Sequences and series 3F

1 a i $\sum_{r=1}^{5}(3r+1) = 4+7+10+13+16$

ii $S_5 = 50$

b i $\sum_{r=1}^{6}3r^2 = 3+12+27+48+75+108$

ii $S_6 = 273$

c i $\sum_{r=1}^{5}\sin(90r°) = 1+0+(-1)+0+1$

ii $S_5 = 1$

d i $\sum_{r=5}^{8}2\left(-\frac{1}{3}\right)^r = -\frac{2}{243}+\frac{2}{729}-\frac{2}{2187}+\frac{2}{6561}$

ii $S_4 = -\frac{40}{6561}$

2 a i $2+4+6+8 = \sum_{r=1}^{4}2r$

ii $S_4 = 20$

b i $2+6+18+54+162 = \sum_{r=1}^{5}\left(2\times 3^{r-1}\right)$

ii $S_5 = 242$

c i

$6+4.5+3+1.5+0-1.5 = \sum_{r=1}^{6}\left(-\frac{3}{2}r+\frac{15}{2}\right)$

ii $S_6 = 13.5$

3 a i $7+13+19+\ldots+157 = \sum_{r=1}^{n}(6r+1)$

$6n + 1 = 157$

$n = 26$

ii $\sum_{r=1}^{26}(6r+1)$

b i

$\frac{1}{3}+\frac{2}{15}+\frac{4}{75}+\ldots+\frac{64}{46\,875} = \sum_{r=1}^{n}\left(\frac{1}{3}\times\left(\frac{2}{5}\right)^{r-1}\right)$

$\frac{1}{3}\times\left(\frac{2}{5}\right)^{n-1} = \frac{64}{46\,875}$

$\left(\frac{2}{5}\right)^{n-1} = \frac{64}{15\,625}$

$n = \frac{\log(0.004096)}{\log(0.4)}+1 = 7$

ii $\sum_{r=1}^{7}\left(\frac{1}{3}\times\left(\frac{2}{5}\right)^{r-1}\right)$

c i $8-1-10-19-\ldots-127 = \sum_{r=1}^{n}(17-9r)$

$17-9n = -127$

$n = 16$

ii $\sum_{r=1}^{16}(17-9r)$

4 a $\sum_{r=1}^{20}(7-2r) = 5+3+1+\ldots-33$

$a = 5,\ l = -33,\ n = 20$

$S_{20} = \frac{20}{2}(5-33)$

$= -280$

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

4 b $\sum_{r=1}^{10} 3\times 4^r = 12+48+192+\ldots+3145\,728$

$a=12,\ r=4,\ n=10$

$S_{10}=\dfrac{12(4^{10}-1)}{4-1}$

$= 4\,194\,300$

c $\sum_{r=1}^{100}(2r-8) = -6-4-2+\ldots+192$

$a=-6,\ l=192,\ n=100$

$S_{100}=\dfrac{100}{2}(-6+192)$

$= 9300$

d $\sum_{r=1}^{\infty} 7\left(-\dfrac{1}{3}\right)^r = -\dfrac{7}{3}+\dfrac{7}{9}-\dfrac{7}{27}+\ldots$

$a=-\dfrac{7}{3},\ r=-\dfrac{1}{3}$

$S_{\infty}=\dfrac{-\frac{7}{3}}{1+\frac{1}{3}}$

$= -\dfrac{7}{4}$

5 a $\sum_{r=9}^{30}\left(5r-\dfrac{1}{2}\right) = 44\dfrac{1}{2}+49\dfrac{1}{2}+\ldots+149\dfrac{1}{2}$

