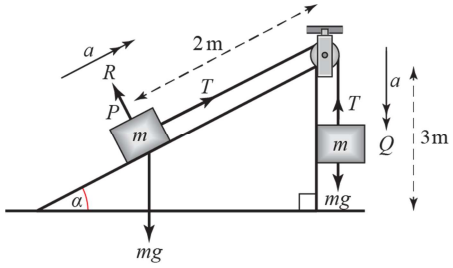
## Applications of forces, 7F

**1** For $P$: $R(\nearrow)$

$$T - mg\sin\alpha = ma$$

$$T - \frac{3mg}{5} = ma \quad \textbf{(1)}$$




For $Q$: $R(\downarrow)$

$$mg - T = ma \quad \textbf{(2)}$$

$$\textbf{(1)} + \textbf{(2)}: mg - \frac{3mg}{5} = 2ma$$

$$\frac{g}{5} = a$$

For $P$:

$$u = 0,\ a = \frac{g}{5},\ s = 2,\ v = ?$$

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + \frac{2g}{5} \times 2$$

$$v = \sqrt{\frac{4g}{5}} = 2.8\,\text{m s}^{-1}$$

$P$ hits the pulley with speed $2.8\,\text{m s}^{-1}$.

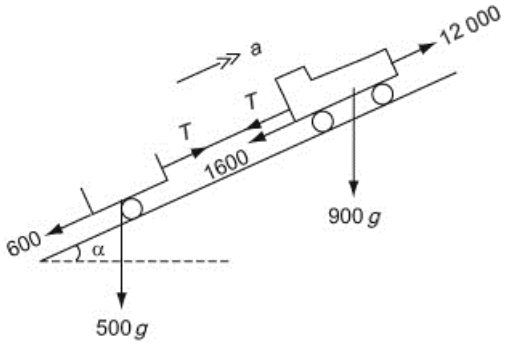
**2** $R(\nearrow)$ For the Van:

$$F = ma$$

$$12\,000 - T - 1600 - 900g\sin\alpha = 900a$$

$$10400 - 900 \times 9.8 \times \frac{3}{5} - T = 900a$$

$$5108 - T = 900a \quad \textbf{(1)}$$




$R(\nearrow)$ For the Trailer:

$$F = ma$$

$$T - 600 - 500g\sin\alpha = 500a$$

$$T - 600 - 500 \times 9.8 \times \frac{3}{5} = 500a$$

$$T - 3540 = 500a \quad \textbf{(2)}$$

**a** $\textbf{(1)} + \textbf{(2)} \Rightarrow 1568 = 1400a$

$$a = 1.12\ \text{ms}^{-2}$$

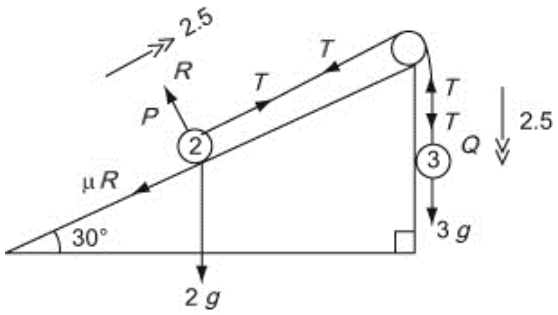
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**2 b** Sub $a = \dfrac{1568}{1400}$ ms$^{-2}$ in (**2**)

$$T = 3540 + 500 \times \frac{1568}{1400}$$
$$= 4100 \text{ N}$$

**c** The resistance forces are unlikely to be constant: it is more probable that they will increase as the speed increases.

**3 a** For $P$:




$R(\nwarrow)$

$$R - 2g\cos 30° = 0$$
$$R = g\sqrt{3}$$

$R(\nearrow)$

$$F = ma$$
$$T - \mu R - 2g\cos 60° = 2 \times 2.5$$
$$T - \mu g\sqrt{3} - g = 5 \qquad (\mathbf{1})$$

For $Q$:

$R(\downarrow)$

$$F = ma$$
$$3g - T = 3 \times 2.5$$
$$3g - T = 7.5 \qquad (\mathbf{2})$$
$$\therefore T = 21.9$$

The tension is 21.9 N.

