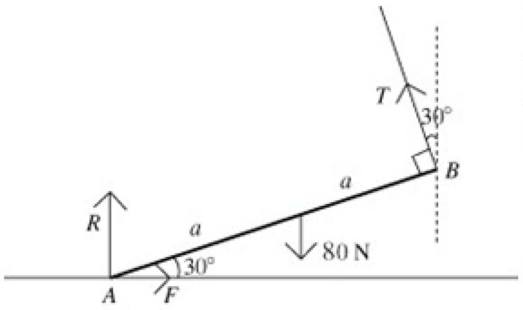
Applications of forces 7D

1 Suppose that the rod has length $2a$.



Taking moments about A:

$$2aT = 80 \times a\cos 30°$$

$$2T = 80 \times \frac{\sqrt{3}}{2}$$

$$T = 20\sqrt{3}$$

$$= 34.6 \text{ N}$$

$$R(\rightarrow), \quad F = T\sin 30° = 10\sqrt{3} = 17.3 \text{ N}$$

$$R(\uparrow), \quad T\cos 30° + R = 80$$

$$R = 80 - 20\sqrt{3} \times \frac{\sqrt{3}}{2}$$

$$= 50 \text{ N}$$

In order for the rod to remain in equilibrium, we must have $F \leq \mu R$:

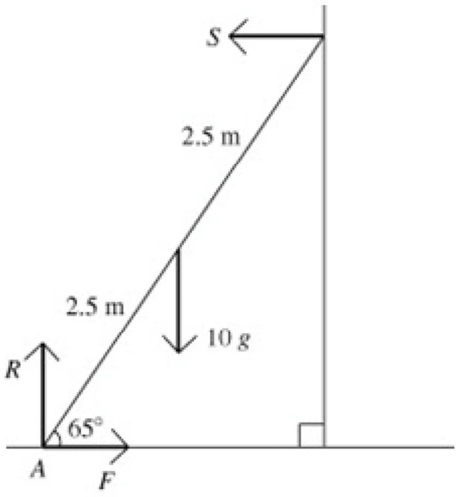
$$10\sqrt{3} \leqslant \mu \times 50$$

$$\mu \geqslant \frac{10\sqrt{3}}{50}$$

$$\mu \geqslant \frac{\sqrt{3}}{5}$$

$\therefore$ minimum $\mu = 0.35$ (2 s.f.)

2 Let A be the end of the ladder on the ground.
Let F be the frictional force at A.



a Taking moments about A:

$$10g \times 2.5\cos 65° = S \times 5\sin 65°$$

$$S = \frac{25g\cos 65°}{5\sin 65°}$$

$$= \frac{5g}{\tan 65°}$$

$$= 22.8 \text{ N}$$

b $R(\rightarrow), \quad F = S = 22.8 \text{ N}$

$R(\uparrow), \quad R = 10g = 98 \text{ N}$

c To ensure ladder remains in equilibrium, we must have

$$F \leqslant \mu R$$

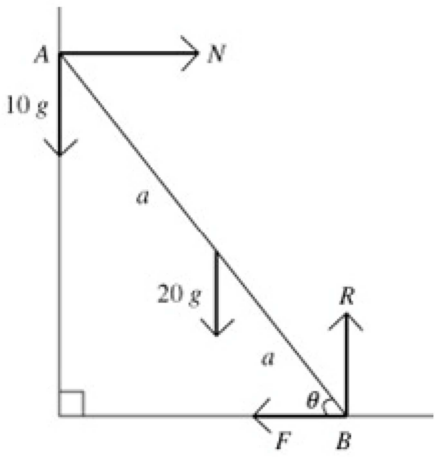
$$22.8 \leqslant \mu \times 98$$

$$\mu \geqslant 0.233 \text{ (3 s.f.)}$$

d The weight is shown as acting through the midpoint of the ladder because of the assumption that the ladder is uniform.

