Please check the examination details below before entering your candidate information

Candidate surname: StemUp Model Solution | Other names:

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Monday 13 May 2019

Afternoon (Time: 1 hour 40 minutes) | Paper Reference **8FM0-01**

Further Mathematics

Advanced Subsidiary

Paper 1: Core Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1.

$$\mathbf{M} = \begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}$$

(a) Show that the matrix **M** is non-singular.

(2)

The transformation T of the plane is represented by the matrix **M**.

The triangle R is transformed to the triangle S by the transformation T.

Given that the area of S is 63 square units,

(b) find the area of R.

(2)

(c) Show that the line $y = 2x$ is invariant under the transformation T.

(2)

(a) $\det M = (4)(-7) - (-5)(2) = -28 + 10 = -18$

$\det M = -18 \neq 0$, so M is non-singular.

(b) Area of $R = \dfrac{\text{Area of } S}{|\det M|} = \dfrac{63}{18} = \dfrac{7}{2}$

Area of R = 3.5 square units

(c) $\begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}\begin{pmatrix} x \\ 2x \end{pmatrix} = \begin{pmatrix} 4x - 10x \\ 2x - 14x \end{pmatrix} = \begin{pmatrix} -6x \\ -12x \end{pmatrix} = -6\begin{pmatrix} x \\ 2x \end{pmatrix}$

So any point (x, 2x) on y = 2x maps to (−6x, −12x), and −12x = 2(−6x),
so every image point also lies on y = 2x.
Hence the line y = 2x is invariant under T.

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Question 1 continued

(Total for Question 1 is 6 marks)

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2. The cubic equation

$$2x^3 + 6x^2 - 3x + 12 = 0$$

has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(\alpha + 3)$, $(\beta + 3)$ and $(\gamma + 3)$, giving your answer in the form $pw^3 + qw^2 + rw + s = 0$, where p, q, r and s are integers to be found.

(5)

$w = x + 3 \quad \Rightarrow \quad x = w - 3$

$2(w-3)^3 + 6(w-3)^2 - 3(w-3) + 12 = 0$

$2(w^3 - 9w^2 + 27w - 27) + 6(w^2 - 6w + 9) - 3w + 9 + 12 = 0$

$2w^3 - 18w^2 + 54w - 54 + 6w^2 - 36w + 54 - 3w + 21 = 0$

$2w^3 - 12w^2 + 15w + 21 = 0$

$p = 2, \quad q = -12, \quad r = 15, \quad s = 21$

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Question 2 continued

(Total for Question 2 is 5 marks)

3. Prove by mathematical induction that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$$

(6)

$n = 1$: LHS $= \sum_{r=1}^{1} \frac{1}{(2r-1)(2r+1)} = \frac{1}{1 \times 3} = \frac{1}{3}$, RHS $= \frac{1}{2(1)+1} = \frac{1}{3}$

LHS = RHS, so the result is true for $n = 1$.

Assume true for $n = k$: $\sum_{r=1}^{k} \frac{1}{(2r-1)(2r+1)} = \frac{k}{2k+1}$

Then $\sum_{r=1}^{k+1} \frac{1}{(2r-1)(2r+1)} = \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$

$= \frac{k(2k+3)+1}{(2k+1)(2k+3)} = \frac{2k^2+3k+1}{(2k+1)(2k+3)}$

$= \frac{(2k+1)(k+1)}{(2k+1)(2k+3)} = \frac{k+1}{2k+3} = \frac{k+1}{2(k+1)+1}$

which is the result with $n = k + 1$.

The result is true for $n = 1$, and if it is true for $n = k$ then it is true for $n = k + 1$. Hence, by mathematical induction, the result is true for all $n \in \mathbb{N}$.

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Question 3 continued

(Total for Question 3 is 6 marks)

4. The line l has equation

$$\frac{x+2}{1} = \frac{y-5}{-1} = \frac{z-4}{-3}$$

The plane Π has equation

$$\mathbf{r}.(\mathbf{i}-2\mathbf{j}+\mathbf{k}) = -7$$

Determine whether the line l intersects Π at a single point, or lies in Π, or is parallel to Π without intersecting it.

