Trigonometric Functions 6C

1 a $\frac{1}{\sin^3 q} = \left(\frac{1}{\sin q}\right)^3 = \text{cosec}^3 q$

b $\frac{4}{\tan^6 q} = 4 \times \left(\frac{1}{\tan q}\right)^6 = 4\cot^6 q$

c $\frac{1}{2\cos^2 q} = \frac{1}{2} \times \left(\frac{1}{\cos q}\right)^2 = \frac{1}{2}\sec^2 q$

d $\frac{1 - \sin^2 q}{\sin^2 q} = \frac{\cos^2 q}{\sin^2 q}$

(using $\sin^2 q + \cos^2 q \equiv 1$)

So $\frac{1 - \sin^2 q}{\sin^2 q} = \left(\frac{\cos q}{\sin q}\right)^2 = \cot^2 q$

e $\frac{\sec q}{\cos^4 q} = \frac{1}{\cos q} \times \frac{1}{\cos^4 q} = \frac{1}{\cos^5 q}$

$= \left(\frac{1}{\cos q}\right)^5 = \sec^5 q$

f $\sqrt{\text{cosec}^3 q \cot q \sec q}$

$= \sqrt{\frac{1}{\sin^3 q} \times \frac{\cos q}{\sin q} \times \frac{1}{\cos q}} = \sqrt{\frac{1}{\sin^4 q}}$

$= \frac{1}{\sin^2 q} = \left(\frac{1}{\sin q}\right)^2 = \text{cosec}^2 q$

g $\frac{2}{\sqrt{\tan q}} = 2 \times \frac{1}{(\tan q)^{\frac{1}{2}}} = 2\cot^{\frac{1}{2}} q$

h $\frac{\text{cosec}^2 q \tan^2 q}{\cos q} = \frac{1}{\sin^2 q} \times \frac{\sin^2 q}{\cos^2 q} \times \frac{1}{\cos q}$

$= \left(\frac{1}{\cos q}\right)^3 = \sec^3 q$

2 a $5\sin x = 4\cos x$

$\Rightarrow 5 = \frac{4\cos x}{\sin x}$ (divide by $\sin x$)

$\Rightarrow \frac{5}{4} = \cot x$ (divide by 4)

b $\tan x = -2$

$\Rightarrow \frac{1}{\tan x} = \frac{1}{-2}$

$\Rightarrow \cot x = -\frac{1}{2}$

c $3\frac{\sin x}{\cos x} = \frac{\cos x}{\sin x}$

$\Rightarrow 3\sin^2 x = \cos^2 x$

(multiply by $\sin x \cos x$)

$\Rightarrow 3 = \frac{\cos^2 x}{\sin^2 x}$

(divide by $\sin^2 x$)

$\Rightarrow \left(\frac{\cos x}{\sin x}\right)^2 = 3$

$\Rightarrow \cot^2 x = 3$

$\Rightarrow \cot x = \pm\sqrt{3}$

3 a $\sin q \cot q = \sin q \times \frac{\cos q}{\sin q} = \cos q$

b $\tan q \cot q = \tan q \times \frac{1}{\tan q} = 1$

c $\tan 2q \,\text{cosec}\, 2q = \frac{\sin 2q}{\cos 2q} \times \frac{1}{\sin 2q}$

$= \frac{1}{\cos 2q} = \sec 2q$

d $\cos q \sin q(\cot q + \tan q)$

$= \cos q \sin q\left(\frac{\cos q}{\sin q} + \frac{\sin q}{\cos q}\right)$

$= \cos^2 q + \sin^2 q = 1$

e $\sin^3 x \,\text{cosec}\, x + \cos^3 x \sec x$

$= \sin^3 x \times \frac{1}{\sin x} + \cos^3 x \times \frac{1}{\cos x}$

$= \sin^2 x + \cos^2 x = 1$

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

3 f $\sec A - \sec A \sin^2 A$

$= \sec A(1 - \sin^2 A)$ (factorise)

$= \frac{1}{\cos A} \times \cos^2 A$

(using $\sin^2 A + \cos^2 A \equiv 1$)

$= \cos A$

g $\sec^2 x \cos^5 x + \cot x \operatorname{cosec} x \sin^4 x$

$= \frac{1}{\cos^2 x} \times \cos^5 x + \frac{\cos x}{\sin x} \times \frac{1}{\sin x} \times \sin^4 x$

