Exam-style practice: Paper 1

1 $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{2\sec^2 t}{2\sin t\cos t} = \dfrac{1}{\sin t\cos^3 t} = \operatorname{cosec} t\sec^3 t$

2 a $2(7x-5)-6x<10x-7$

$\Rightarrow 14x - 10 - 6x < 10x - 7$

$\Rightarrow -3 < 2x$

So $x > -\dfrac{3}{2}$

b $|2x+5|-3>0$

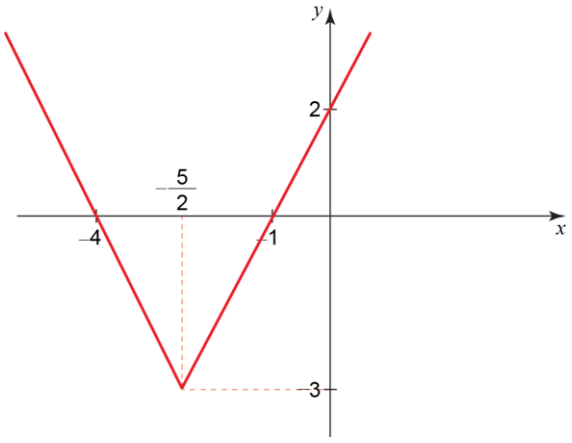
Solve $|2x+5|-3=0$

$x < -\dfrac{5}{2}$: $-(2x+5)-3=0$

$\Rightarrow 2x = -8 \Rightarrow x = -4$

$x > -\dfrac{5}{2}$: $(2x+5)-3=0$

$\Rightarrow 2x = -2 \Rightarrow x = -1$



From the graph, we see that the inequality holds when $x < -4$ or $x > -1$

c For both inequalities to hold, x must satisfy both $x > -\frac{3}{2}$ and $x < -4$ or $x > -1$ so the solution is $x > -1$

3 a $2x + y - 3 = 0 \Rightarrow y = 3 - 2x$

Substitute this equation for y in the equation of the circle

$x^2 + kx + (3-2x)^2 + 4(3-2x) = 4$

$5x^2 + kx - 20x + 17 = 0$

If this equation has solutions, the line will intersect the circle. As the equation is a positive quadratic, there will be no solutions, and the line will not intersect the circle, if

$5x^2 + kx - 20x + 17 > 0$

b As there are no solutions to the equation

$5x^2 + (k-20)x + 17 = 0$

the discriminant must be less than zero

$\Rightarrow (k-20)^2 - 4(5)(17) < 0$

$\Rightarrow k^2 - 40k + 400 - 340 < 0$

$\Rightarrow k^2 - 40k + 60 < 0$

Solve $k^2 - 40k + 60 = 0$

Using the quadratic formula,

$$k = \frac{40 \pm \sqrt{(-40)^2 - 4(1)(60)}}{2(1)} = 20 \pm 2\sqrt{85}$$

Since $f(k) = k^2 - 40k + 60$ is a positive quadratic in k, the set of values of k for which $f(k)$ is negative must be

$20 - 2\sqrt{85} < k < 20 + 2\sqrt{85}$

4 Let $f(x) = \cos x$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h\to 0}\frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$

$$= \lim_{h\to 0}\left[\left(\frac{\cos h - 1}{h}\right)\cos x - \left(\frac{\sin h}{h}\right)\sin x\right]$$

$$= -\sin x$$

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5 a Using the binomial expansion, the coefficient of x^2 in the expansion of $(3+px)^6$ is