(5)

l: $\mathbf{r} = \begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix}$ so a general point on l is $\begin{pmatrix} -2+\lambda \\ 5-\lambda \\ 4-3\lambda \end{pmatrix}$

Π: $\mathbf{r}.(\mathbf{i}-2\mathbf{j}+\mathbf{k}) = -7 \Rightarrow x - 2y + z = -7$

Substitute: $\begin{pmatrix} -2+\lambda \\ 5-\lambda \\ 4-3\lambda \end{pmatrix} . \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = -7$

$(-2+\lambda) - 2(5-\lambda) + (4-3\lambda) = -7$

$-2 + \lambda - 10 + 2\lambda + 4 - 3\lambda = -7$

$0\lambda - 8 = -7 \Rightarrow -8 = -7$, which is a contradiction.

So there is no value of λ for which l meets Π: no intersection.

(Check: $\begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix} . \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 1 + 2 - 3 = 0$, so l is parallel to Π)

Hence l is parallel to Π without intersecting it (l does not lie in Π).

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Question 4 continued

(Total for Question 4 is 5 marks)

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5.

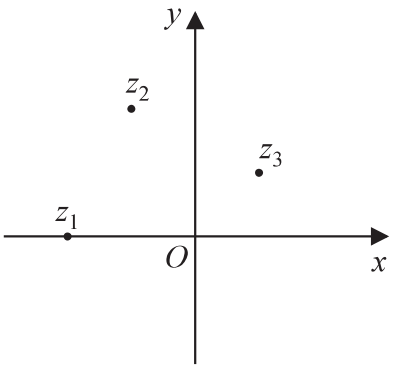


Figure 1

The complex numbers $z_1 = -2$, $z_2 = -1 + 2i$ and $z_3 = 1 + i$ are plotted in Figure 1, on an Argand diagram for the complex plane with $z = x + iy$

(a) Explain why z_1, z_2 and z_3 cannot all be roots of a quartic polynomial equation with real coefficients.

(2)

(b) Show that $\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \frac{\pi}{4}$

(3)

(c) Hence show that $\arctan(2) - \arctan\left(\frac{1}{3}\right) = \frac{\pi}{4}$

(2)

A copy of Figure 1, labelled Diagram 1, is given on page 12.

(d) Shade, on Diagram 1, the set of points of the complex plane that satisfy the inequality

$$|z + 2| \leqslant |z - 1 - i|$$

(2)

(a) Complex roots of a polynomial with real coefficients occur in conjugate pairs, so $z_2^* = -1 - 2i$ and $z_3^* = 1 - i$ would also be roots. That gives at least 5 roots (z_1, z_2, z_2^*, z_3, z_3^*), but a quartic has at most 4 roots, so z_1, z_2 and z_3 cannot all be roots.

(b) $\frac{z_2 - z_1}{z_3 - z_1} = \frac{-1 + 2i - (-2)}{1 + i - (-2)} = \frac{1 + 2i}{3 + i} \times \frac{3 - i}{3 - i}$

$= \frac{3 - i + 6i - 2i^2}{9 + 1} = \frac{5 + 5i}{10} = \frac{1}{2} + \frac{1}{2}i$

This lies in the first quadrant (real and imaginary parts both positive),

so $\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \arctan\left(\frac{1/2}{1/2}\right) = \arctan(1) = \frac{\pi}{4}$

(c) $\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \arg(z_2 - z_1) - \arg(z_3 - z_1) = \arg(1 + 2i) - \arg(3 + i)$

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Question 5 continued

$= \arctan(2) - \arctan\left(\frac{1}{3}\right)$ (both in the first quadrant)

So, using (b), $\arctan(2) - \arctan\left(\frac{1}{3}\right) = \frac{\pi}{4}$

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Question 5 continued

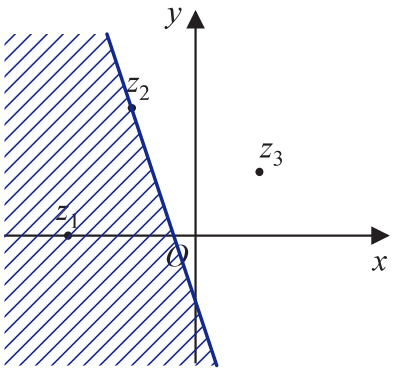


Diagram 1

(d) $|z + 2| = |z - 1 - i|$ is the perpendicular bisector of z_1 and z_3:

$(x + 2)^2 + y^2 = (x - 1)^2 + (y - 1)^2$

$4x + 4 = -2x - 2y + 2 \quad \Rightarrow \quad y = -3x - 1$ (passes through z_2)

$|z + 2| \leq |z - 1 - i|$: points nearer z_1 than z_3, shaded on Diagram 1

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Question 5 continued

(Total for Question 5 is 9 marks)

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6. An art display consists of an arrangement of n marbles.

