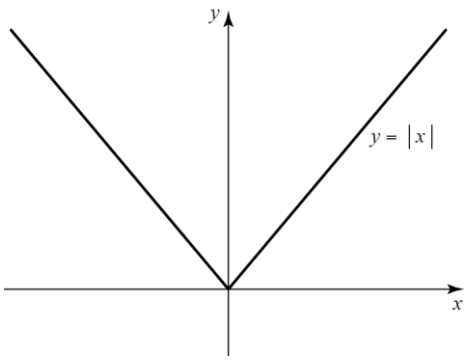
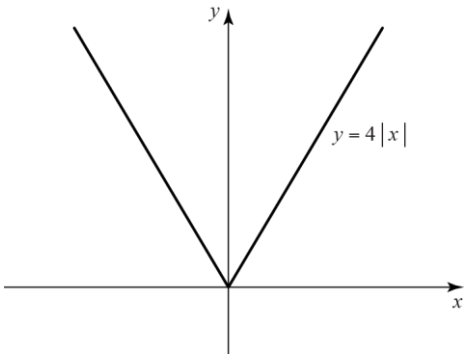
Functions and graphs 2G

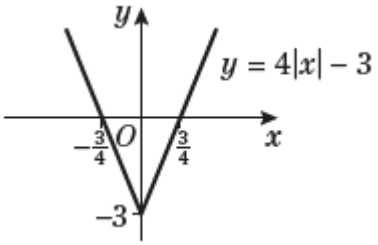
1 a i Start with $y = |x|$



$y = 4|x|$ is a vertical stretch by scale factor 4

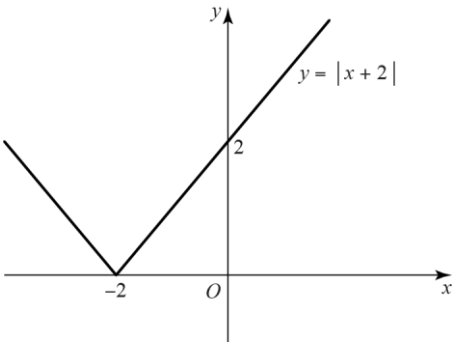


$y = 4|x| - 3$ is a horizontal translation by –3



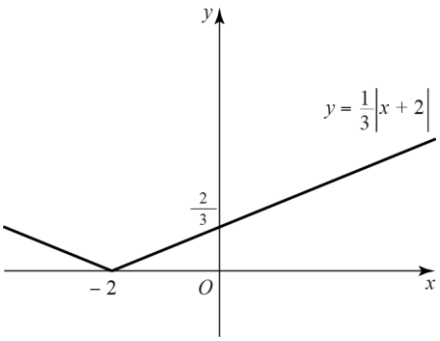
ii The range is $f(x) \geq -3$

b i Start with $y = |x|$

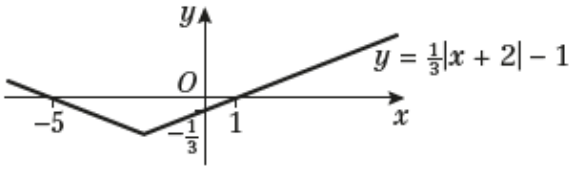


$y = |x + 2|$ is a horizontal translation by –2

$y = \frac{1}{3}|x + 2|$ is a vertical stretch by scale factor $\frac{1}{3}$



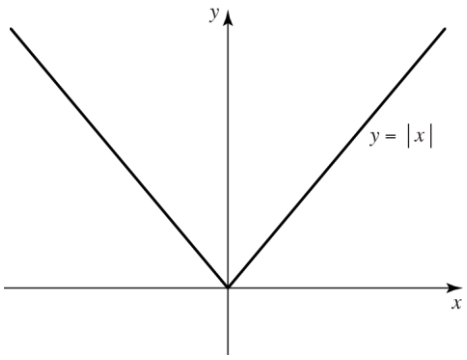
$y = \frac{1}{3}|x + 2| - 1$ is a vertical translation by –1