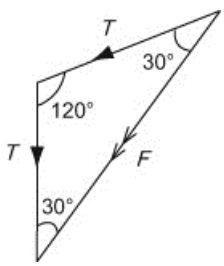
**b** $(\mathbf{1}) + (\mathbf{2}) \Rightarrow 2g - \mu g\sqrt{3} = 12.5$

$$\mu g\sqrt{3} = 7.1$$
$$\mu = \frac{7.1}{g\sqrt{3}}$$
$$= 0.418 \text{ (3 s.f.)}$$

The coefficient of friction is 0.42 (2 s.f.).

**c** $F = 2T\cos 30°$

$$= 43.8\cos 30°$$
$$= 37.9 \text{ N (3 s.f.)}$$




The force exerted by the string on the pulley is 38N (2 s.f.).

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**4 a** For $B$:

$R(\downarrow)$

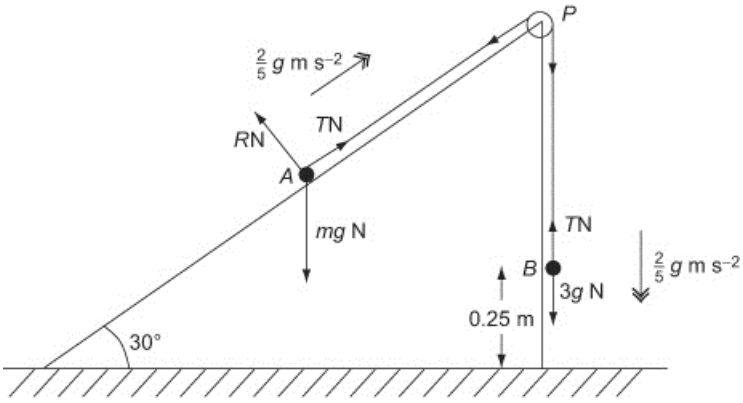
$$F = ma$$

$$3g - T = 3 \times \frac{2}{5}g$$

$$T = 3g - \frac{6}{5}g$$

$$= \frac{9}{5}g$$

$$= 17.64$$




The tension in the string while $B$ is descending is 18 N (2 s.f.).

**b** For $A$:

$R(\nearrow)$

$$F = ma$$

$$T - mg\sin 30° = m \times \frac{2}{5}g$$

$$\frac{9}{5}g - \frac{1}{2}mg = \frac{2}{5}mg$$

$$\left(\frac{1}{2} + \frac{2}{5}\right)m = \frac{9}{5}$$

$$\frac{9}{10}m = \frac{9}{5}$$

$$\Rightarrow m = 2$$

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**4 c** Whilst $A$ is still ascending,

$u = 0,\ a = \frac{2}{5}g,\ s = 0.25,\ v = ?$

$v^2 = u^2 + 2as$

$v^2 = \frac{4}{5}g \times 0.25$

$v = 1.4 \text{ ms}^{-2}$

After $B$ strikes the ground, there is no tension in the string and the only force acting on $A$ parallel to the plane is the component of its weight acting down the plane.

For $A$:

$R(\nearrow)$

$-mg\sin 30° = ma$

$a = -\frac{1}{2}g$

$u = 1.4,\ v = 0,\ a = -\frac{1}{2}g,\ t = ?$

$v = u + at$

$0 = 1.4 - \frac{1}{2}gt$

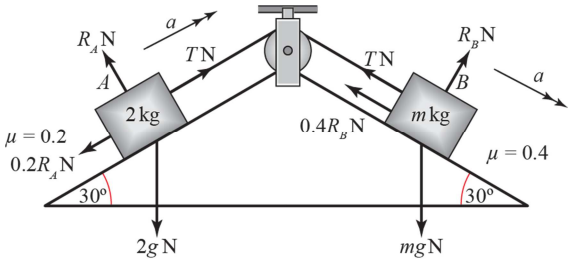
$\Rightarrow t = \frac{2.8}{9.8} = \frac{2}{7}$

The approximate answer, 0.28 s, would also be acceptable.