When arranged in ascending order of mass, the mass of the first marble is 10 grams. The mass of each subsequent marble is 3 grams more than the mass of the previous one, so that the rth marble has mass $(7 + 3r)$ grams.

(a) Show that the mean mass, in grams, of the marbles in the display is given by

$$\frac{1}{2}(3n+17)$$

(3)

Given that there are 85 marbles in the display,

(b) use the standard summation formulae to find the standard deviation of the mass of the marbles in the display, giving your answer, in grams, to one decimal place.

(6)

(a) Mean $= \frac{1}{n}\sum_{r=1}^{n}(7 + 3r)$

$\sum_{r=1}^{n}(7 + 3r) = 7\sum_{r=1}^{n}1 + 3\sum_{r=1}^{n}r = 7n + \frac{3}{2}n(n + 1)$

Mean $= 7 + \frac{3}{2}(n + 1) = \frac{14 + 3n + 3}{2} = \frac{1}{2}(3n + 17)$

(b) $n = 85$: mean $= \frac{1}{2}(3 \times 85 + 17) = 136$

$\sum_{r=1}^{n}(7 + 3r)^2 = \sum_{r=1}^{n}(49 + 42r + 9r^2)$

$= 49n + 42 \times \frac{1}{2}n(n + 1) + 9 \times \frac{1}{6}n(n + 1)(2n + 1)$

$= 49(85) + 21(85)(86) + \frac{3}{2}(85)(86)(171)$

$= 4165 + 153510 + 1875015 = 2032690$

Variance $= \frac{\Sigma x^2}{n} - (\text{mean})^2 = \frac{2032690}{85} - 136^2$

$= 23914 - 18496 = 5418$

s.d. $= \sqrt{5418} = 73.607...$

Standard deviation = 73.6 g (1 d.p.)

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Question 6 continued

(Total for Question 6 is 9 marks)

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7. $$f(z) = z^3 - 8z^2 + pz - 24$$

where p is a real constant.

Given that the equation $f(z) = 0$ has distinct roots

$$\alpha,\ \beta \text{ and } \left(\alpha + \frac{12}{\alpha} - \beta\right)$$

(a) solve completely the equation $f(z) = 0$

(6)

(b) Hence find the value of p.

(2)

(a) Sum of roots = 8: $\alpha + \beta + \left(\alpha + \frac{12}{\alpha} - \beta\right) = 8 \Rightarrow 2\alpha + \frac{12}{\alpha} = 8$

$2\alpha^2 - 8\alpha + 12 = 0 \Rightarrow \alpha^2 - 4\alpha + 6 = 0$

$\alpha = \frac{4 \pm \sqrt{(16 - 24)}}{2} = \frac{4 \pm \sqrt{(-8)}}{2} = \frac{4 \pm 2\sqrt{2}\,i}{2} = 2 \pm i\sqrt{2}$

f has real coefficients, so the roots α and β are the conjugate pair

$\alpha = 2 + i\sqrt{2}, \quad \beta = 2 - i\sqrt{2}$

Third root $= \alpha + \frac{12}{\alpha} - \beta = 2i\sqrt{2} + \frac{12}{2 + i\sqrt{2}} \times \frac{2 - i\sqrt{2}}{2 - i\sqrt{2}}$

$= 2i\sqrt{2} + \frac{12(2 - i\sqrt{2})}{6} = 2i\sqrt{2} + 4 - 2i\sqrt{2} = 4$

(Check: product of roots $= (2 + i\sqrt{2})(2 - i\sqrt{2}) \times 4 = 6 \times 4 = 24$ ✓)

Roots: $z = 2 + i\sqrt{2}, \quad z = 2 - i\sqrt{2}, \quad z = 4$

(b) p = sum of products of pairs of roots

$= (2 + i\sqrt{2})(2 - i\sqrt{2}) + 4(2 + i\sqrt{2}) + 4(2 - i\sqrt{2})$

$= 6 + 8 + 4i\sqrt{2} + 8 - 4i\sqrt{2}$

$p = 22$

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 8 marks)



8. A gas company maintains a straight pipeline that passes under a mountain.

The pipeline is modelled as a straight line and one side of the mountain is modelled as a plane.

There are accessways from a control centre to two access points on the pipeline.

Modelling the control centre as the origin O, the two access points on the pipeline have coordinates $P(-300, 400, -150)$ and $Q(300, 300, -50)$, where the units are metres.

(a) Find a vector equation for the line PQ, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where λ is a scalar parameter.

(2)

The equation of the plane modelling the side of the mountain is $2x + 3y - 5z = 300$

The company wants to create a new accessway from this side of the mountain to the pipeline.

