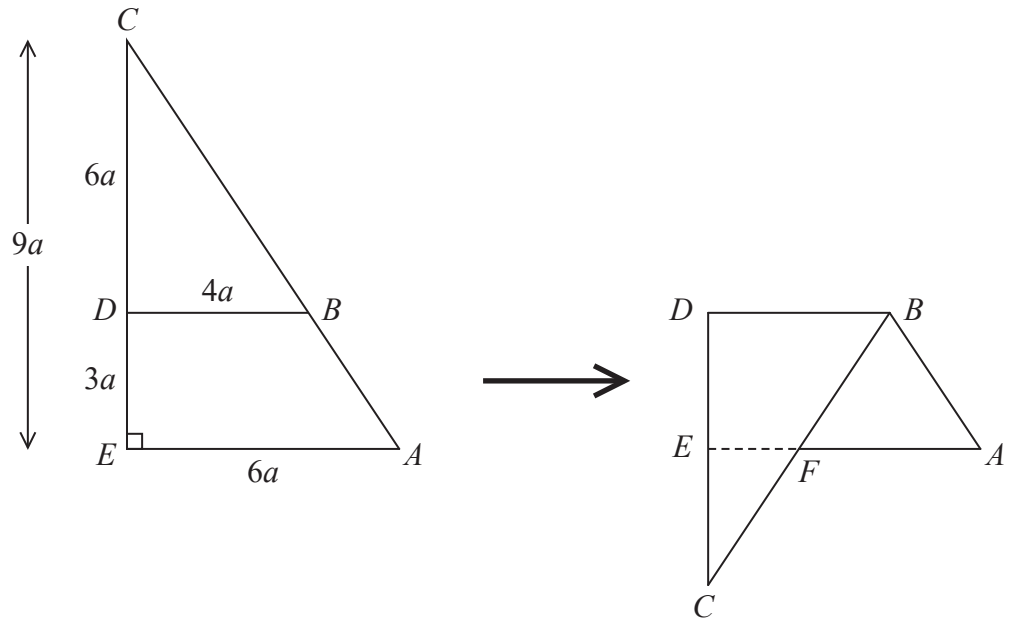
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Question 3 continued

(Total for Question 3 is 9 marks)

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4.



Triangular lamina Folded lamina

Figure 2

The uniform triangular lamina $ABCDE$ is such that angle $CEA = 90°$, $CE = 9a$ and $EA = 6a$. The point D lies on CE, with $DE = 3a$. The point B on CA is such that DB is parallel to EA and $DB = 4a$. The triangular lamina is folded along the line DB to form the folded lamina $ABDECF$, as shown in Figure 2.

The distance of the centre of mass of the triangular lamina from DC is d_1

The distance of the centre of mass of the folded lamina from DC is d_2

(a) Explain why $d_1 = d_2$

(1)

The folded lamina is freely suspended from B and hangs in equilibrium with BA inclined at an angle α to the downward vertical through B.

(b) Find, to the nearest degree, the size of angle α.

(9)

(a) Folding along DB moves points of BCD but each point stays the same distance from the line DC, so the mass distribution relative to DC is unchanged and $d_1 = d_2$.

(b) Triangle ACE: centroid distance from DC $= \frac{6a}{3} = 2a$, so $\bar{x} = d_2 = 2a$

Distances from EA (E origin):

	ACE	BCD removed	BCD added	Folded
Area	$27a^2$	$12a^2$	$12a^2$	$27a^2$
From EA	$3a$	$5a$	a	$\bar{y}$

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Question 4 continued

Moments about EA: $27a^2 \times 3a - 12a^2 \times 5a + 12a^2 \times a = 27a^2 \times \bar{y}$

$81a - 60a + 12a = 27\bar{y} \quad \Rightarrow \quad 33a = 27\bar{y}$

$\bar{y} = \frac{11a}{9}$ above EA

$G = (2a, \frac{11a}{9}), \quad B = (4a, 3a), \quad A = (6a, 0)$

Hanging freely from B, BG is vertical (G directly below B).

From B to G: horizontal $4a - 2a = 2a$, vertical $3a - \frac{11a}{9} = \frac{16a}{9}$

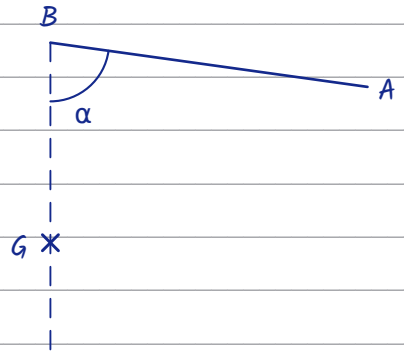
Angle of BG to the downward vertical: $\tan^{-1}\left(\frac{2a}{16a/9}\right) = \tan^{-1}\left(\frac{9}{8}\right) = 48.366...°$

Angle of BA to the downward vertical: $\tan^{-1}\left(\frac{2a}{3a}\right) = \tan^{-1}\left(\frac{2}{3}\right) = 33.690...°$

BG and BA are on opposite sides of the vertical through B:

$\alpha = 48.366...° + 33.690...° = 82.056...°$

$\alpha = 82°$ (nearest degree)



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Question 4 continued

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Question 4 continued

Question 4 continued

(Total for Question 4 is 10 marks)

TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS

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