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3 Let the ladder have length $2a$, and be inclined at θ to the horizontal.



a $R(\uparrow), \quad R = 30g$

Taking moments about A:

$20g \times a\cos\theta + F \times 2a\sin\theta = R \times 2a\cos\theta$

$20g\cos\theta + 2F\sin\theta = 60g\cos\theta \quad$ (using $R = 30g$)

$2F\sin\theta = 40g\cos\theta$

$$F = \frac{20g}{\tan\theta}$$

The ladder is on the point of slipping, so $F = \mu R$

$$\frac{20g}{\tan\theta} = \frac{3}{4} \times 30g$$

$$\therefore \tan\theta = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

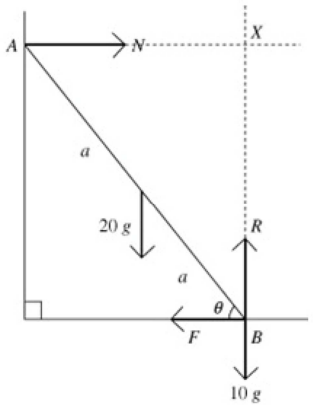
$$\therefore \theta = 41.6°$$

N is the normal reaction at A,
R is the normal reaction at B,
F is the frictional force at B.

b $R(\uparrow), \quad R = 30g$

$R(\rightarrow), \quad N - F = 0$

$N = F$



Taking moments about B:

$20g \times a\cos\theta = N \times 2a\sin\theta$

$20g \times a\cos\theta = F \times 2a\sin\theta$

$$F = \frac{10g\cos\theta}{\sin\theta}$$

$$F = \frac{10g}{\tan\theta}$$

The ladder is on the point of slipping, so $F = \mu R$

$$\frac{10g}{\tan\theta} = \frac{3}{4} \times 30g$$

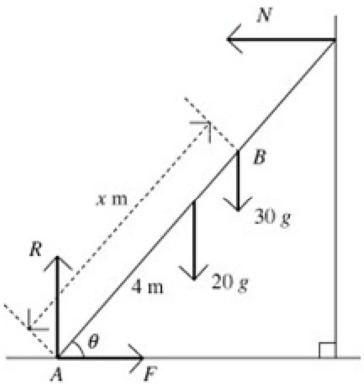
$$\tan\theta = \frac{4}{9}$$

$$\theta = 24.0°$$

c The assumption that the wall is smooth means there is no friction between the ladder and the wall.

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4 a Suppose that the boy reaches the point B, a distance x from A, whilst the end of the ladder is still in contact with the ground.



$R(\rightarrow),\quad F = N$

$R(\uparrow),\quad R = 50g$

Taking moments about A:

$$20g \times 4\cos\theta + 30g \times x\cos\theta = N \times 8\sin\theta$$

$$80g + 30gx = 8N\tan\theta$$

$$N = \frac{80g + 30gx}{8\tan\theta}$$

$$N = \frac{80g + 30gx}{16} \quad (\text{since } \tan\theta = 2)$$

$$F = \frac{80g + 30gx}{16} \quad (\text{since } F = N)$$

$$\mu R = \frac{80g + 30gx}{16} \quad (\text{in limiting equilibrium})$$

$$0.3 \times 50g = \frac{80g + 30gx}{16}$$

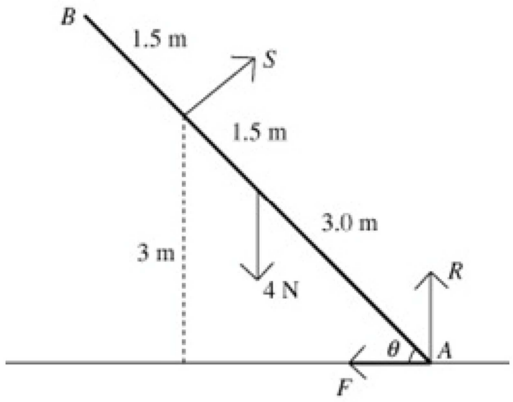
$$240 = 80 + 30x$$

$$x = 5\frac{1}{3} \text{ m}$$

b i The ladder may not be uniform.

ii There would be friction between the ladder and the wall.