$= \cos^3 x + \sin^2 x \cos x$

$= \cos x(\cos^2 x + \sin^2 x)$

$= \cos x$ (since $\cos^2 x + \sin^2 x \equiv 1$)

4 a LHS $\equiv \cos\theta + \sin\theta \tan\theta$

$\equiv \cos\theta + \sin\theta \frac{\sin\theta}{\cos\theta}$

$\equiv \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$

$\equiv \frac{1}{\cos\theta}$ (using $\sin^2\theta + \cos^2\theta \equiv 1$)

$\equiv \sec\theta \equiv$ RHS

b LHS $\equiv \cot\theta + \tan\theta$

$\equiv \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$

$\equiv \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$

$\equiv \frac{1}{\sin\theta\cos\theta}$

$\equiv \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$

$\equiv \operatorname{cosec}\theta\sec\theta \equiv$ RHS

c LHS $\equiv \operatorname{cosec}\theta - \sin\theta$

$\equiv \frac{1}{\sin\theta} - \sin\theta$

$\equiv \frac{1 - \sin^2\theta}{\sin\theta}$

$\equiv \frac{\cos^2\theta}{\sin\theta}$

$\equiv \cos\theta \times \frac{\cos\theta}{\sin\theta}$

$\equiv \cos\theta\cot\theta \equiv$ RHS

d LHS $\equiv (1 - \cos x)(1 + \sec x)$

$\equiv 1 - \cos x + \sec x - \cos x \sec x$

(multiplying out)

$\equiv \sec x - \cos x$ (as $\cos x \sec x = 1$)

$\equiv \frac{1}{\cos x} - \cos x$

$\equiv \frac{1 - \cos^2 x}{\cos x}$

$\equiv \frac{\sin^2 x}{\cos x}$

$\equiv \sin x \times \frac{\sin x}{\cos x}$

$\equiv \sin x \tan x \equiv$ RHS

e LHS $\equiv \frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x}$

$\equiv \frac{\cos^2 x + (1 - \sin x)^2}{(1 - \sin x)\cos x}$

$\equiv \frac{\cos^2 x + (1 - 2\sin x + \sin^2 x)}{(1 - \sin x)\cos x}$

$\equiv \frac{2 - 2\sin x}{(1 - \sin x)\cos x}$

(using $\sin^2 x + \cos^2 x \equiv 1$)

$\equiv \frac{2(1 - \sin x)}{(1 - \sin x)\cos x}$

(factorising)

$= \frac{2}{\cos x}$

$\equiv 2\sec x \equiv$ RHS

f LHS $\equiv \frac{\cos\theta}{1 + \cot\theta}$

$\equiv \frac{\cos\theta}{1 + \frac{1}{\tan\theta}}$

$\equiv \frac{\cos\theta}{\frac{\tan\theta + 1}{\tan\theta}}$

$\equiv \frac{\cos\theta\tan\theta}{1 + \tan\theta}$

$\equiv \frac{\cos\theta \times \frac{\sin\theta}{\cos\theta}}{1 + \tan\theta}$

$\equiv \frac{\sin\theta}{1 + \tan\theta} \equiv$ RHS

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

5 a $\sec\theta = \sqrt{2}$

$\Rightarrow \frac{1}{\cos\theta} = \sqrt{2}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

Calculator value is $\theta = 45°$

$\cos\theta$ is positive

$\Rightarrow \theta$ is in 1st and 4th quadrants

Solutions are 45°, 315°

b $\operatorname{cosec}\theta = -3$

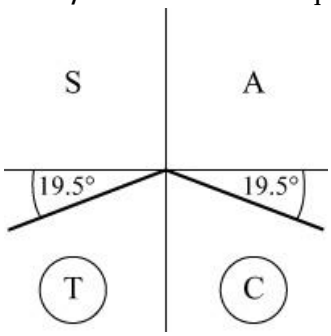
$\Rightarrow \frac{1}{\sin\theta} = -3$

$\Rightarrow \sin\theta = -\frac{1}{3}$

Calculator value is $\theta = -19.47°$ (2 d.p.)

