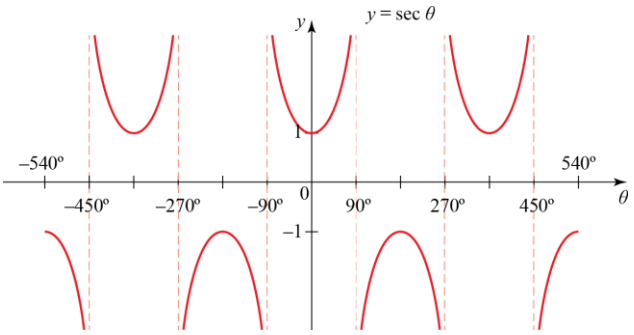
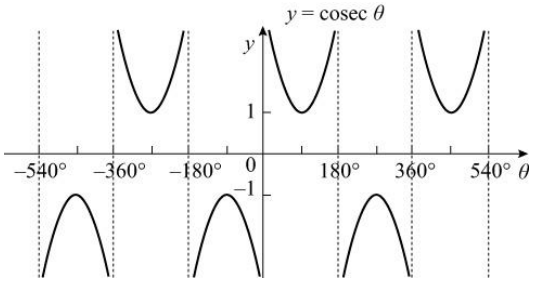
Trigonometric Functions 6B

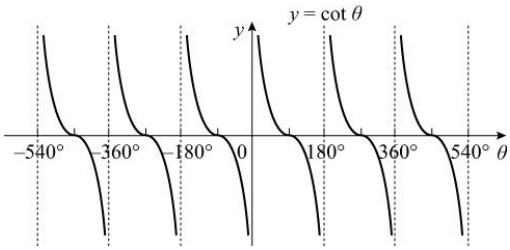
1 a



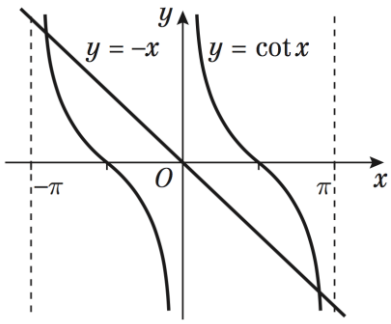
b



c

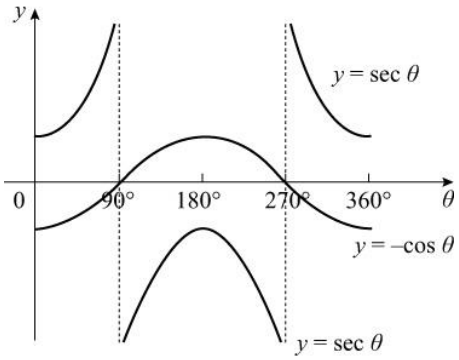


2 a



b 2 solutions

3 a



b You can see that the graphs of $y = \sec q$ and $y = -\cos q$ do not meet, so $\sec q = -\cos q$ has no solutions.

The same result can be found algebraically

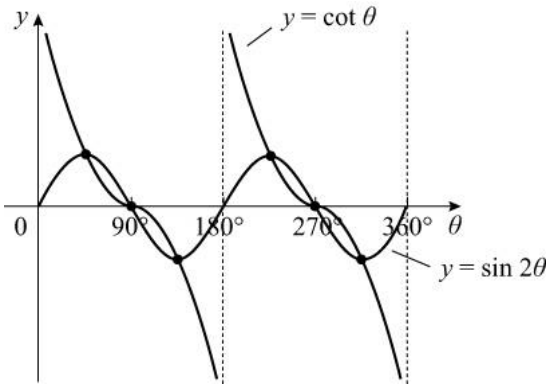
$\sec q = -\cos q$

$\Rightarrow \dfrac{1}{\cos q} = -\cos q$

$\Rightarrow \cos^2 q = -1$

There are no solutions of this equation for real q.