5 Let:
S be the normal reaction of the rail on the pole at C,
R be the normal reaction of the ground on the pole at A,
F be the friction between the pole and the ground at A.
θ be the angle between the pole and the ground.



From the diagram,

$$\sin\theta = \frac{3}{4.5} = \frac{2}{3} \text{ and hence } \cos\theta = \frac{\sqrt{9-4}}{3} = \frac{\sqrt{5}}{3}$$

a Taking moments about A:

$$4.5S = 4 \times 3\cos\theta$$

$$= \frac{12\sqrt{5}}{3}$$

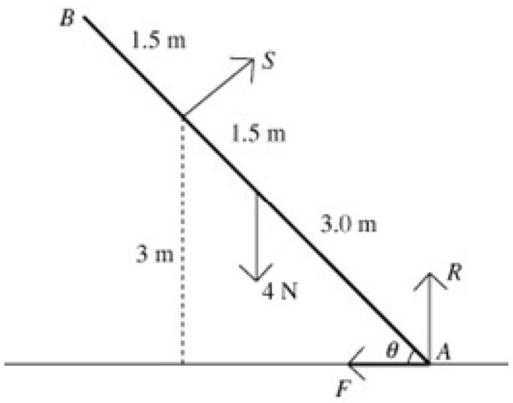
$$= 4\sqrt{5}$$

$$S = \frac{8\sqrt{5}}{9} \text{ N}$$

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5 b $R(\rightarrow)$

$$F = S\sin\theta$$
$$= \frac{8\sqrt{5}}{9} \times \frac{2}{3}$$
$$= \frac{16\sqrt{5}}{27}$$



$R(\uparrow)$

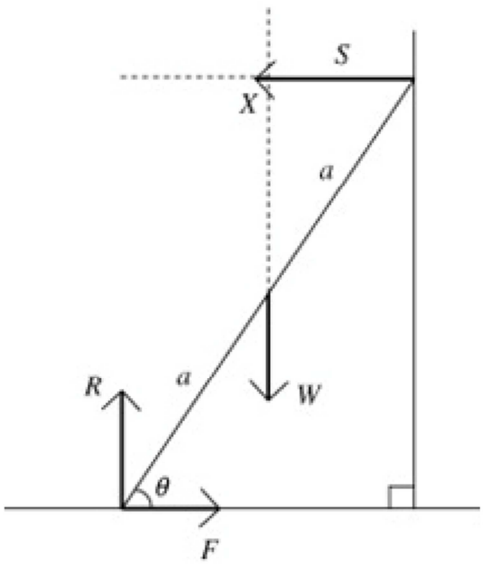
$$R + S\cos\theta = 4$$
$$R = 4 - \frac{8\sqrt{5}}{9} \times \frac{\sqrt{5}}{3}$$
$$= 4 - \frac{40}{27}$$
$$= \frac{68}{27}$$

Pole is in limiting equilibrium, so $F = \mu R$

$$\frac{16\sqrt{5}}{27} = \mu \times \frac{68}{27}$$
$$\therefore \mu = \frac{16\sqrt{5}}{68}$$
$$= \frac{4\sqrt{5}}{17}$$
$$= 0.526 \text{ (3 s.f.)}$$

5 c The assumption that the rail is smooth means there is no friction between the rail and the pole.

6 Suppose that the ladder has length $2a$ and weight W.
Let:
S be the normal reaction of the wall on the ladder,
R be the normal reaction of the floor on the ladder,
F be the friction between the floor and the ladder.
X be the point where the lines of action of W and S meet.



