Hypothesis testing 7D

1 Using method 1

Distribution, B(30, 0.50)

$H_0 : p = 0.50$ $H_1 : p \neq 0.50$

$P(X \leq 10) = 0.0494 > 0.025$ (two-tailed)

There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.5$

2 Using method 2

H_0: $p = 0.3$ H_1: $p \neq 0.3$

If H_0 is true $X \sim B(25, 0.3)$

Let c_1 and c_2 be the two critical values so $P(X \leq c_1) \leq 0.05$ and $P(X \geq c_2) \leq 0.05$ (two-tailed)

For the lower tail:

$P(X \leq 3) = 0.0332 < 0.05$
$P(X \leq 4) = 0.0905 > 0.05$

So $c_1 = 3$

For the upper tail:

$P(X \geq 11) = 1 - P(X \leq 10) = 1 - 0.9022 = 0.0978 > 0.05$
$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9558 = 0.0442 < 0.05$

So $c_2 = 12$

The observed value of 10 does not lie in the critical region so H_0 is not rejected. Therefore, there is no reason to doubt $p = 0.3$.

3 Using method 2

H_0: $p = 0.75$ H_1: $p \neq 0.75$

If H_0 is true $X \sim B(10, 0.75)$

Let c_1 and c_2 be the two critical values so $P(X \leq c_1) \leq 0.025$ and $P(X \geq c_2) \leq 0.025$ (two-tailed)

For the lower tail:

$P(X \leq 4) = 0.0197 < 0.025$
$P(X \leq 5) = 0.0781 > 0.025$

So $c_1 = 4$

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

3 For the upper tail:

$P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.9437 = 0.0563 > 0.025$

So there is no upper tail.

The observed value of 9 does not lie in the critical region so H_0 is not rejected. Therefore, there is no reason to doubt p = 0.75.

4 Using method 2

$H_0: p = 0.6$ $H_1: p \neq 0.6$

If H_0 is true $X \sim B(20, 0.6)$

Let c_1 and c_2 be the two critical values so $P(X \leq c_1) \leq 0.005$ and $P(X \geq c_2) \leq 0.005$ (two-tailed)

For the lower tail:

$P(X \leq 5) = 0.001612 < 0.005$
$P(X \leq 6) = 0.006466 > 0.005$

So $c_1 = 5$

For the upper tail:

$P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.984 = 0.016 > 0.005$
$P(X \geq 18) = 1 - P(X \leq 17) = 1 - 0.9964 = 0.0036 < 0.005$

So $c_2 = 18$

The observed value of 1 lies in the critical region so H_0 is rejected. Therefore, there is reason to doubt $p = 0.6$.

5 Using method 2

$H_0: p = 0.02$ $H_1: p \neq 0.02$

If H_0 is true $X \sim B(50, 0.02)$

Let c_1 and c_2 be the two critical values so $P(X \leq c_1) \leq 0.01$ and $P(X \geq c_2) \leq 0.01$ (two-tailed)

For the lower tail:

$P(X \leq 0) = 0.3642 > 0.01$

So there is no lower tail.

For the upper tail:

$P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.9822 = 0.0178 > 0.01$
$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9968 = 0.0033 < 0.01$

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

5 So $c_2 = 5$

The observed value of 4 does not lie in the critical region so H_0 is not rejected. Therefore, there is no reason to doubt $p = 0.02$.

6 Using method 1

Distribution, B(20, 0.50)

$H_0 : p = 0.50 \qquad H_1 : p \neq 0.50$

$P(X \leq 6) = 0.0577 > 0.025$ (two-tailed)

There is insufficient evidence to reject H_0 so we conclude there is no evidence the coin is biased.

7 $X \sim B(20, 0.20)$

a Using method 2

$H_0 : p = 0.20 \qquad H_1 : p \neq 0.20$

$P(X \leq 1) = 0.0692$
$P(X \leq 0) = 0.0115$ (closer to 0.025)

critical value $= 0$

$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9900 = 0.0100$
$P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9679 = 0.0321$ (closer to 0.025)

Critical region $X = 0$ and $X \geq 8$

b Actual significance level is 0.0115 + 0.0321 = 0.0436 = 4.36%

c $X = 8$ is in the critical region. There is enough evidence to reject H_0. The hospital's proportion of complications differs from the national figure.

8 Using method 2

The test statistic is the number of cracked bowls.

H_0: $p = 0.1 \qquad H_1$: $p \neq 0.1$

If H_0 is true $X \sim B(20, 0.1)$

Let c_1 and c_2 be the two critical values so $P(X \leq c_1) \leq 0.05$ and $P(X \geq c_2) \leq 0.05$ (two-tailed)

For the lower tail:

$P(X \leq 0) = 0.1216 > 0.05$

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

8 So there is no lower tail.

For the upper tail:

$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.8670 = 0.133 > 0.05$
$P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.9568 = 0.0432 < 0.05$

So $c_2 = 6$

The observed value of 1 does not lie in the critical region so H_0 is not rejected. Therefore, there is no reason to doubt $p = 0.1$. So the proportion of cracked bowls has not changed.

9 Using method 2

The test statistic is the number of carrots longer than 7 cm.

$H_0: p = 0.25$ $H_1: p \neq 0.25$

If H_0 is true $X \sim B(30, 0.25)$

Let c_1 and c_2 be the two critical values so $P(X \leq c_1) \leq 0.025$ and $P(X \geq c_2) \leq 0.025$ (two-tailed)

For the lower tail:

$P(X \leq 2) = 0.0106 < 0.025$
$P(X \leq 3) = 0.0374 > 0.025$

So $c_1 = 2$

For the upper tail:

$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9493 = 0.0507 > 0.025$
$P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0.9784 = 0.0216 < 0.025$

So $c_2 = 13$

The observed value of 13 lies in the critical region so H_0 is rejected. Therefore, there is reason to doubt $p = 0.25$. So the probability of a carrot being longer than 7 cm has increased.

10 Using method 2

The test statistic is the number of patients correctly diagnosed.

$H_0: p = 0.96$ $H_1: p \neq 0.96$

If H_0 is true $X \sim B(75, 0.96)$

Let c_1 and c_2 be the two critical values so $P(X \leq c_1) \leq 0.05$ and $P(X \geq c_2) \leq 0.05$

For the lower tail:

$P(X \leq 68) = 0.03046 < 0.05$
$P(X \leq 69) = 0.07978 > 0.05$

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

10 So $c_1 = 68$

For the upper tail:

$P(X \geq 74) = 1 - P(X \leq 73) = 1 - 0.8069 = 0.1931 > 0.05$
$P(X \geq 75) = 1 - P(X \leq 74) = 1 - 0.9532 = 0.0468 < 0.05$

So $c_2 = 75$

The observed value of 63 lies in the critical region so H_0 is rejected. Therefore, there is reason to doubt $p = 0.96$. So the new test does not have the same probability of success as the old test.

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.