The time between the instants is $\frac{2}{7}$ s

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**5 a** Let the reaction forces on the blocks be $R_A$ and $R_B$.
If the system is in limiting equilibrium for the maximum value of $m$, object $B$ will move down the right-hand slope and object $A$ will move up the left-hand slope.




For $A$:

$R(\nwarrow)$:

$$R_A = 2g\cos 30^\circ = \sqrt{3}g$$

$R(\nearrow)$

$$T - 2g\sin 30^\circ - 0.2R_A = 0$$

$$T = g + \frac{\sqrt{3}}{5}g$$

$$T = \left(1 + \frac{\sqrt{3}}{5}\right)g \qquad \textbf{(1)}$$

For $B$

$R(\nearrow)$:

$$R_B = mg\cos 30^\circ = \frac{\sqrt{3}}{2}mg$$

$R(\nwarrow)$:

$$mg\sin 30 - T - 0.4R_B = 0$$

$$T = \frac{1}{2}mg - \frac{4}{10}\times\frac{\sqrt{3}}{2}mg$$

$$T = mg\left(\frac{1}{2} - \frac{\sqrt{3}}{5}\right) \qquad \textbf{(2)}$$

$$\left(\frac{1}{2} - \frac{\sqrt{3}}{5}\right)m = 1 + \frac{\sqrt{3}}{5}$$

$$\left(5 - 2\sqrt{3}\right)m = 10 + 2\sqrt{3}$$

$$m = \frac{10 + 2\sqrt{3}}{5 - 2\sqrt{3}}$$

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**5 b** Since $m = 10\text{ kg}$, $R_B = 5\sqrt{3}g$

$R(\nearrow)$ for $A$, using Newton's second law:

$$2a = T - 2g\sin 30^\circ - 0.2R_A$$

$$2a = T - g - \frac{\sqrt{3}}{5}g \qquad \textbf{(1)}$$

$R(\searrow)$ for $B$, using Newton's second law:

$$10a = 5g - 2\sqrt{3}g - T \qquad \textbf{(2)}$$

$5\times(\textbf{1}) = (\textbf{2}) \Rightarrow$

$$5T - 5g - \sqrt{3}g = 5g - 2\sqrt{3}mg - T$$

$$6T = 10g - \sqrt{3}g$$

$$T = \frac{5}{3}g - \frac{\sqrt{3}}{6}g$$

Substituting this value into (**1**):

$$2a = \left(\frac{5}{3}g - \frac{\sqrt{3}}{6}g\right) - g - \frac{\sqrt{3}}{5}g$$

$$2a = \frac{2}{3}g - \frac{11\sqrt{3}}{30}g$$

$$a = \left(\frac{1}{3} - \frac{11\sqrt{3}}{60}\right)g = 0.15474...$$

The acceleration is $0.155\text{ ms}^{-2}$ (3s.f.).

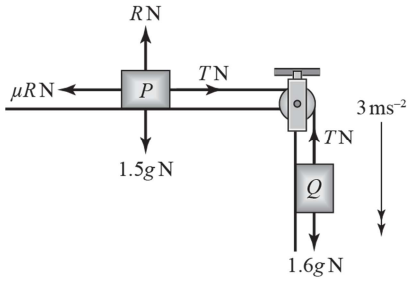
**6 a** $u = 0\text{ ms}^{-1}$, $v = 6\text{ ms}^{-1}$, $t = 2\text{ s}$, $a = ?$

$$v = u + at$$

$$6 = 0 + 2a$$

$$a = \frac{6}{3} = 3$$

The acceleration is $3\text{ ms}^{-2}$




**b** Considering the box, $Q$, and using Newton's second law:

$$F = ma$$

$$1.6g - T = 1.6\times 3$$

$$T = 1.6g - 1.6\times 3$$

$$T = 1.6\times(9.8 - 3)$$

$$T = 10.88$$

The tension in the string is 10.88 N.