Taking moments about X:

$$2a\sin\theta \times F = R \times a\cos\theta$$
$$2F\sin\theta = R\cos\theta \qquad \textbf{(1)}$$

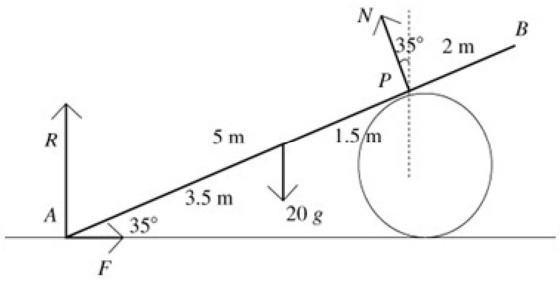
The ladder is in limiting equilibrium, so $F = \mu R$

Substituting $F = \mu R$ in **(1)**:

$$2\mu R\sin\theta = R\cos\theta$$
$$2\mu\sin\theta = \cos\theta$$
$$\frac{2\mu\sin\theta}{\cos\theta} = 1$$
$$2\mu\tan\theta = 1$$

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7 Let:
N be the normal reaction of the drum on the ladder at P,
R be the normal reaction of the ground on the ladder at A,
F be the friction between the ground and the ladder at A.



Taking moments about A:
$20g \times 3.5\cos 35° = 5N$

$$N = \frac{20g \times 3.5\cos 35°}{5}$$
$$= 14g\cos 35°$$

$R(\uparrow)$
$N\cos 35° + R = 20g$
$$R = 20g - 14g\cos 35° \times \cos 35°$$
$$= 103.9...\text{ N}$$

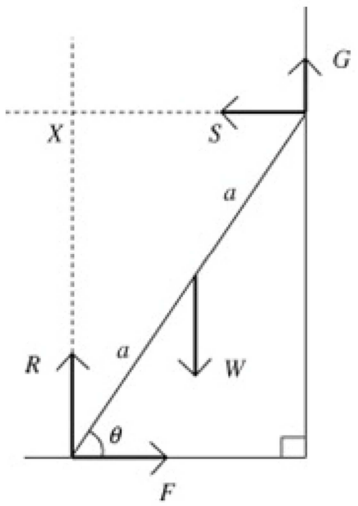
$R(\rightarrow)$
$F = N\sin 35°$
$= 14g\cos 35° \times \sin 35°$
$= 64.46...\text{ N}$

$F \le \mu R$ to maintain equilibrium:
$14g\cos 35°\sin 35° \le \mu(20g - 14g\cos^2 35°)$

$$\mu \ge \frac{14\cos 35°\sin 35°}{20 - 14\cos^2 35°}$$
$$\mu \ge 0.620 \text{ (3 s.f.)}$$

Least possible μ is 0.620 (3 s.f.)

8 Let:
R be the reaction of the ground on the ladder
F be the friction between the ground and the ladder
S be the reaction of the wall on the ladder
G be the friction between the wall and the ladder.
X be the point where the lines of action R and S meet.



Suppose that the ladder has length $2a$ and weight W.

As the ladder rests in limiting equilibrium, $F = \mu_1 R$ and $G = \mu_2 S$.

Taking moments about X:
$W \times a\cos\theta = F \times 2a\sin\theta + G \times 2a\cos\theta$

$$W = 2F\tan\theta + 2G \qquad \textbf{(1)}$$

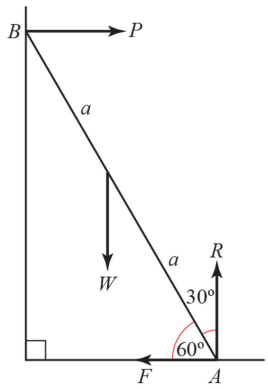
$R(\rightarrow)$, $F = S$
$R(\uparrow)$, $W = R + G$