The accessway will consist of a tunnel of shortest possible length between the pipeline and the point $M(100, k, 100)$ on this side of the mountain, where k is a constant.

(b) Using the model, find

(i) the coordinates of the point at which this tunnel will meet the pipeline,

(ii) the length of this tunnel.

(7)

It is only practical to construct the new accessway if it will be significantly shorter than both of the existing accessways, OP and OQ.

(c) Determine whether the company should build the new accessway.

(2)

(d) Suggest one limitation of the model.

(1)

(a) $\overrightarrow{PQ} = \begin{pmatrix} 300 \\ 300 \\ -50 \end{pmatrix} - \begin{pmatrix} -300 \\ 400 \\ -150 \end{pmatrix} = \begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix} = 100\begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix}$

$\mathbf{r} = \begin{pmatrix} -300 \\ 400 \\ -150 \end{pmatrix} + \lambda\begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix}$

(b)(i) M on the plane: $2(100) + 3k - 5(100) = 300 \Rightarrow 3k = 600 \Rightarrow k = 200$

Let X be a point on PQ: $\overrightarrow{MX} = \begin{pmatrix} -300 + 6\lambda \\ 400 - \lambda \\ -150 + \lambda \end{pmatrix} - \begin{pmatrix} 100 \\ 200 \\ 100 \end{pmatrix} = \begin{pmatrix} -400 + 6\lambda \\ 200 - \lambda \\ -250 + \lambda \end{pmatrix}$

Shortest tunnel when MX is perpendicular to the line:

$\begin{pmatrix} -400 + 6\lambda \\ 200 - \lambda \\ -250 + \lambda \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix} = 0$

$-2400 + 36\lambda - 200 + \lambda - 250 + \lambda = 0 \Rightarrow 38\lambda = 2850 \Rightarrow \lambda = 75$

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Question 8 continued

$$\overrightarrow{OX} = \begin{pmatrix} -300 \\ 400 \\ -150 \end{pmatrix} + 75\begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 150 \\ 325 \\ -75 \end{pmatrix}$$

The tunnel meets the pipeline at (150, 325, −75)

(ii) $\overrightarrow{MX} = \begin{pmatrix} 150 - 100 \\ 325 - 200 \\ -75 - 100 \end{pmatrix} = \begin{pmatrix} 50 \\ 125 \\ -175 \end{pmatrix}$

Length $= \sqrt{(50^2 + 125^2 + (-175)^2)} = \sqrt{48750} = 220.794...$

Length of tunnel = 221 m (3 s.f.)

(c) $OP = \sqrt{((-300)^2 + 400^2 + (-150)^2)} = \sqrt{272500} = 522$ m (3 s.f.)

$OQ = \sqrt{(300^2 + 300^2 + (-50)^2)} = \sqrt{182500} = 427$ m (3 s.f.)

221 m is over 200 m shorter than both OP and OQ, so the new tunnel is significantly shorter: the company should build the new accessway.

(d) The side of the mountain is unlikely to be flat, so modelling it as a plane may not be accurate (the point M may not be where predicted).

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 12 marks)

9. $f(x) = 2x^{\frac{1}{3}} + x^{-\frac{2}{3}} \qquad x > 0$

The finite region bounded by the curve $y = f(x)$, the line $x = \frac{1}{8}$, the x-axis and the line $x = 8$ is rotated through θ radians about the x-axis to form a solid of revolution.

Given that the volume of the solid formed is $\frac{461}{2}$ units cubed, use algebraic integration to find the angle θ through which the region is rotated.

(8)

$y^2 = (2x^{1/3} + x^{-2/3})^2 = 4x^{2/3} + 4x^{-1/3} + x^{-4/3}$

$\int y^2\,dx = \int (4x^{2/3} + 4x^{-1/3} + x^{-4/3})\,dx = \frac{12}{5}x^{5/3} + 6x^{2/3} - 3x^{-1/3}$

Full turn (2π): $V = \pi\int_{1/8}^{8} y^2\,dx$, so through θ: $V = \frac{\theta}{2\pi} \times \pi\int_{1/8}^{8} y^2\,dx = \frac{\theta}{2}\int_{1/8}^{8} y^2\,dx$

$\left[\frac{12}{5}x^{5/3} + 6x^{2/3} - 3x^{-1/3}\right]_{1/8}^{8} = \left(\frac{12}{5}(32) + 6(4) - \frac{3}{2}\right) - \left(\frac{12}{5} \times \frac{1}{32} + 6 \times \frac{1}{4} - 3(2)\right)$

$= (76.8 + 24 - 1.5) - (0.075 + 1.5 - 6)$

$= 99.3 - (-4.425) = 103.725 = \frac{4149}{40}$

$\frac{\theta}{2} \times \frac{4149}{40} = \frac{461}{2} \Rightarrow \theta = \frac{461 \times 40}{4149} = \frac{18440}{4149}$

$\theta = \frac{40}{9}$ radians

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Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 8 marks)

10. The population of chimpanzees in a particular country consists of juveniles and adults. Juvenile chimpanzees do not reproduce.