$\sin\theta$ is negative

$\Rightarrow \theta$ is in 3rd and 4th quadrants



Solutions are 199°, 341° (3 s.f.)

c $5\cot\theta = -2$

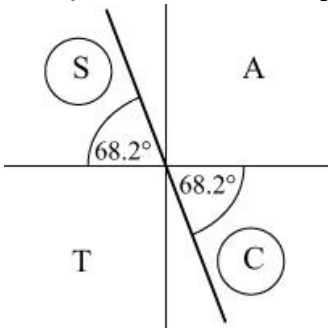
$\Rightarrow \cot\theta = -\frac{2}{5}$

$\Rightarrow \tan\theta = -\frac{5}{2}$

Calculator value is $\theta = -68.20°$ (2 d.p.)

$\tan\theta$ is negative

$\Rightarrow \theta$ is in 2nd and 4th quadrants



Solutions are 112°, 292° (3 s.f.)

d $\operatorname{cosec}\theta = 2$

$\Rightarrow \frac{1}{\sin\theta} = 2$

$\Rightarrow \sin\theta = \frac{1}{2}$

Calculator value is $\theta = 30°$

$\sin\theta$ is positive

$\Rightarrow \theta$ is in 1st and 2nd quadrants

Solutions are 30°, 150°

e $3\sec^2\theta = 4$

$\Rightarrow \sec^2\theta = \frac{4}{3}$

$\Rightarrow \cos^2\theta = \frac{3}{4}$

$\Rightarrow \cos\theta = \pm\frac{\sqrt{3}}{2}$

Calculator value for $\cos\theta = \frac{\sqrt{3}}{2}$ is $\theta = 30°$

As $\cos\theta$ is $\pm$, θ is in all four quadrants

Solutions are 30°, 150°, 210°, 330°

f $5\cos\theta = 3\cot\theta$

$\Rightarrow 5\cos\theta = 3\frac{\cos\theta}{\sin\theta}$

Do not cancel $\cos\theta$ on each side.

Multiply through by $\sin\theta$.

$\Rightarrow 5\cos\theta\sin\theta = 3\cos\theta$

$\Rightarrow 5\cos\theta\sin\theta - 3\cos\theta = 0$

$\Rightarrow \cos\theta(5\sin\theta - 3) = 0$ (factorise)

So $\cos\theta = 0$ or $\sin\theta = \frac{3}{5}$

When $\cos\theta = 0$, $\theta = 90°, 270°$

When $\sin\theta = \frac{3}{5}$, $\theta = 36.9°, 143°$ (3 s.f.)

Solutions are 36.9°, 90°, 143°, 270°

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

5 g $\cot^2\theta - 8\tan\theta = 0$

$\Rightarrow \dfrac{1}{\tan^2\theta} - 8\tan\theta = 0$

$\Rightarrow 1 - 8\tan^3\theta = 0$

$\Rightarrow 8\tan^3\theta = 1$

$\Rightarrow \tan^3\theta = \dfrac{1}{8}$

$\Rightarrow \tan\theta = \dfrac{1}{2}$

Calculator value is $\theta = 26.57°$ (2 d.p.)

$\tan\theta$ is positive

$\Rightarrow \theta$ is in 1st and 3rd quadrants

Solutions are $26.57°$ and $(180° + 26.57°)$

So solutions are $26.6°,\ 207°$ (3 s.f.)

h $2\sin\theta = \operatorname{cosec}\theta$

$\Rightarrow 2\sin\theta = \dfrac{1}{\sin\theta}$

$\Rightarrow \sin^2\theta = \dfrac{1}{2}$

$\Rightarrow \sin\theta = \pm\dfrac{1}{\sqrt{2}}$

Calculator value for $\sin\theta = \dfrac{1}{\sqrt{2}}$ is $\theta = 45°$

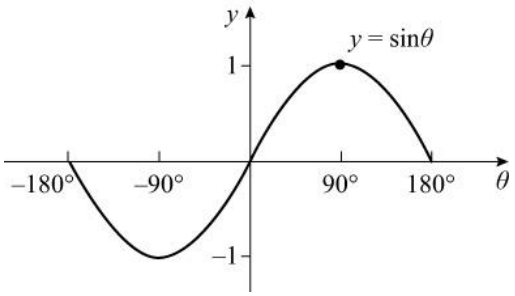
Solutions are in all four quadrants

Solutions are $45°, 135°, 225°, 315°$

6 a $\operatorname{cosec}\theta = 1$

$\Rightarrow \sin\theta = 1$

$\Rightarrow \theta = 90°$



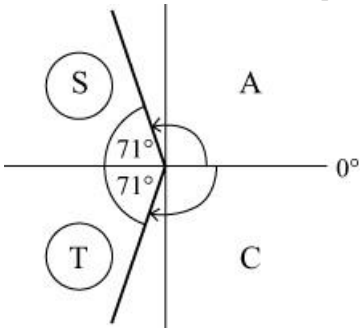
b $\sec\theta = -3$

$\Rightarrow \cos\theta = -\dfrac{1}{3}$

Calculator value is $\theta = 109°$ (3 s.f.)