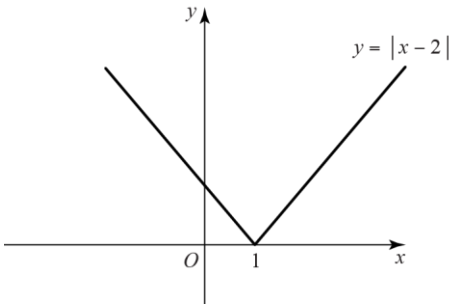
ii The range is $f(x) \geq -1$

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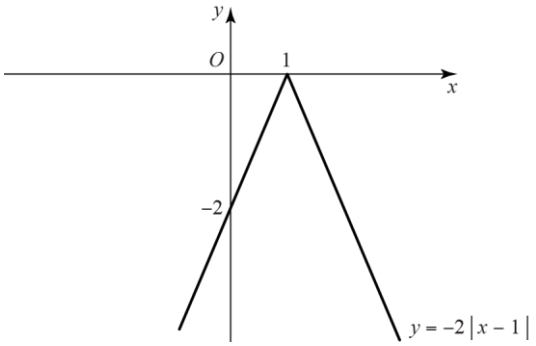
1 c i Start with $y = |x|$



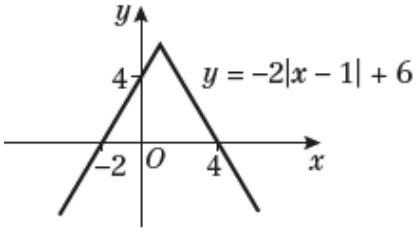
$y = |x-1|$ is a horizontal translation by +1