4 a

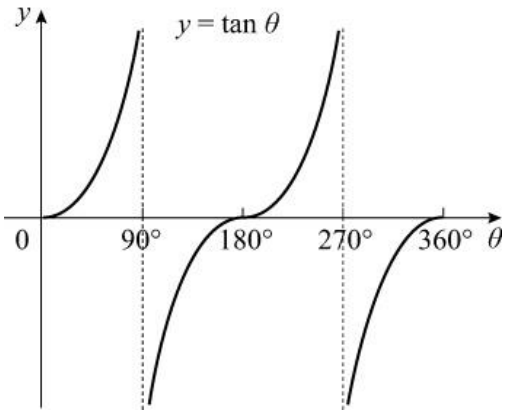


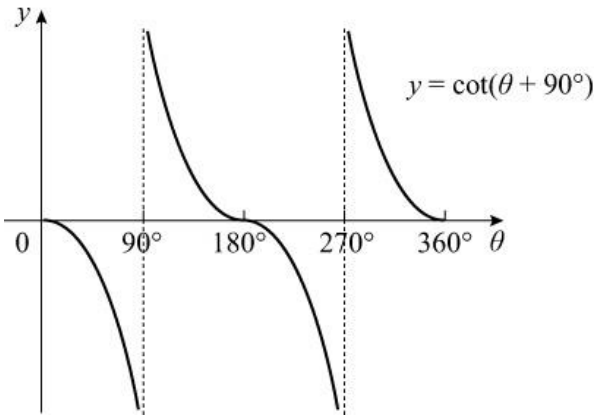
b The curves meet at the maxima and minima of $y = \sin 2q$, and on the q-axis at odd integer multiples of 90°.

In the interval $0 \leq q \leq 360°$ there are 6 intersections. So there are 6 solutions of $\cot q = \sin 2q$ in the interval $0 \leq q \leq 360°$

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5 a





b $y = \cot(q + 90°)$ is a reflection in the q-axis of $y = \tan q$, so
$\cot(q + 90°) = -\tan q$

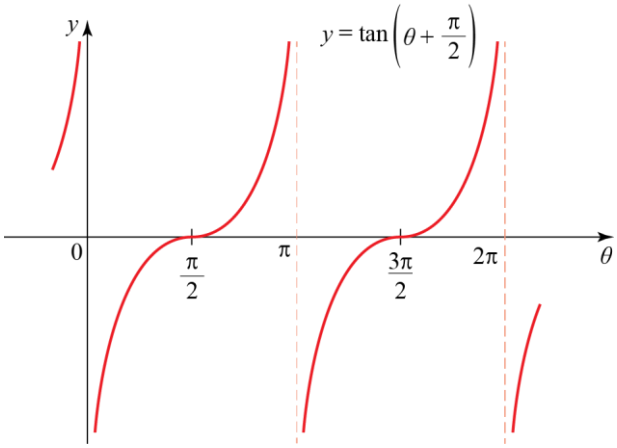
6 a i The graph of $y = \tan\left(q + \frac{p}{2}\right)$ is the same as that of $y = \tan\theta$ translated by the vector $\begin{pmatrix} -\frac{\pi}{2} \\ 0 \end{pmatrix}$, i.e by $\frac{\pi}{2}$ to the left.

ii The graph of $y = \cot(-q)$ is the same as that of $y = \cot q$ reflected in the y-axis.

iii The graph of $y = \text{cosec}\left(q + \frac{p}{4}\right)$ is the same as that of $y = \text{cosec}\,\theta$ translated by the vector $\begin{pmatrix} -\frac{\pi}{4} \\ 0 \end{pmatrix}$

iv The graph of $\sec\left(q - \frac{p}{4}\right)$ is the same as that of $y = \sec\theta$ translated by the vector $\begin{pmatrix} \frac{\pi}{4} \\ 0 \end{pmatrix}$