$\cos\theta$ is negative

$\Rightarrow \theta$ is in 2nd and 3rd quadrants



Solutions are $109°,\ -109°$ (3 s.f.)

c $\cot\theta = 3.45$

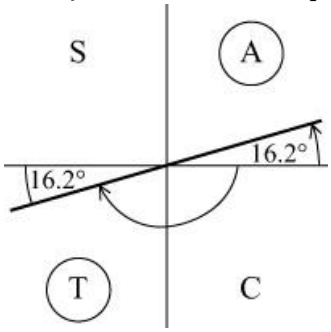
$\Rightarrow \dfrac{1}{\tan\theta} = 3.45$

$\Rightarrow \tan\theta = \dfrac{1}{3.45} = 0.2899$ (4 d.p.)

Calculator value is $\theta = 16.16°$ (2 d.p.)

$\tan\theta$ is positive

$\Rightarrow \theta$ is in 1st and 3rd quadrants



Solutions are $16.2°$ and $(-180° + 16.2°)$

So solutions are $16.2°,\ -164°$ (3 s.f.)

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

6 d $2\text{cosec}^2\theta - 3\text{cosec}\theta = 0$

$\Rightarrow \text{cosec}\theta(2\text{cosec}\theta - 3) = 0$ (factorise)

$\Rightarrow \text{cosec}\theta = 0$ or $\text{cosec}\theta = \frac{3}{2}$

$\Rightarrow \sin\theta = \frac{2}{3}$

$\text{cosec}\theta = 0$ has no solutions

Calculator value for $\sin\theta = \frac{2}{3}$ is $\theta = 41.8°$

θ is in 1st and 2nd quadrants

Solutions are $41.8°$, $(180 - 41.8)°$

So solutions are $41.8°$, $138°$ (3 s.f.)

e $\sec\theta = 2\cos\theta$

$\Rightarrow \frac{1}{\cos\theta} = 2\cos\theta$

$\Rightarrow \cos^2\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}}$

Calculator value for $\cos\theta = \frac{1}{\sqrt{2}}$ is $\theta = 45°$

θ is in all quadrants, but remember that solutions required for $-180° \leq \theta \leq 180°$

Solutions are $\pm 45°$, $\pm 135°$

f $3\cot\theta = 2\sin\theta$

$\Rightarrow 3\frac{\cos\theta}{\sin\theta} = 2\sin\theta$

$\Rightarrow 3\cos\theta = 2\sin^2\theta$

$\Rightarrow 3\cos\theta = 2(1 - \cos^2\theta)$

(use $\sin^2\theta + \cos^2\theta \equiv 1$)

$\Rightarrow 2\cos^2\theta + 3\cos\theta - 2 = 0$

$\Rightarrow (2\cos\theta - 1)(\cos\theta + 2) = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$ or $\cos\theta = -2$

As $\cos\theta = -2$ has no solutions, $\cos\theta = \frac{1}{2}$

Solutions are $\pm 60°$

g $\text{cosec}\,2\theta = 4$

$\Rightarrow \sin 2\theta = \frac{1}{4}$

Remember that solutions are required in the interval $-180° \leq \theta \leq 180°$

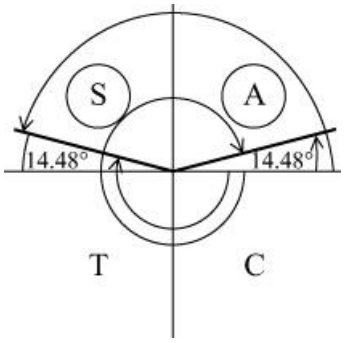
So $-360° \leq 2\theta \leq 360°$

Calculator value for $\sin 2\theta = \frac{1}{4}$ is

$2\theta = 14.48°$ (2 d.p.)