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**6 c** For P:

$R(\uparrow): R = 1.5g$

$R(\rightarrow)$:

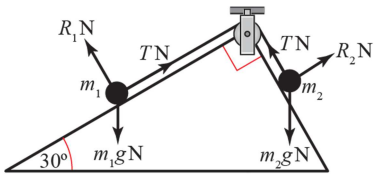
$$F = ma$$
$$T - \mu R = ma$$
$$10.88 - 1.5\mu g = 1.5 \times 3$$
$$1.5\mu g = 10.88 - 4.5$$
$$\mu = \frac{6.38}{1.5 \times 9.8} = 0.43401...$$

To 3 s.f. the coefficient of friction is 0.434, as required.

**6 d** The tension in the two parts of the string can be assumed to be the same because the string is inextensible.

**Challenge**

**a** With wedge smooth, let the reaction forces on the blocks be $R_1$ and $R_2$ respectively.
Resolving parallel to the slope on each side:




$T = m_1 g \sin 30^\circ$

$T = m_2 g \cos 30^\circ$

Since the string is inextensible, both values of T are the same, so:

$m_1 g \sin 30^\circ = m_2 g \cos 30^\circ$

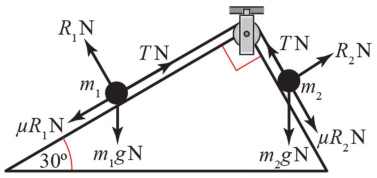
$$\frac{m_1}{m_2} = \frac{\cos 30^\circ}{\sin 30^\circ} = \frac{1}{\tan 30^\circ}$$

$\frac{m_1}{m_2} = \sqrt{3}$ as required.

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**Challenge**

**b** Resolving perpendicular to the slope on each side:




$R_1 = m_1 g\cos 30^\circ$

$R_2 = m_2 g\sin 30^\circ$

Case 1: $m_1$ is about to move down

Resolving parallel to each slope to find tension in string if $m_1$ is about to move down:

$T = m_1 g\sin 30^\circ - \mu R_1 = m_1 g\sin 30^\circ - \mu m_1 g\cos 30^\circ$

$T = m_2 g\cos 30^\circ + \mu R_2 = m_2 g\cos 30^\circ + \mu m_2 g\sin 30^\circ$

Since the string is inextensible, both values of T are the same, so:

$m_1 g\left(\sin 30^\circ - \mu\cos 30^\circ\right) = m_2 g\left(\cos 30^\circ + \mu\sin 30^\circ\right)$

$$\frac{m_1}{m_2} = \frac{\cos 30^\circ + \mu\sin 30^\circ}{\sin 30^\circ - \mu\cos 30^\circ}$$

$$\frac{m_1}{m_2} = \frac{\sqrt{3} + \mu}{1 - \mu\sqrt{3}}$$

Case 1: $m_2$ is about to move down

Resolving parallel to each slope to find tension in string if $m_2$ is about to move down:

$T = m_1 g\sin 30^\circ + \mu R_1 = m_1 g\sin 30^\circ + \mu m_1 g\cos 30^\circ$

$T = m_2 g\cos 30^\circ - \mu R_2 = m_2 g\cos 30^\circ - \mu m_2 g\sin 30^\circ$

Since the string is inextensible, both values of T are the same, so:

$m_1 g\left(\sin 30^\circ + \mu\cos 30^\circ\right) = m_2 g\left(\cos 30^\circ - \mu\sin 30^\circ\right)$

$$\frac{m_1}{m_2} = \frac{\cos 30^\circ - \mu\sin 30^\circ}{\sin 30^\circ + \mu\cos 30^\circ}$$

$$\frac{m_1}{m_2} = \frac{\sqrt{3} - \mu}{1 + \mu\sqrt{3}}$$

$\frac{m_1}{m_2}$ must lie between these two values, since they are the values of limiting equilibrium.

So:

$\frac{\sqrt{3} - \mu}{1 + \mu\sqrt{3}} \le \frac{m_1}{m_2} \le \frac{\sqrt{3} + \mu}{1 - \mu\sqrt{3}}$ as required.

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