$y = -2|x-1|$ is a vertical stretch by scale factor –2

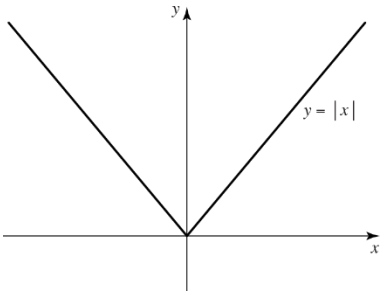


$y = -2|x-1|+6$ is a vertical translation by +6

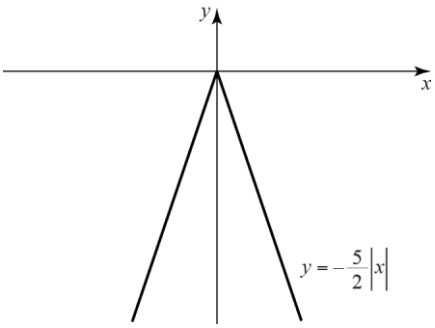


ii The range is f(x) ≤ 6

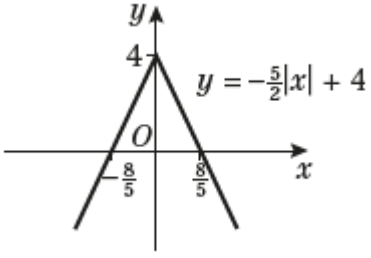
d i Start with $y = |x|$



$y = -\frac{5}{2}|x|$ is a vertical stretch by scale factor $-\frac{5}{2}$



$y = -\frac{5}{2}|x|+4$ is a horizontal translation by –3

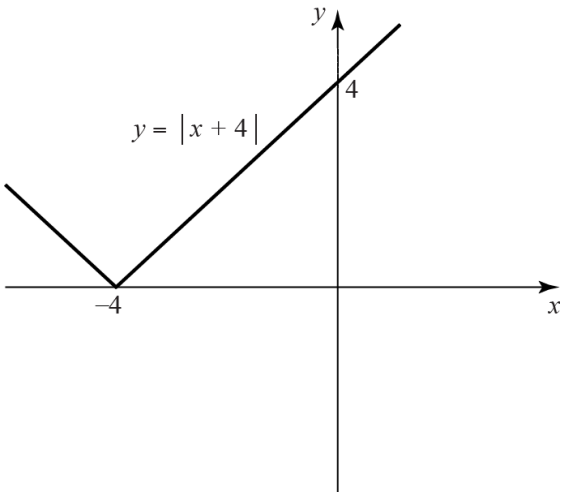


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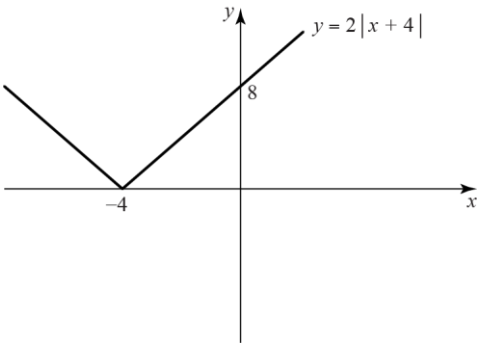
1 d ii The range is f(x) ≤ 4

2 a Start with $y = |x|$

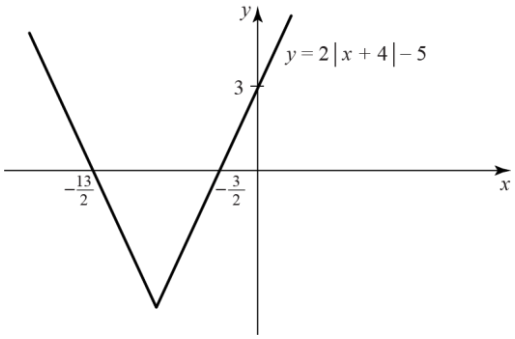
$y = |x + 4|$ is a horizontal translation of –4



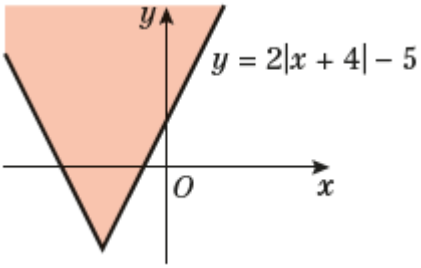
$y = 2|x + 4|$ is a vertical stretch scale factor 2



$y = 2|x + 4| - 5$ is a vertical translation of –5

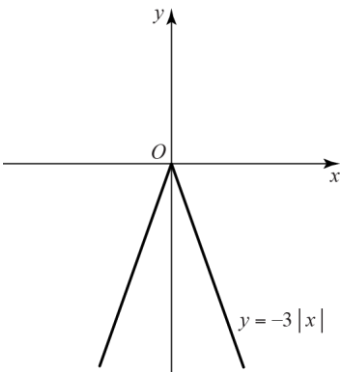


b The region where $y \geq p(x)$ is the region which lies on and above the line $y = 2|x + 4| - 5$