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8 Substituting for W and F in equation (**1**):

$$R + G = 2\mu_1 R\tan\theta + 2G$$
$$R - G = 2\mu_1 R\tan\theta$$
$$R - \mu_1\mu_2 R = 2\mu_1 R\tan\theta \qquad \text{(Since } G = \mu_2 S = \mu_2 F = \mu_2\mu_1 R\text{)}$$
$$\text{Hence } \frac{1-\mu_1\mu_2}{2\mu_1} = \tan\theta$$

9 Let:
A and B be the ends of the ladder.
P be the normal reaction of the wall on the ladder at B,
R the normal reaction of the ground on the ladder at A
F be the friction at between the ladder and the ground at A



Let the length of the ladder be $2a$.

a Taking moments about A:
$W \times a\cos 60° = P \times 2a\cos 30°$

$$P = \frac{Wa\cos 60°}{2a\cos 30°}$$
$$P = \frac{W \times \frac{1}{2}}{2 \times \frac{\sqrt{3}}{2}}$$
$$P = \frac{W}{2\sqrt{3}} \qquad (\mathbf{1})$$

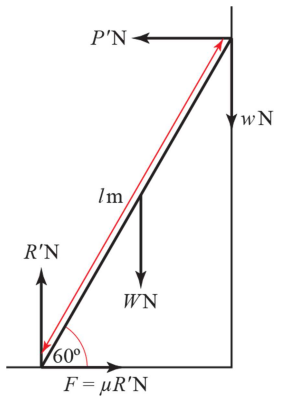
b $R(\uparrow), \quad R = W \qquad (\mathbf{2})$
$R(\rightarrow), \quad F = P \qquad (\mathbf{3})$

Now $F \le \mu R$ since the ladder is in equilibrium (if not, ladder would slide)
Hence, $P \leqslant \mu R$ (by (**3**))

$$\frac{W}{2\sqrt{3}} \leqslant \mu R \quad \text{(by (1))}$$
$$\frac{W}{2\sqrt{3}} \leqslant \mu W \quad \text{(by (2))}$$
$$\mu \geqslant \frac{\sqrt{3}}{6}$$

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9 c Let:
R' be the normal reaction of the ground on the ladder at A
P' be the normal reaction of the wall on the ladder at B,
l be the length of the ladder



Since the ladder is in limiting equilibrium, $F' = \mu R'$

$R(\uparrow)$, $R' = W + w$

R($\rightarrow$), $\mu R' = P'$

Taking moments about B:

$$\frac{Wl\cos 60^\circ}{2} + \left(F' \times l \sin 60^\circ\right) = \left(R' \times l \cos 60^\circ\right)$$

$$\frac{W}{4} + \left(\mu R' \times \frac{\sqrt{3}}{2}\right) = \frac{R'}{2}$$

$$\frac{W}{4} + \left(\frac{\sqrt{3}}{5}(W+w) \times \frac{\sqrt{3}}{2}\right) = \frac{W+w}{2} \qquad \text{(since } R' = W + w \text{ and } \mu = \frac{\sqrt{3}}{5}\text{)}$$

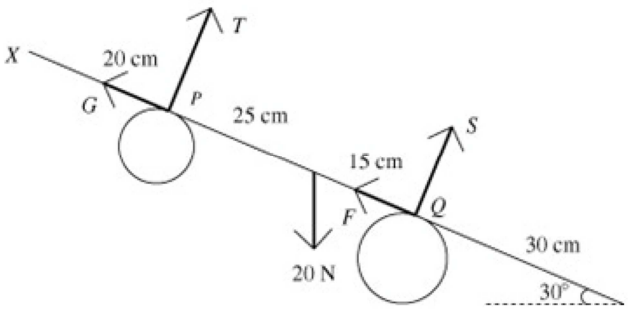
$$W + \frac{6}{5}(W+w) = 2(W+w)$$

$$5W + 6W + 6w = 10W + 10w$$

$$W = 4w$$

$$\Rightarrow w = \frac{W}{4}$$

10 Let:
T be the normal force of the peg on the rod at P,
G be the frictional force at P,
S be the normal force of the peg on the rod at Q,
F be the frictional force at Q.