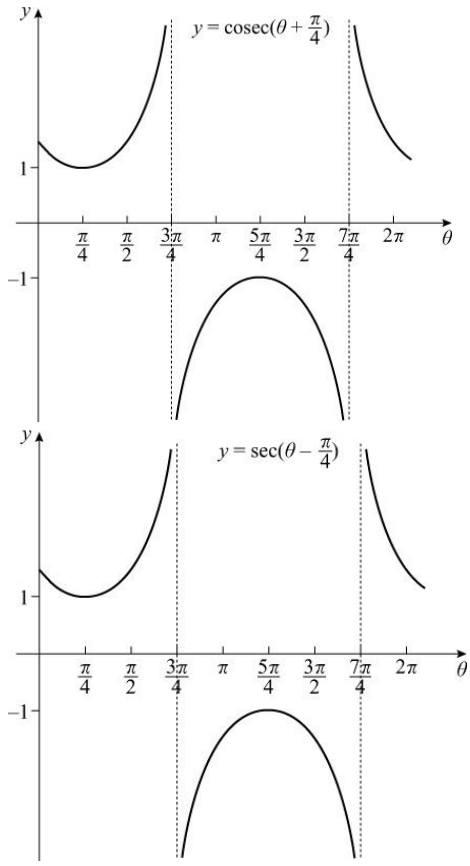
b



(reflection of $y = \cot q$ in the y-axis)
$\tan\left(q + \frac{p}{2}\right) = \cot(-q)$

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6 b



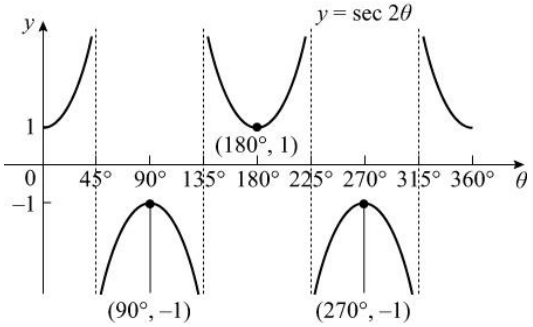
$$\text{cosec}\left(q+\frac{\text{p}}{4}\right)=\sec\left(q-\frac{\text{p}}{4}\right)$$

7 a A stretch of $y=\sec q$ in the q direction with scale factor $\frac{1}{2}$

Minimum at (180°,1)

Maxima at (90°,- 1) and (270°,- 1)

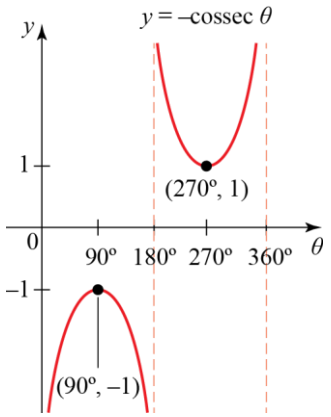
It meets the y-axis at (0, 1)



b Reflection in q-axis of $y=\text{cosec}\,q$

Minimum at (270°, 1)

Maximum at (90°, - 1)

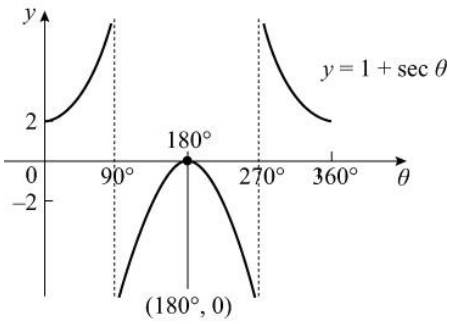


c Translation of $y=\sec q$ by the vector $\begin{pmatrix}0\\1\end{pmatrix}$, i.e. +1 in the y direction.

