Sequences and series 3D

1 a $1+2+4+8+\ldots$ (8 terms)

In this series $a=1, r=2, n=8.$

As $|r|>1$ use $S_n=\dfrac{a(r^n-1)}{r-1}$.

$$S_8=\frac{a(r^8-1)}{r-1}=\frac{1\times(2^8-1)}{2-1}=256-1=255$$

b $32+16+8+\ldots$ (10 terms)

In this series $a=32, r=\dfrac{1}{2}, n=10.$

As $|r|<1$ use $S_n=\dfrac{a(1-r^n)}{1-r}$.

$$S_{10}=\frac{a(1-r^{10})}{1-r}$$

$$=\frac{32\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}}=63.938\ (3\text{ d.p.})$$

c $a=\dfrac{2}{3}, r=\dfrac{2}{5}, n=8$

$$S_8=\frac{\frac{2}{3}\left(1-\left(\frac{2}{5}\right)^8\right)}{1-\frac{2}{5}}$$

$= 1.110$

d $4-12+36-108+\ldots$ (6 terms)

In this series $a=4, r=-3, n=6.$

As $|r|>1$ use $S_n=\dfrac{a(r^n-1)}{r-1}$.

$$S_6=\frac{a(r^6-1)}{r-1}=\frac{4\left((-3)^6-1\right)}{-3-1}=-728$$

e $729-243+81-\ldots-\dfrac{1}{3}$

Here, $a=729, r=\dfrac{-243}{729}=-\dfrac{1}{3}$

and the nth term is $-\dfrac{1}{3}$.

Using nth term $=ar^{n-1}$

$$-\frac{1}{3}=729\times\left(-\frac{1}{3}\right)^{n-1}$$

$$-\frac{1}{2187}=\left(-\frac{1}{3}\right)^{n-1}$$

$$\left(-\frac{1}{3}\right)^7=\left(-\frac{1}{3}\right)^{n-1}$$

So $n-1=7$

$\Rightarrow n=8$

There are 8 terms in the series.

As $|r|<1$ use $S_n=\dfrac{a(1-r^n)}{1-r}$ with $a=729, r=-\dfrac{1}{3}$ and $n=8$.

$$S_8=\frac{729\left(1-\left(-\frac{1}{3}\right)^8\right)}{1-\left(-\frac{1}{3}\right)}=546\frac{2}{3}$$

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1 f $a = -\frac{5}{2}$, $r = -\frac{1}{2}$, $n = 15$

$$S_{15} = \frac{-\frac{5}{2}\left(1-\left(-\frac{1}{2}\right)^{15}\right)}{1+\frac{1}{2}}$$

$= -1.67$

2 $a = 3$, $r = 0.4$, $n = 10$

$$S_{10} = \frac{3\left(1-0.4^{10}\right)}{1-0.4}$$

$= 4.9995$

3 $a = 5$, $r = \frac{2}{3}$, $n = 8$

$$S_8 = \frac{5\left(1-\left(\frac{2}{3}\right)^8\right)}{1-\frac{2}{3}}$$

$= 14.4147$

4 Let the common ratio be r.

The first three terms are 8, $8r$ and $8r^2$.

Given that the first three terms add up to 30.5,

$$8+8r+8r^2 = 30.5 \quad (\times 2)$$
$$16+16r+16r^2 = 61$$
$$16r^2+16r-45 = 0$$
$$(4r-5)(4r+9) = 0$$
$$r = \frac{5}{4}, \frac{-9}{4}$$

Possible values of r are $\frac{5}{4}$ and $\frac{-9}{4}$.

5 $3 + 6 + 12 + 24 + \ldots$ is a geometric series with $a = 3$, $r = 2$.

So $S_n = \frac{a\left(r^n-1\right)}{r-1} = \frac{3\left(2^n-1\right)}{2-1} = 3\left(2^n-1\right)$

We want $S_n > 1.5\,\text{million}$

$$S_n > 1500\,000$$
$$3\left(2^n-1\right) > 1500\,000$$
$$2^n - 1 > 500\,000$$
$$2^n > 500\,001$$
$$\log 2^n > \log 500\,001$$
$$n \log 2 > \log 500\,001$$
$$n > \frac{\log 500\,001}{\log 2}$$
$$n > 18.9$$

Least value of n is 19.