a Taking moments about P:
$S \times 40 = 20 \times 25 \times \cos 30°$

$$S = \frac{20 \times 25 \times \frac{\sqrt{3}}{2}}{40}$$

$$S = \frac{25\sqrt{3}}{4}\text{N}$$

Taking moments about Q:
$T \times 40 = 20 \times 15 \times \cos 30°$

$$T = \frac{20 \times 15 \times \frac{\sqrt{3}}{2}}{40}$$

$$T = \frac{15\sqrt{3}}{4}\text{N}$$

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10 b $R(\searrow)$

$G + F = 20\cos 60° = 10$ **(1)**

Since the rod is about to slip, friction is limiting and hence $G = \mu T$, $F = \mu S$.

From part **a,**

$G + F = \mu T + \mu S = \mu \times \dfrac{40\sqrt{3}}{4} = 10\sqrt{3}\mu$ **(2)**

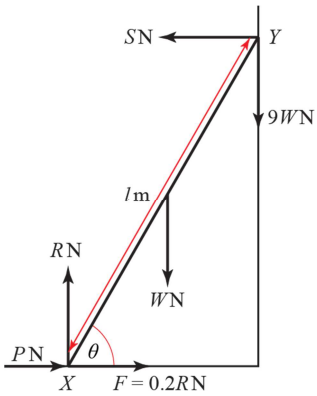
$(1) = (2) \Rightarrow \mu = \dfrac{1}{\sqrt{3}}$

11 a Let:

S be the normal reaction of the wall on the ladder at Y,

R be the normal reaction of the ground on the ladder at X

F be the friction at between the ladder and the ground at X



$\tan\theta = \sqrt{3}$ so $\sin\theta = \dfrac{\sqrt{3}}{3}$ and $\cos\theta = \dfrac{1}{2}$

Ladder is in equilibrium.

Taking moments about X:

$\dfrac{Wl\cos\theta}{2} + 9Wl\cos\theta = Sl\sin\theta$

$\dfrac{W}{4} + \dfrac{9W}{2} = \dfrac{\sqrt{3}S}{2}$

$\sqrt{3}S = \dfrac{W}{2} + 9W$

$\sqrt{3}S = \dfrac{19W}{2}$

$S = \dfrac{19W}{2\sqrt{3}}$

b R(↑): $R = W + 9W = 10W$

For the ladder to be in limiting equilibrium,

$F = \mu R$

$F = \dfrac{1}{5} \times 10W$

$F = 2W$

R(→):

If $P + F > S$, ladder will slide towards and up the wall

If $P < S - F$, ladder will slide away from and down the wall

Therefore $S - F \leq P \leq S + F$

Substituting values for S & F from part **a** and above:

$\dfrac{19W}{2\sqrt{3}} - 2W \leq P \leq \dfrac{19W}{2\sqrt{3}} + 2W$

$\left(\dfrac{19}{2\sqrt{3}} - 2\right)W \leq P \leq \left(\dfrac{19}{2\sqrt{3}} + 2\right)W$

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11 c Modelling the ladder as uniform allows us to assume the weight acts through the midpoint.

d i The reaction of the wall on the ladder will decrease. To understand why, consider how we took moments about X in part **a**

$$\frac{Wl\cos\theta}{2} + 9Wl\cos\theta = Sl\sin\theta$$

The first term in this equation is the turning moment of the weight of the ladder, which acts at a distance $\frac{l}{2}$ from X. If the centre of mass of the ladder is more towards X, say $\frac{l}{a}$ where $a > 2$, then this first term would decrease and hence S would also decrease.

ii Ladder remains in equilibrium when $S - F \leq P \leq S + F$
If S were to decrease, then this range of values for P would also decrease.

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