It meets x-axis at (180°, 0)

There is a maximum at (180°, 0)

It meets the y-axis at (0, 2)



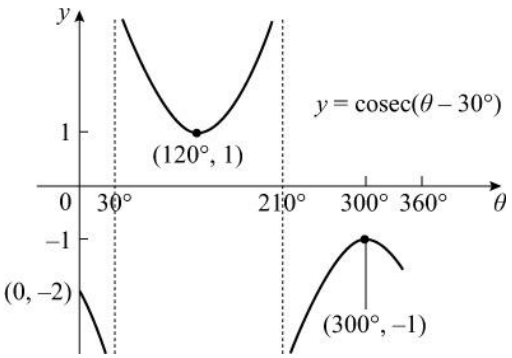
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d Translation of $y = \text{cosec}\,q$ by the vector $\begin{pmatrix} 30 \\ 0 \end{pmatrix}$

Minimum at (120°, 1)

Maximum at (300°, –1)

It meets the y-axis at $(0, -2)$

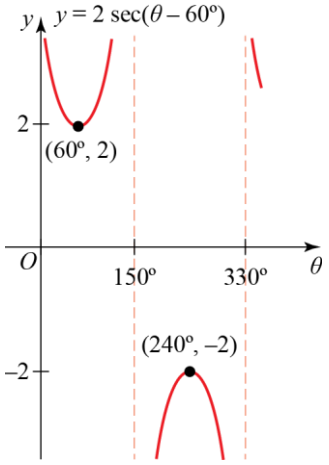


7 e $y = 2\sec(\theta - 60^\circ)$ is $y = \sec\theta$ translated by the vector $\begin{pmatrix} 60 \\ 0 \end{pmatrix}$ and then stretched by a scale factor 2 in the y direction.

Minimum at (60°, 2)

Maximum at (240°, - 2)

It meets the y-axis at (0, 4)

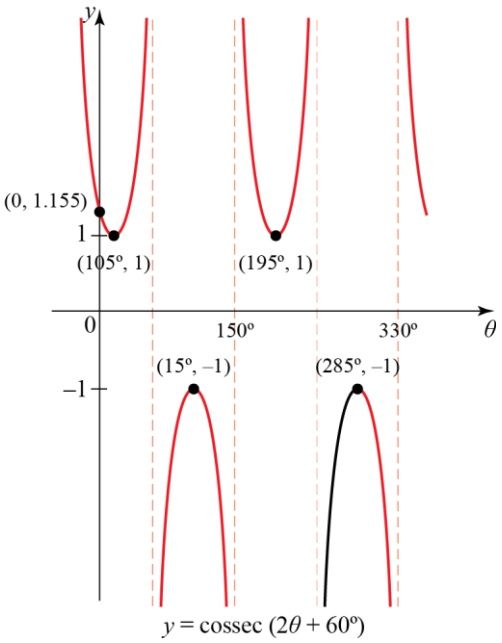


f $y = \text{cosec}(2\theta + 60^\circ)$ is $y = \text{cosec}\,q$ translated by the vector $\begin{pmatrix} -60 \\ 0 \end{pmatrix}$ and then stretched by a scale factor $\frac{1}{2}$ in the θ direction.

Minima at (15°, 1), (195°, 1)

Maxima at (105°, - 1), (285°, - 1)

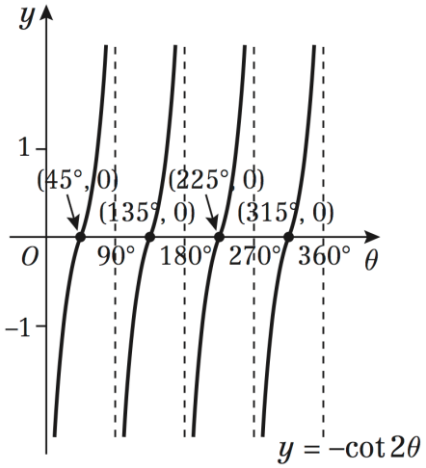
It meets the y-axis at (0, 1.155)



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g $y = -\cot 2q$ is $y = \cot q$ stretched by a scale factor $\frac{1}{2}$ in the q direction and then reflected in the x-axis.