$\sin 2\theta$ is positive

$\Rightarrow 2\theta$ is in 1st and 2nd quadrants



$2\theta = -194.48°, -345.52°, 14.48°, 165.52°$

$\theta = -97.2°, -172.8°, 7.24°, 82.76°$

$= -173°, -97.2°, 7.24°, 82.8°$ (3 s.f.)

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

6 h $2\cot^2\theta - \cot\theta - 5 = 0$

As this quadratic in $\cot\theta$ does not factorise, use the quadratic formula

$$\cot\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

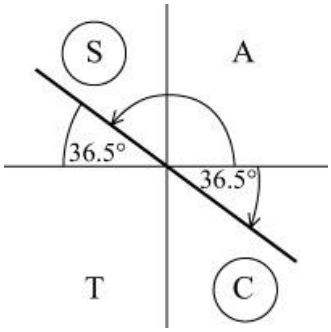
(You could change $\cot\theta$ to $\frac{1}{\tan\theta}$ and work with the quadratic $5\tan^2\theta + \tan\theta - 2 = 0$)

So $\cot\theta = \dfrac{1 \pm \sqrt{41}}{4}$

$= -1.3508,\ 1.8508$ (4 d.p.)

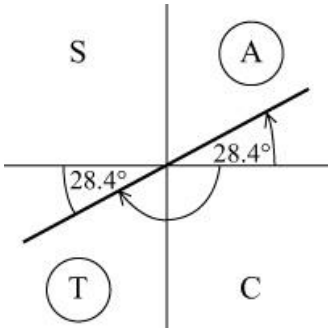
So $\tan\theta = -0.7403,\ 0.5403$ (4 d.p.)

The calculator value for $\tan\theta = -0.7403$ is $\theta = -36.51°$ (2 d.p.)



Solutions are $-36.5°,\ 143°$ (3 s.f.).

The calculator value for $\tan\theta = 0.5403$ is $\theta = 28.38°$ (2 d.p.)



Solutions are $28.4°,\ (-180 + 28.4)°$

Total set of solutions is

$-152°,\ -36.5°,\ 28.4°,\ 143°$ (3 s.f.)

7 a $\sec\theta = -1$

$\Rightarrow \cos\theta = -1$

$\Rightarrow \theta = \pi$

(refer to graph of $y = \cos\theta$)

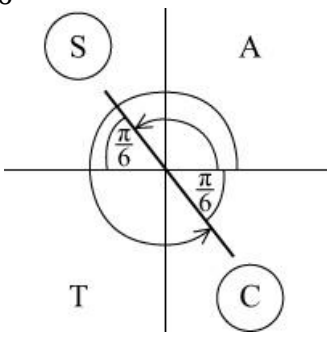
b $\cot\theta = -\sqrt{3}$

$\Rightarrow \tan\theta = -\dfrac{1}{\sqrt{3}}$

Calculator solution is $-\dfrac{\pi}{6}$

$\left(\text{you should know that } \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}\right)$

$-\dfrac{\pi}{6}$ is not in the interval



Solutions are $\pi - \dfrac{\pi}{6},\ 2\pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6},\ \dfrac{11\pi}{6}$

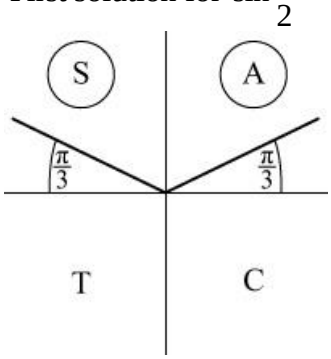
c $\operatorname{cosec}\dfrac{\theta}{2} = \dfrac{2\sqrt{3}}{3}$

$\Rightarrow \sin\dfrac{\theta}{2} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

Remember that $0 \leq \theta \leq 2\pi$

so $0 \leq \dfrac{\theta}{2} \leq \pi$

First solution for $\sin\dfrac{\theta}{2} = \dfrac{\sqrt{3}}{2}$ is $\dfrac{\theta}{2} = \dfrac{\pi}{3}$



