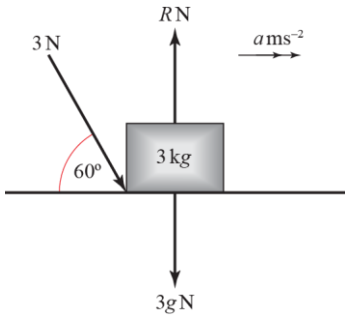
Forces and friction Mixed exercise 5

1 a Resolving vertically:



$$R = 3g + 3\sin 30^o$$
$$= (3\times 9.8) + 3\frac{\sqrt{3}}{2}$$
$$= 31.998...$$

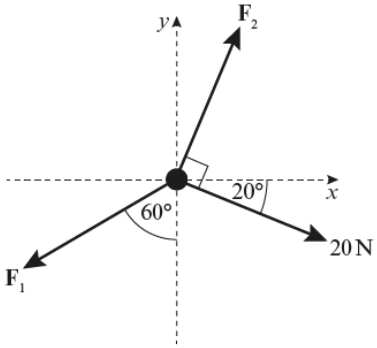
The normal reaction of the floor on the box is 32.0 N (3s.f.).

b Resolving horizontally and using $F = ma$:

$$3\cos 60^o = 3a$$
$$a = 0.5$$

The acceleration of the box is 0.5 ms^{-2}

2 Resolving vertically (**j** components):



$$F_2\cos 20^\circ - F_1\cos 60^\circ - 20\sin 20^\circ = 2$$
$$F_2\cos 20^\circ - F_1\cos 60^\circ = 2 + 20\sin 20^\circ$$
$$F_2\cos 20^\circ - F_1\cos 60^\circ = 8.8404... \quad (1)$$

Resolving horizontally (**i** components):

$$F_2\sin 20^\circ + 20\cos 20^\circ - F_1\sin 60^\circ = 3$$
$$20\cos 20^\circ - 3 + F_2\sin 20^\circ = F_1\sin 60^\circ$$
$$\frac{20\cos 20^\circ - 3 + F_2\sin 20^\circ}{\sin 60^\circ} = F_1$$
$$18.237 + 0.39493F_2 = F_1 \quad (2)$$

Substituting value for F_1 from (**2**) into (**1**):

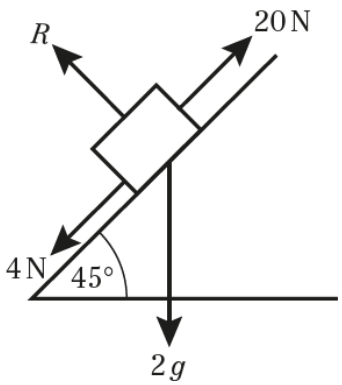
$$8.8404 = F_2\cos 20^\circ - (18.237 + 0.39493F_2)\cos 60^\circ$$
$$8.8404 = 0.93969F_2 - 9.1185 - 0.19746F_2$$
$$8.8404 + 9.1185 = (0.93969 - 0.19746)F_2$$
$$F_2 = \frac{17.958...}{0.7422...} = 24.196...$$

Substituting $F_2 = 24.196...$ into (**2**)

$$F_1 = 18.237 + (0.39493\times 24.196)$$
$$F_1 = 27.792...$$

The forces F_1 and F_2 are 27.8 N and 24.2 N respectively (both to 3s.f.).

3 a



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3 b R(↖):

$$R = 2g\cos 45^\circ$$
$$= 2\times 9.8\cos 45^\circ$$
$$= 13.859...$$

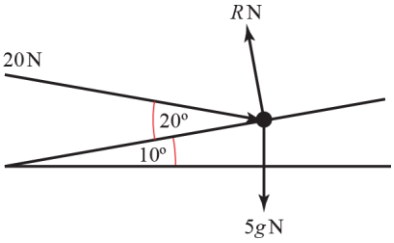
The normal reaction between the particle and the plane is 13.9 N (3s.f.).

c Using Newton's second law of motion and R(↗):

$$F = ma$$
$$20 - 4 - 2g\sin 45^\circ = 2a$$
$$a = 8 - g\sin 45^\circ$$
$$a = 1.0703...$$
$$a = 1.1 \text{ ms}^{-2} \text{ (2 s.f.)}$$

4 Using Newton's second law of motion and R(↗):

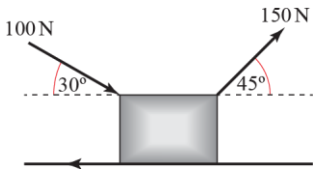
$$F = ma$$
$$20\cos 20^\circ - 5g\sin 10^\circ = 5a$$
$$4\cos 20^\circ - g\sin 10^\circ = a$$
$$a = 2.0570...$$



The acceleration of the particle is 2.06 ms^{-2} (3 s.f.) up the slope.