It meets the q-axis at (45°, 0), (135°, 0), (225°, 0) and (315°, 0)

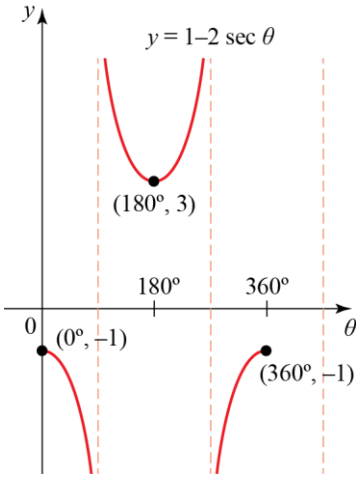


h $y = 1 - 2\sec q = -2\sec q + 1$ is $y = \sec\theta$ stretched by a scale factor 2 in the y direction, reflected in the x-axis and then translated by the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Minima at (180°, 3)

Maxima at $(0, -1), (360°, -1)$

It meets the y-axis at $(0, -1)$



8 a The period of $\sec q$ is 2π radians

$y = \sec 3q$ is a stretch of $y = \sec q$ with scale factor $\frac{1}{3}$ in the q direction.

So the period of $\sec 3q$ is $\dfrac{2\pi}{3}$

b $\operatorname{cosec} q$ has a period of 2π

$\operatorname{cosec} \frac{1}{2}q$ is a stretch of $\operatorname{cosec} q$ in the q direction with scale factor 2.

So the period of $\operatorname{cosec} \frac{1}{2}q$ is 4π

c $\cot q$ has a period of π

$2\cot q$ is a stretch in the y direction by scale factor 2. So the periodicity is not affected.

The period of $2\cot q$ is π

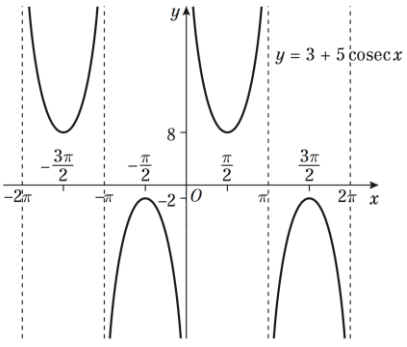
d $\sec q$ has a period of 2π

$\sec(-q)$ is a reflection in the y-axis.

So the periodicity is not affected.

The period of $\sec(-q)$ is 2π

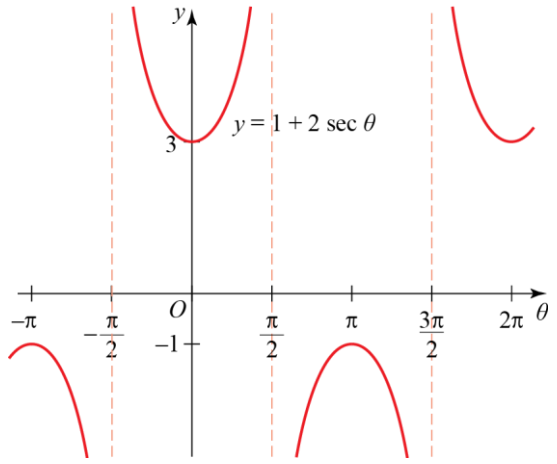
9 a $y = 3 + 5\operatorname{cosec} q$ is $y = \operatorname{cosec} q$ stretched by a scale factor 5 in the y direction and then translated by the vector $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$



b $-2 < k < 8$

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10 a



b The q coordinates at points at which the gradient is zero are at the maxima and minima. These are $q = -p, 0, p, 2p$

c Minimum value of $\dfrac{1}{1+2\sec q}$

is where $1+2\sec q$ is a maximum.

So minimum value of $\dfrac{1}{1+2\sec q}$

is $\dfrac{1}{-1} = -1$

The first positive value of q where this occurs is when $q = p$

(see diagram)

Maximum value of $\dfrac{1}{1+2\sec q}$

is where $1+2\sec q$ is a minimum.

So maximum value of $\dfrac{1}{1+2\sec q}$ is $\dfrac{1}{3}$

The first positive value of q where this occurs is when $q = 2p$

(see diagram)

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