So $\dfrac{\theta}{2} = \dfrac{\pi}{3},\ \dfrac{2\pi}{3}$

$\Rightarrow \theta = \dfrac{2\pi}{3},\ \dfrac{4\pi}{3}$

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

7 d $\sec\theta = \sqrt{2}\tan\theta$

$\Rightarrow \dfrac{1}{\cos\theta} = \sqrt{2}\dfrac{\sin\theta}{\cos\theta}$

$\Rightarrow 1 = \sqrt{2}\sin\theta \qquad (\cos\theta \neq 0)$

$\Rightarrow \sin\theta = \dfrac{1}{\sqrt{2}}$

Solutions are $\dfrac{\pi}{4}, \dfrac{3\pi}{4}$

8 a In the right-angled triangle ABD

$\dfrac{AB}{AD} = \cos\theta$

$\Rightarrow AD = \dfrac{6}{\cos\theta} = 6\sec\theta$

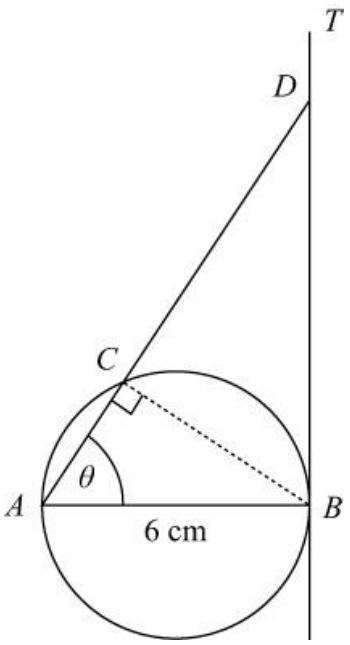
In the right-angled triangle ACB

$\dfrac{AC}{AB} = \cos\theta$

$\Rightarrow AC = 6\cos\theta$

$CD = AD - AC$

$= 6\sec\theta - 6\cos\theta = 6(\sec\theta - \cos\theta)$



b As $16 = 6\sec\theta - 6\cos\theta$

$\Rightarrow 8 = \dfrac{3}{\cos\theta} - 3\cos\theta$

$\Rightarrow 8\cos\theta = 3 - 3\cos^2\theta$

$\Rightarrow 3\cos^2\theta + 8\cos\theta - 3 = 0$

$\Rightarrow (3\cos\theta - 1)(\cos\theta + 3) = 0$

$\Rightarrow \cos\theta = \dfrac{1}{3}$ as $\cos\theta \neq -3$

From (a) $AC = 6\cos\theta = 6 \times \dfrac{1}{3} = 2$ cm

9 a $\dfrac{\operatorname{cosec} x - \cot x}{1 - \cos x} \equiv \dfrac{\dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}}{1 - \cos x}$

$\equiv \dfrac{1}{\sin x} \times \dfrac{1 - \cos x}{1 - \cos x}$

$\equiv \operatorname{cosec} x$

b By part a equation becomes

$\operatorname{cosec} x = 2$

$\Rightarrow \dfrac{1}{\sin x} = 2$

$\Rightarrow \sin x = \dfrac{1}{2}$

$\sin x$ is positive, so x is in 1st and 2nd quadrants

$x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

10 a $\dfrac{\sin x \tan x}{1 - \cos x} - 1 \equiv \dfrac{\sin^2 x}{\cos x(1 - \cos x)} - 1$

$\equiv \dfrac{\sin^2 x - \cos x + \cos^2 x}{\cos x(1 - \cos x)}$

$\equiv \dfrac{1 - \cos x}{\cos x(1 - \cos x)}$

$\equiv \dfrac{1}{\cos x}$

$\equiv \sec x$

b Need to solve $\sec x = -\dfrac{1}{2}$

$\Rightarrow \cos x = -2$

which has no solutions.

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

11 $\dfrac{1+\cot x}{1+\tan x}=5$

$$\Rightarrow \frac{1+\dfrac{\cos x}{\sin x}}{1+\dfrac{\sin x}{\cos x}}=5$$

$$\Rightarrow \frac{\dfrac{\sin x+\cos x}{\sin x}}{\dfrac{\cos x+\sin x}{\cos x}}=5$$

$$\Rightarrow \frac{\sin x+\cos x}{\sin x}\times\frac{\cos x}{\cos x+\sin x}=5$$

$$\Rightarrow \frac{\cos x}{\sin x}=5$$

$$\Rightarrow \cot x=5$$

$$\Rightarrow \tan x=\frac{1}{5}$$

Calculator solution is 11.3° (1 d.p.)

$\tan x$ is positive, so x is in

1st and 3rd quadrants

Solutions are $11.3°, 191.3°$ (1 d.p.)

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.