5 Since the box is moving at constant speed, the horizontal component of the resultant force is zero.
Resolving horizontally:

$$F = 150\cos 45^\circ + 100\cos 30^\circ$$
$$F = \frac{150}{\sqrt{2}} + \frac{100\sqrt{3}}{2}$$
$$F = \frac{50}{2}(3\sqrt{2} + 2\sqrt{3})$$
$$F = 25(3\sqrt{2} + 2\sqrt{3}) \text{ N as required.}$$

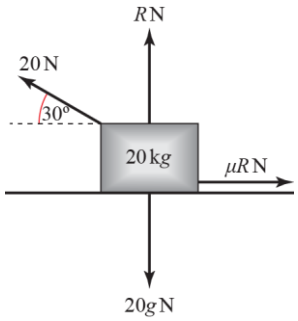


6 Resolving vertically:

$$R = 20g$$

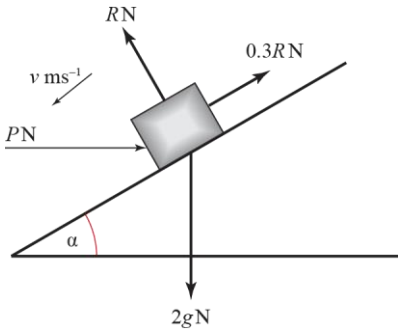
Resolving horizontally:

$$20\cos 30^\circ = \mu R$$
$$20\cos 30^\circ = 20\mu g$$
$$\mu = \frac{\cos 30^\circ}{g}$$
$$\mu = \frac{\sqrt{3}}{2\times 9.8} = \frac{5\sqrt{3}}{98}$$



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7 $\tan\alpha = \frac{3}{4}$ so $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$



R(↖):

$$R = 2g\cos\alpha + P\sin\alpha$$

$$R = \frac{8}{5}g + \frac{3}{5}P$$

Particle moving at a constant velocity means that forces parallel to the slope are balanced.

R(↗):

$$P\cos\alpha + 0.3R = 2g\sin a$$

$$\frac{4}{5}P + \frac{3}{10}\left(\frac{8}{5}g + \frac{3}{5}P\right) = \frac{6}{5}g$$

$$40P + 3(8g + 3P) = 60g$$

$$40P + 9P = 60g - 24g$$

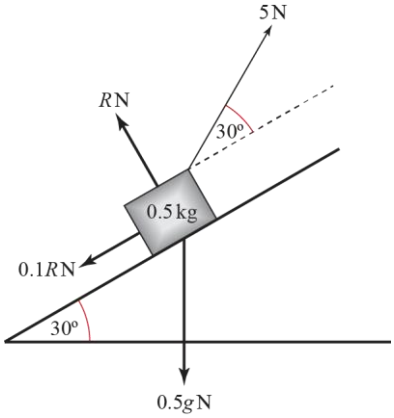
$$P = \frac{36\times 9.8}{49} = 7.2$$

The force P is 7.2 N.

8 R(↖):

$$R + 5\sin 30^\circ = 0.5g\cos 30^\circ$$

$$R = \frac{g\sqrt{3}}{4} - \frac{5}{2}$$



Using Newton's second law of motion and R(↗):

$$ma = F$$

$$0.5a = 5\cos 30^\circ - 0.1R - 0.5g\sin 30^\circ$$

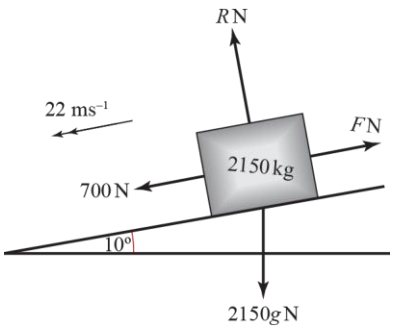
$$a = 5\sqrt{3} - \frac{1}{5}\left(\frac{g\sqrt{3}}{4} - \frac{5}{2}\right) - \frac{g}{2}$$

$$a = 5\sqrt{3} + \frac{1}{2} - \left(\frac{\sqrt{3}}{20} + \frac{1}{2}\right)\times 9.8$$

$$a = 3.4115...$$

The acceleration of the particle is 3.41 ms^{-2} (3s.f.).

9 **a** Since the car is travelling at constant speed, the resultant force parallel to the road is zero.



R(↗):

$$F = 700 + 2150g\sin 10^\circ$$

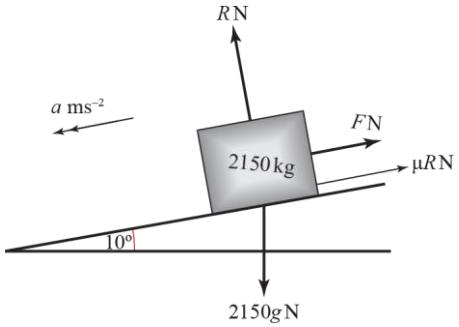
$$F = 700 + (2150\times 9.8\sin 10^\circ)$$

$$F = 4358.7...$$

The air resistance, F, is 4400 N (2s.f.).