$\frac{6(6-1)}{2!}(3)^4 p^2 = 1215p^2$

As $1215p^2 = 19440 \Rightarrow p^2 = 16$

So solutions are $p = 4$, $p = -4$

b The coefficient of x^5 is

$\frac{6(6-1)(6-2)(6-3)(6-4)}{5!}3p^5 = 1215p^2$

Since this negative, use $p = -4$, so the coefficient is

$6 \times 3 \times (-4)^5 = -18432$

6 First find the y-coordinate of R

$y = (2)^2 + 4(2) - 2 = 10$ so $R(2,10)$

To find the normal line to the curve at R, find the gradient at R

$\frac{dy}{dx} = 2x + 4$, so at $x = 2$, $\frac{dy}{dx} = 8$

At R, the normal line will therefore have a gradient of $-\frac{1}{8}$

So the equation of the normal line at R is

$(y - 10) = -\frac{1}{8}(x - 2) \Rightarrow y = -\frac{x}{8} + \frac{41}{4}$

To find T, solve

$-\frac{x}{8} + \frac{41}{4} = x^2 + 4x - 2$

$\Rightarrow 8x^2 + 33x - 98 = 0$

In factorising this equation, remember that $x = 2$ is a solution

So $(x - 2)(8x + 49) = 0$

The normal also meets the curve at $x = -\frac{49}{8}$

And when $y = -\frac{-\frac{49}{8}}{8} + \frac{41}{4} = \frac{705}{64}$

Required coordinates are $\left(-\frac{49}{8}, \frac{705}{64}\right)$

7 a $u_1 = a, u_2 = ar = 96, S_\infty = 600$

$a = \frac{96}{r}, S_\infty = \frac{\frac{96}{r}}{1-r} = 600$

$\Rightarrow 600 - 600r = \frac{96}{r}$

$\Rightarrow 600r^2 - 600r + 96 = 0$

$\Rightarrow 25r^2 - 25r + 4 = 0$ (dividing by 24)

b $25r^2 - 25r + 4 = 0$

Factorise $(5r - 1)(5r - 4) = 0$

Or use the quadratic formula

$r = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(4)(25)}}{2(25)}$

$= \frac{25 \pm 15}{50}$

Solutions are $r = \frac{1}{5} = 0.2$, $r = \frac{4}{5} = 0.8$

c The larger value of r is $r = 0.8$. The corresponding value of a is

$a = \frac{96}{0.8} = 120$

d $S_n = \frac{a(1-r^n)}{1-r} > 599.9$

$\Rightarrow \frac{120\left(1-(0.8)^n\right)}{1-0.8} > 599.9$

$\Rightarrow 1 - (0.8)^n > \frac{599.9}{600}$

$\Rightarrow (0.8)^n < \frac{0.1}{600}$

$\Rightarrow n \ln 0.8 < \ln \frac{0.1}{600}$

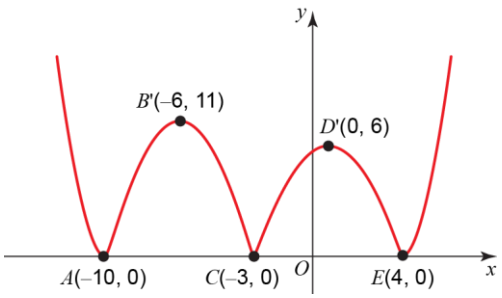
$\Rightarrow n > \frac{\ln \frac{0.1}{600}}{\ln 0.8}$ (as ln0.8 is negative)

$\Rightarrow n > 38.986$ (3 d.p.)

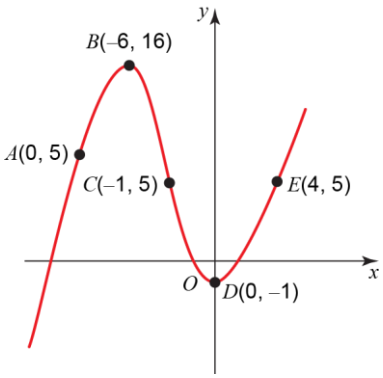
So $n = 39$

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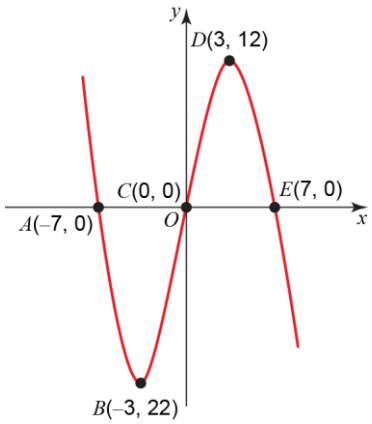
8 a Reflect graph of f(x) the x-axis in regions where f(x) < 0, i.e. $-10 < x < -3$ and $x > 4$



b First reflect the graph in the x-axis to obtain $y = -f(x)$ then translate this graph by the vector $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$



c Translate by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ to obtain $y = f(x - 3)$ and then stretch in the y-direction by a scale factor of 2



9 $31 - 25\cos x = 19 - 12\sin^2 x$

$\Rightarrow 31 - 25\cos x = 19 - 12(1 - \cos^2 x)$

$\Rightarrow 12\cos^2 x + 25\cos x - 24 = 0$

$$\Rightarrow \cos x = \frac{-25 \pm \sqrt{(25)^2 - 4(12)(-24)}}{2(12)}$$

$$= 0.7148, -2.7981 \text{ (4 d.p.)}$$

$\cos x = -2.798...$ has no solutions,

since $|\cos x| \leq 1$

So there are two solutions in the required interval

$\cos^{-1}(0.7148) = 0.77$ (2 d.p.)

and $2\pi - \cos^{-1}(0.7148) = 5.51$ (2 d.p.)

