Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 16 May 2019

Afternoon | Paper Reference **8FM0-22**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
22: Further Pure Mathematics 2
(Part of option A only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 2 & 2 \end{pmatrix}$$

(a) find the characteristic equation for the matrix $\mathbf{A}$, simplifying your answer.

(2)

(b) Hence find an expression for the matrix $\mathbf{A}^{-1}$ in the form $\lambda\mathbf{A} + \mu\mathbf{I}$, where λ and μ are constants to be found.

(3)

(a)

$\det(A - \lambda I) = (3 - \lambda)(2 - \lambda) - 2 \times 2$

$= 6 - 5\lambda + \lambda^2 - 4$

$\therefore$ *characteristic equation:* $\lambda^2 - 5\lambda + 2 = 0$

(b)

By the Cayley-Hamilton theorem: $A^2 - 5A + 2I = 0$

Multiply through by A^{-1}*:* $A - 5I + 2A^{-1} = 0$

$\Rightarrow 2A^{-1} = 5I - A$

$\Rightarrow A^{-1} = -\frac{1}{2}A + \frac{5}{2}I$

i.e. $\lambda = -\frac{1}{2}, \quad \mu = \frac{5}{2}$

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Question 1 continued

(Total for Question 1 is 5 marks)

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2. (i) Determine all the possible integers a, where $a > 3$, such that

$$15 \equiv 3 \bmod a$$

(2)

(ii) Show that if p is prime, x is an integer and $x^2 \equiv 1 \bmod p$ then either

$$x \equiv 1 \bmod p \qquad \text{or} \qquad x \equiv -1 \bmod p$$

(3)

(iii) A company has £13 940 220 to share between 11 charities.

Without performing any division and showing all your working, decide if it is possible to share this money equally between the 11 charities.

(2)

(i)

$15 \equiv 3 \bmod a \Rightarrow a$ divides $15 - 3 = 12$

Factors of 12 greater than 3: $a = 4, 6, 12$

(ii)

$x^2 \equiv 1 \bmod p \Rightarrow x^2 - 1 \equiv 0 \bmod p$

$\Rightarrow (x - 1)(x + 1) \equiv 0 \bmod p$, i.e. $p \mid (x - 1)(x + 1)$

Since p is prime, p must divide one of the factors:

either $p \mid (x - 1)$ or $p \mid (x + 1)$

$\therefore x \equiv 1 \bmod p$ or $x \equiv -1 \bmod p$

(iii)

Divisibility test for 11: alternating sum of the digits of 13 940 220

$1 - 3 + 9 - 4 + 0 - 2 + 2 - 0 = 3$

3 is not divisible by 11 ($3 \not\equiv 0 \bmod 11$),

so 13 940 220 is not divisible by 11.

$\therefore$ It is not possible to share the money equally between the 11 charities.

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Question 2 continued

(Total for Question 2 is 7 marks)

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3. A curve C in the complex plane is described by the equation

$$|z-1-8\mathrm{i}| = 3|z-1|$$

(a) Show that C is a circle, and find its centre and radius.

(4)

(b) Using the answer to part (a), determine whether $z = 3 - 3\mathrm{i}$ satisfies the inequality

$$|z - 1 - 8\mathrm{i}| \geqslant 3|z - 1|$$

(2)

(c) Shade, on an Argand diagram, the set of points that satisfies both

$$|z - 1 - 8\mathrm{i}| \geqslant 3|z - 1| \quad \text{and} \quad 0 \leqslant \arg(z+\mathrm{i}) \leqslant \frac{\pi}{4}$$

(4)

(a) Let $z = x + iy$

$|(x - 1) + (y - 8)i| = 3|(x - 1) + yi|$

$(x - 1)^2 + (y - 8)^2 = 9[(x - 1)^2 + y^2]$

$x^2 - 2x + 1 + y^2 - 16y + 64 = 9x^2 - 18x + 9 + 9y^2$

$\Rightarrow 8x^2 - 16x + 8y^2 + 16y - 56 = 0$

$\Rightarrow x^2 - 2x + y^2 + 2y - 7 = 0$

$\Rightarrow (x - 1)^2 + (y + 1)^2 = 9$

$\therefore$ C is a circle, centre $(1, -1)$, radius 3

(b) $|z - 1 - 8i| \geq 3|z - 1| \Leftrightarrow (x - 1)^2 + (y + 1)^2 \leq 9$,

i.e. on or inside the circle C.

$z = 3 - 3i$: distance from $(1, -1) = \sqrt{((3 - 1)^2 + (-3 + 1)^2)}$

$= \sqrt{8} = 2.828... < 3$

$\therefore$ $z = 3 - 3i$ lies inside C, so it satisfies the inequality.