6 $5 + 4.5 + 4.05 + \ldots$ is a geometric series with $a = 5$ and $r = \frac{4.5}{5} = 0.9$.

$$\text{Using } S_n = \frac{a\left(1-r^n\right)}{1-r} = \frac{5\left(1-0.9^n\right)}{1-0.9}$$
$$= 50\left(1-0.9^n\right)$$

We want $S_n > 45$

$$50\left(1-0.9^n\right) > 45$$
$$\left(1-0.9^n\right) > \frac{45}{50}$$
$$1-0.9^n > 0.9$$
$$0.9^n < 0.1$$

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6 (continued)

$$\log(0.9)^n < \log(0.1)$$
$$n\log(0.9) < \log(0.1)$$
$$n > \frac{\log(0.1)}{\log(0.9)}$$
$$n > 21.85$$
So $n = 22$

7 a $a = 25$, $r = \frac{3}{5}$, $S_k > 61$

$$\frac{25\left(1-\left(\frac{3}{5}\right)^k\right)}{1-\frac{3}{5}} > 61$$
$$\frac{25\left(1-0.6^k\right)}{0.4} > 61$$
$$25(1 - 0.6^k) > 24.4$$
$$1 - 0.6^k > 0.976$$
$$0.6^k > 0.024$$
$$k\log(0.6) > \log(0.024)$$
$$k > \frac{\log(0.024)}{\log(0.6)}$$

b $k > 7.301$
$k = 8$

8
$$S_2 = \frac{a\left(1-r^2\right)}{1-r} = 4.48$$
$$a\left(1-r^2\right) = 4.48(1-r)$$
$$a = \frac{4.48(1-r)}{1-r^2}$$
$$S_4 = \frac{a\left(1-r^4\right)}{1-r} = 5.1968$$
$$a\left(1-r^4\right) = 5.1968(1-r)$$
$$a = \frac{5.1968(1-r)}{1-r^4}$$
$$\frac{4.48(1-r)}{1-r^2} = \frac{5.1968(1-r)}{1-r^4}$$
$$\frac{1}{1-r^2} = \frac{1.16}{1-r^4}$$
$$\frac{1}{1-r^2} = \frac{1.16}{(1-r^2)(1+r^2)}$$
$$1 = \frac{1.16}{(1+r^2)}$$
$$1 + r^2 = 1.16$$
$$r^2 = 0.16$$
$$r = \pm 0.4$$

9 $a = a$, $r = \sqrt{3}$
$$S_{10} = \frac{a\left(\sqrt{3}^{10}-1\right)}{\sqrt{3}-1}$$
$$= \frac{a\left(243-1\right)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$
$$= \frac{242a(\sqrt{3}+1)}{3-1}$$
$$= 121a(\sqrt{3}+1)$$

10 First series:
$a = a$, $r = 2$
$$S_4 = \frac{a\left(2^4-1\right)}{2-1}$$
$$S_4 = 15a$$
Second series:
$a = b$, $r = 3$
$$S_4 = \frac{b\left(3^4-1\right)}{3-1}$$
$$S_4 = 40b$$
$$15a = 40b$$
$$a = \frac{8}{3}b$$

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11 a $\dfrac{2k+5}{k} = \dfrac{k}{k-6}$

$(2k + 5)(k - 6) = k^2$

$2k^2 + 7k - 30 = k^2$

$k^2 + 7k - 30 = 0$

b $(k + 3)(k - 10) = 0$

$k = -3$ or $k = 10$

As $k > 0$, $k = 10$

c $r = \dfrac{10}{10-6} = \dfrac{5}{2} = 2.5$

d $S_{10} = \dfrac{4\left(2.5^{10} - 1\right)}{2.5-1}$

$= 25\,429$

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