$a=44\dfrac{1}{2},\ l=149\dfrac{1}{2},\ n=22$

$S_{22}=\dfrac{22}{2}\left(44\dfrac{1}{2}+149\dfrac{1}{2}\right)$

$= 2134$

b $\sum_{r=100}^{200}(3r+4) = 304+307+310+\ldots+604$

$a=304,\ l=604,\ n=101$

$S_{101}=\dfrac{101}{2}(304+604)$

$= 45\,854$

c

$\sum_{r=5}^{100} 3\times 0.5^r = 0.09375+0.046875+0.0234375+\ldots$

$a=0.09375,\ r=0.5,\ n=96$

$S_{96}=\dfrac{0.09375(1-0.5^{96})}{1-0.5}$

$= 0.1875$

d $\sum_{i=5}^{100} 1 = 1+1+1+\ldots+1$

$a=1,\ l=1,\ n=96$

$S_{96}=\dfrac{96}{2}(1+1)$

$= 96$

6 $\sum_{r=1}^{n} 2r = 2+4+6+\ldots+2n$

$a=2,\ l=2n$

$S_n=\dfrac{n}{2}(2+2n)$

$= n+n^2$

7 $\sum_{r=1}^{n} 2r = n+n^2$

$\sum_{r=1}^{n}(2r-1) = 1+3+5+\ldots+(2n-1)$

$a=1,\ l=2n-1$

$S_n=\dfrac{n}{2}(1+2n-1)$

$= n^2$

$\sum_{r=1}^{n} 2r - \sum_{r=1}^{n}(2r-1) = n+n^2-n^2 = n$

8 a $\sum_{r=1}^{k} 4(-2)^r = -8+16+\ldots+4(-2)^k$

$a=-8,\ r=-2$

$S_k=\dfrac{-8(1-(-2)^k)}{1+2}$

$=\dfrac{8}{3}\left((-2)^k-1\right)$

b $\sum_{r=1}^{k}(100-2r) = 98+96+\ldots+(100-2k)$

$a=98,\ l=100-2k$

$S_k=\dfrac{k}{2}(98+100-2k)$

$= 99k-k^2$

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

8 c $\sum_{r=10}^{k}(7-2r) = -13-15-\ldots+(7-2k)$

$a = -13,\ l = 7-2k$

$S_{k-9} = \frac{k-9}{2}(-13+7-2k)$

$= (k-9)(-3-k)$

$= 6k - k^2 + 27$

9 $\sum_{r=10}^{\infty} 200\times\left(\frac{1}{4}\right)^r =$

$\sum_{r=1}^{\infty} 200\times\left(\frac{1}{4}\right)^r - \sum_{r=1}^{r=9} 200\times\left(\frac{1}{4}\right)^r$

$a = 50,\ r = \frac{1}{4}$

$S_\infty - S_9 = \frac{50}{1-\frac{1}{4}} - \frac{50\left(1-\left(\frac{1}{4}\right)^9\right)}{1-\frac{1}{4}}$

$= \frac{50-50\left(1-\left(\frac{1}{4}\right)^9\right)}{\frac{3}{4}}$

$= \frac{200}{3}\left(\frac{1}{4}\right)^9$

$= \frac{25}{98\,304}$

10 a $\sum_{r=1}^{k}(8+3r) = 11+14+17+\ldots+(8+3k)$

$a = 11,\ l = 8 + 3k,\ n = k$

$S_k = \frac{k}{2}(11+8+3k)$

$\frac{k}{2}(19+3k) = 377$

$19k + 3k^2 = 754$

$3k^2 + 19k - 754 = 0$

$(3k+58)(k-13) = 0$

b As $k > 0$, $k = 13$

11 a $\sum_{r=1}^{k} 2\times 3^r = 59\,046$

$a = 6,\ r = 3$

$S_k = \frac{6(3^k-1)}{3-1} = 59\,046$

$3(3^k - 1) = 59\,046$

$3^k = 19\,683$

$k\log 3 = \log 19\,683$

$k = \frac{\log 19\,683}{\log 3}$

b $k = 9$

$\sum_{r=10}^{13} 2\times 3^r = \sum_{r=1}^{13} 2\times 3^r - \sum_{r=1}^{9} 2\times 3^r$

$= \frac{6(3^{13}-1)}{3-1} - 59\,046$

$= 4\,782\,966 - 59\,046$

$= 4\,723\,920$

12 a $r = 3x$

As the series is convergent, $|3x| < 1$

$|x| < \frac{1}{3}$

b $S_\infty = \frac{1}{1-3x} = 2$

$1 = 2(1-3x)$

$6x = 1$

$x = \frac{1}{6}$

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

Challenge

$\sum_{r=1}^{10}(a+(r-1)d) = a+(a+d)+\ldots+(a+9d)$

$= \frac{10}{2}(a+a+9d)$

$= 5(2a+9d)$

$\sum_{r=11}^{14}(a+(r-1)d) = (a+10d)+\ldots+(a+13d)$

$= \frac{4}{2}(a+10d+a+13d)$

$= 2(2a+23d)$

As $\sum_{r=1}^{10}(a+(r-1)d) = \sum_{r=11}^{14}(a+(r-1)d)$

$5(2a+9d) = 2(2a+23d)$

$10a + 45d = 4a + 46d$

$d = 6a$

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.