In a study, the numbers of juvenile and adult chimpanzees were estimated at the start of each year. A model for the population satisfies the matrix system

$$\begin{pmatrix} J_{n+1} \\ A_{n+1} \end{pmatrix} = \begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix} \begin{pmatrix} J_n \\ A_n \end{pmatrix} \qquad n = 0,1,2,\ldots$$

where a is a constant, and J_n and A_n are the respective numbers of juvenile and adult chimpanzees n years after the start of the study.

(a) Interpret the meaning of the constant a in the context of the model.

(1)

At the start of the study, the total number of chimpanzees in the country was estimated to be 64 000

According to the model, after one year the number of juvenile chimpanzees is 15 360 and the number of adult chimpanzees is 43 008

(b) (i) Find, in terms of a

$$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1}$$

(3)

(ii) Hence, or otherwise, find the value of a.

(3)

(iii) Calculate the change in the number of juvenile chimpanzees in the first year of the study, according to this model.

(2)

Given that the number of juvenile chimpanzees is known to be in decline in the country,

(c) comment on the short-term suitability of this model.

(1)

A study of the population revealed that adult chimpanzees stop reproducing at the age of 40 years.

(d) Refine the matrix system for the model to reflect this information, giving a reason for your answer.

(*There is no need to estimate any unknown values for the refined model, but any known values should be made clear.*)

(2)

(a) a is the proportion of juvenile chimpanzees that remain juveniles (survive but do not become adults) from one year to the next.

(b)(i) $\det = 0.82a - 0.08 \times 0.15 = 0.82a - 0.012$

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Question 10 continued

$$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1} = \frac{1}{0.82a - 0.012}\begin{pmatrix} 0.82 & -0.15 \\ -0.08 & a \end{pmatrix}$$

(ii) $$\begin{pmatrix} J_0 \\ A_0 \end{pmatrix} = \frac{1}{0.82a - 0.012}\begin{pmatrix} 0.82 & -0.15 \\ -0.08 & a \end{pmatrix}\begin{pmatrix} 15360 \\ 43008 \end{pmatrix}$$

$$= \frac{1}{0.82a - 0.012}\begin{pmatrix} 0.82 \times 15360 - 0.15 \times 43008 \\ -0.08 \times 15360 + 43008a \end{pmatrix} = \frac{1}{0.82a - 0.012}\begin{pmatrix} 6144 \\ 43008a - 1228.8 \end{pmatrix}$$

$J_0 + A_0 = 64000$:

$6144 + 43008a - 1228.8 = 64000(0.82a - 0.012)$

$4915.2 + 43008a = 52480a - 768$

$9472a = 5683.2 \Rightarrow a = \frac{5683.2}{9472} = 0.6$

(iii) $J_0 = \frac{6144}{0.82(0.6) - 0.012} = \frac{6144}{0.48} = 12800$

Change $= 15360 - 12800 = 2560$

The number of juveniles increases by 2560 in the first year.

(c) The model predicts an increase in juveniles in the first year, not a decline, so the model is not suitable in the short term.

(d) Introduce a third category M_n: mature chimpanzees aged 40 and over.

$$\begin{pmatrix} J_{n+1} \\ A_{n+1} \\ M_{n+1} \end{pmatrix} = \begin{pmatrix} a & b & 0 \\ 0.08 & c & 0 \\ 0 & d & e \end{pmatrix}\begin{pmatrix} J_n \\ A_n \\ M_n \end{pmatrix}$$

The 0 in the top right shows mature chimpanzees (aged 40 or over) do not reproduce, so produce no juveniles. 0.08 is still the proportion of juveniles becoming adults; juveniles cannot become mature directly and mature chimpanzees cannot return to being younger adults (the other 0s).

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Question 10 continued

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DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

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Question 10 continued

Question 10 continued

(Total for Question 10 is 12 marks)

TOTAL FOR CORE PURE MATHEMATICS IS 80 MARKS

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