10 a Let the constant of proportionality be $-k$, where $k > 0$. Therefore

$$\frac{dV}{dt} = -kV \Rightarrow V = Ce^{-kt}$$

where C is a constant

$V = V_0$ at $t = 0$ $\Rightarrow$ $C = V_0$

So $V = V_0 e^{-kt}$

b $25000 = V_0 e^{-2k}$ (1)

$15000 = V_0 e^{-5k}$ (2)

Dividing equations (1) and (2)

$\frac{5}{3} = e^{3k}$ $\Rightarrow$ $k = \frac{1}{3}\ln\left(\frac{5}{3}\right)$

Substituting this value of k into (1)

$V_0 = 25000e^{\frac{2}{3}\ln\frac{5}{3}} = 35143.0$ (6 s.f.)

So $V_0 = 35\,100$ to the nearest hundred

c $V_0 e^{-kt} = 5000$

$$\Rightarrow e^{kt} = \frac{V_0}{5000}$$

$$\Rightarrow kt = \ln\left(\frac{V_0}{5000}\right)$$

$$\Rightarrow t = 3\frac{\ln\left(\frac{35143}{5000}\right)}{\ln\left(\frac{5}{3}\right)} = 11.45 \text{ years (2 d.p.)}$$

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11 a Apply the cosine rule to the triangles ABC and ACD to find $\angle BCA$ and $\angle ACD$

$$\cos(\angle BCA) = \frac{21^2 + 19^2 - 8^2}{2(21)(19)}$$
$$= 0.9248 \text{ (4 d.p.)}$$
$$\therefore \angle BCA = 0.3903 \text{ rad (4 d.p.)}$$
$$\cos(\angle ACD) = \frac{14^2 + 21^2 - 11^2}{2(14)(21)}$$
$$= 0.8776 \text{ (4 d.p.)}$$
$$\therefore \angle ACD = 0.5001 \text{ rad (4 d.p.)}$$
So $\angle BCD = \angle BCA + \angle ACD = 0.890$ rad

Now apply the cosine rule to triangle BCD

$$\cos(\angle BCD) = \frac{14^2 + 19^2 - |BD|^2}{2(14)(19)}$$
$$|BD| = \sqrt{14^2 + 19^2 - 2(14)(19)\cos(\angle BCD)}$$
$$= \sqrt{196 + 361 - 334.847} = 14.9 \text{ (1 d.p.)}$$

b The shortest distance between two points is a straight line, so any other route will be longer.

12 a $y = -0.01x^2 + 0.22x + 1.58$
$$= -0.01(x^2 - 22x - 158)$$
$$= -0.01((x - 11)^2 - 279)$$
$$= 2.79 - 0.01(x - 11)^2$$

b The ball reaches its highest point when its horizontal distance from the goal is 11 metres. Its maximum height is 2.79 metres.

c The ball is kicked when $y = 0$

$$2.79 - 0.01(x - 11)^2 = 0$$
$$\therefore (x - 11)^2 = \frac{2.79}{0.01} = 279$$
$$\therefore x - 11 = \pm 16.703$$
$x > o$, so $x = 27.7$ m (1 d.p.)

d At $x = 0$, $y = 2.79 - 0.01(-11)^2 = 1.58$

As $1.5 < 1.58 < 2.44$, the ball will go not be saved by the keeper but it will go under the crossbar, so it will enter the goal

13 a Surface area of box
$$= 2x^2 + 2(2xh + xh) = 2x^2 + 6xh$$
Surface area of lid
$$= 2x^2 + 2(6x + 3x) = 2x^2 + 18x$$
Total surface area
$$= 4x^2 + 6xh + 18x = 5356$$
So $h = \frac{5356 - 18x - 4x^2}{6x}$

$$V = 2x^2h = \frac{2}{3}\left(2678x - 9x^2 - 2x^3\right)$$

b $V = \frac{2}{3}(2678x - 9x^2 - 2x^3)$

$$\therefore \frac{dV}{dx} = \frac{2}{3}(2678 - 18x - 6x^2)$$

$\frac{dV}{dx} = 0$ at a stationary point so

$$6x^2 + 18x - 2678 = 0$$

Since $x > 0$

$$x = \frac{-18 + \sqrt{(18)^2 - 4(6)(-2678)}}{2(6)}$$
$$= 19.68 \text{ cm (2 d.p.)}$$

c $\frac{d^2V}{dx^2} = \frac{2}{3}(-18 - 12x)$

Since $x > 0$, $\frac{d^2V}{dx^2} < 0 \therefore$ maximum

d $x = 19.68 \therefore V = 22648.7 \text{ cm}^3$ (1 d.p.)

e From part **a**, surface area of lid $= 2x^2 + 18x$
So percentage of cardboard in the lid is
$$\frac{2(19.68)^2 + 18(19.68)}{5356} \times 100 = 21.1\% \text{ (1 d.p.)}$$

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