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9 b R(↖):



$$R = 2150g\cos 10^\circ$$
$$= 2150 \times 9.8\cos 10^\circ$$
$$= 20750$$

$R\left(\swarrow\right)$:

$u = 22$ ms^{-1}, $v = 0$ ms^{-1}, $s = 40$ m, a = ?

$$v^2 = u^2 + 2as$$
$$0 = 22^2 + (2a \times 40)$$
$$a = -\frac{22^2}{80} = -6.05$$

A negative value for acceleration indicates *deceleration* (or acceleration *up* the slope).

R(↗):

$$F = ma$$
$$F + \mu R - 2150g\sin 10^\circ = 2150a$$
$$4358.7 + 20750\mu - 2150g\sin 10^\circ = 2150 \times 6.05 \qquad \text{(taking } F \text{ from part } \mathbf{a}\text{)}$$
$$\mu = \frac{13007.5 - 700}{20750}$$
$$= 0.59313...$$

The coefficient of friction is 0.59 (2s.f.).

c i The force due to air resistance will reduce as the car slows.
If the skid causes the tyres to heat, the value of μ is also likely to vary.

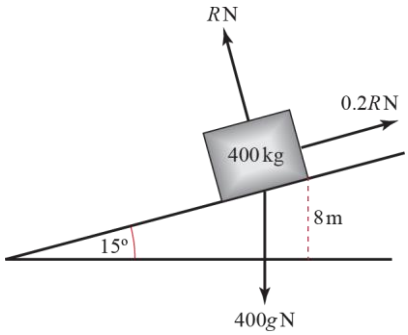
ii If the car skids, it will not be travelling in a straight line so equations of motion do not apply and other forces need to be considered.

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Challenge

At the instant the cable breaks, the boat is still moving up the slope with speed 5 ms^{-1}
At this time, friction acts down the slope and the boat will decelerate to instantaneous rest on the slipway.

When the boat is about to accelerate back down the slipway to the water, the forces acting on the boat are as shown:



To show that the boat will slide back down the slipway, we need to show that the component of weight acting down the slope is greater than the limiting friction:

$R(\nwarrow): R = 400g\cos 15^\circ$

Calculate magnitude of limiting frictional force acting up the slope:

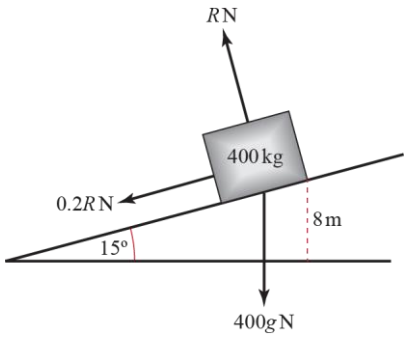
$$F_{MAX} = \mu R$$
$$= 0.2 \times 400g\cos 15^\circ$$
$$= 80g\cos 15^\circ$$
$$= 757.28...$$

Component of weight acting down the slope $= 400g\sin 15^o$
$= 1014.5...$

$F_{MAX} = 757\text{ N} < 1015\text{ N} = \text{Weight down slope}$

Hence, boat will slide back down into the water.

While the boat continues to move up the slope after the cable breaks, μR continues to act down the slope.



Between the time when the cable breaks and the boat comes to instantaneous rest on the slipway,

$R(\swarrow)$

$$F = ma$$
$$400g\sin 15^o + 80g\cos 15^o = 400a$$
$$a = \left(\sin 15^o + 0.2\cos 15^o\right)g$$
$$a = 4.4296...$$

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Challenge (cont.)

$R(\nearrow)$: $u = 5\ \text{ms}^{-1}$, $v = 0$, $a = -4.430\ \text{ms}^{-2}$, $t = t_1$

$$v = u + at$$
$$0 = 5 - 4.43t_1$$
$$t_1 = \frac{5}{4.43} = 1.1287...$$

Once the boat has come to rest, frictional force acts up the slope (see the first diagram).

$R(\swarrow)$:

$$F = ma$$
$$400g\sin 15^o - 80g\cos 15^o = 400a$$
$$a = \left(\sin 15^o - 0.2\cos 15^o\right)g$$
$$a = 0.64321$$

Distance along slipway from where cable breaks to the sea is s where:

$$\sin 15^\circ = \frac{8}{s}$$
$$s = \frac{8}{\sin 15^\circ} = 30.909...$$

$R(\swarrow)$: $u = 0\ \text{ms}^{-1}$, $a = 0.6432\ \text{ms}^{-2}$, $s = 30.91$ m, $t = t_2$

$$s = ut + \frac{1}{2}at^2$$
$$30.91 = 0t_2 + \frac{1}{2}0.6432\left(t_2\right)^2$$
$$30.91 = 0.3216\left(t_2\right)^2$$
$$t_2 = \sqrt{\frac{30.91}{0.3216}}$$
$$t_2 = 9.8035...$$

The boat returns to the sea t seconds after the cable snaps, where

$$t = t_1 + t_2$$
$$= 1.13 + 9.80$$
$$= 10.9\ \text{s} \quad (3\ \text{s.f.})$$

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