(c) see Argand diagram on the next page

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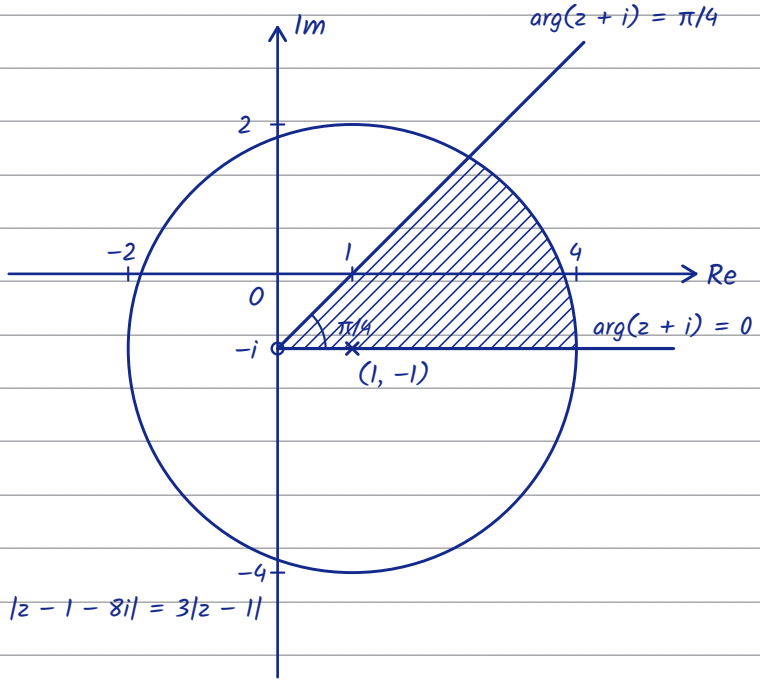
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Question 3 continued

(c)



Shaded region (including boundaries, excluding the point −i) satisfies both conditions.

Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)

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4. The set $\{e, p, q, r, s\}$ forms a group, A, under the operation $*$

Given that e is the identity element and that

$$p*p = s \qquad s*s = r \qquad p*p*p = q$$

(a) show that

(i) $p*q = r$

(ii) $s*p = q$

(2)

(b) Hence complete the Cayley table below.

*	e	p	q	r	s
e	e	p	q	r	s
p	p	s	r	e	q
q	q	r	p	s	e
r	r	e	s	q	p
s	s	q	e	p	r

A spare table can be found on page 11 if you need to rewrite your Cayley table.

(2)

(c) Use your table to find $p*q*r*s$

(1)

A student states that there is a subgroup of A of order 3

(d) Comment on the validity of this statement, giving a reason for your answer.

(2)

(a)(i) $p*q = p*(p*p*p) = (p*p)*(p*p) = s*s = r$

(a)(ii) $s*p = (p*p)*p = p*p*p = q$

(b) see table

(c) $p*q*r*s = (p*q)*r*s = r*r*s = q*s = e$

(d) By Lagrange's theorem, the order of a subgroup must be a factor of the order of the group.
A has order 5 and 3 is not a factor of 5,
so the student's statement is not valid (it is wrong).

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Question 4 continued

Only use this grid if you need to rewrite the Cayley table.

*	*e*	*p*	*q*	*r*	*s*
e					
p					
q					
r					
s					

(Total for Question 4 is 7 marks)

5. On Jim's 11th birthday his parents invest £1000 for him in a savings account.

The account earns 2% interest each year.

On each subsequent birthday, Jim's parents add another £500 to this savings account.

Let U_n be the amount of money that Jim has in his savings account n years after his 11th birthday, once the interest for the previous year has been paid and the £500 has been added.

(a) Explain, in the context of the problem, why the amount of money that Jim has in his savings account can be modelled by the recurrence relation of the form

$$U_n = 1.02U_{n-1} + 500 \qquad U_0 = 1000 \qquad n \in \mathbb{Z}^+$$

(3)

(b) State an assumption that must be made for this model to be valid.

(1)

(c) Solve the recurrence relation

$$U_n = 1.02U_{n-1} + 500 \qquad U_0 = 1000 \qquad n \in \mathbb{Z}^+$$

(5)

Jim hopes to be able to buy a car on his 18th birthday.

(d) Use the answer to part (c) to find out whether Jim will have enough money in his savings account to buy a car that costs £4 500

(2)

(a) U_{n-1} is the amount in the account $n-1$ years after his 11th birthday. 2% interest is added each year, so this is multiplied by 1.02 to give $1.02U_{n-1}$.
His parents add £500 on each subsequent birthday, so 500 is added.
$U_0 = 1000$ since £1000 was invested on his 11th birthday.

(b) The interest rate stays at 2% each year (and Jim does not withdraw any money from the account).

(c) U_n = complementary function + particular solution
CF: $U_n = c(1.02)^n$
PS: try $U_n = \lambda$: $\lambda = 1.02\lambda + 500$
$\Rightarrow -0.02\lambda = 500 \Rightarrow \lambda = -25\,000$
$\therefore U_n = c(1.02)^n - 25\,000$
$U_0 = 1000$: $1000 = c(1.02)^0 - 25\,000 \Rightarrow c = 26\,000$
$\therefore U_n = 26\,000(1.02)^n - 25\,000, \quad n \geq 0$

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Question 5 continued

(d) 18th birthday is 7 years after his 11th birthday: $n = 7$

$U_7 = 26\,000(1.02)^7 - 25\,000$

$= 29\,865.827... - 25\,000 = 4865.827...$

$= £4865.83$

£4865.83 > £4500, so Jim will have enough money in his savings account to buy the car.

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Question 5 continued

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Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 2 IS 40 MARKS

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