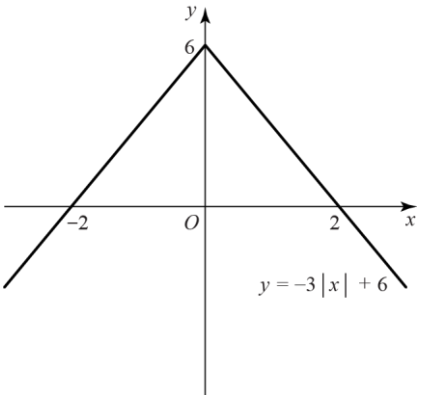


3 a Start with $y = |x|$

$y = -3|x|$ is a vertical stretch scale factor –3

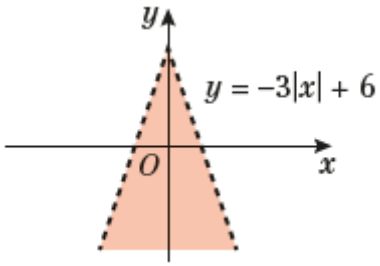


$y = -3|x| + 6$ is a vertical translation of +6



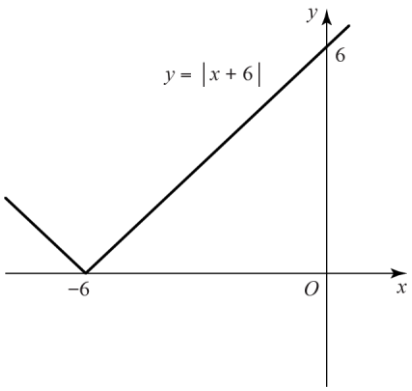
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3 b The region where $y < q(x)$ is the region which lies below the line $y = -3|x| + 6$

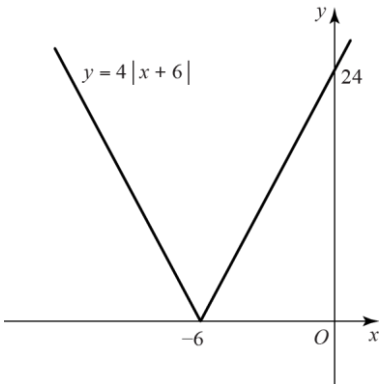


4 a Start with $y = |x|$

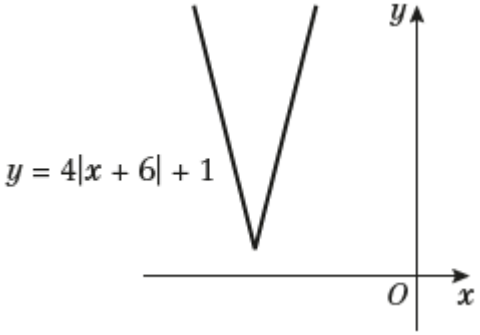
$y = |x + 6|$ is a horizontal translation of –6



$y = 4|x + 6|$ is a vertical stretch scale factor 4



$y = 4|x + 6| + 1$ is a vertical translation of +1



b The range is $f(x) \geq 1$

c At one point of intersection:

$$-4(x+6)+1 = -\frac{1}{2}x + 1$$
$$-4x - 23 = -\frac{1}{2}x + 1$$
$$-8x - 46 = -x + 2$$
$$-48 = 7x$$
$$x = -\frac{48}{7}$$

At other point of intersection:

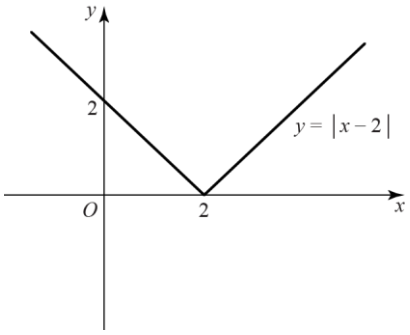
$$4(x+6)+1 = -\frac{1}{2}x + 1$$
$$4x + 25 = -\frac{1}{2}x + 1$$
$$8x + 50 = -x + 2$$
$$9x = -48$$
$$x = -\frac{16}{3}$$

So the solutions are $x = -\frac{48}{7}$ and $x = -\frac{16}{3}$

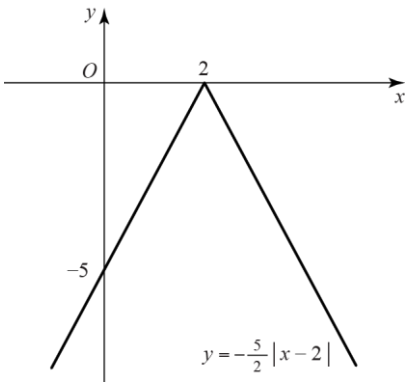
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5 a Start with $y = |x|$

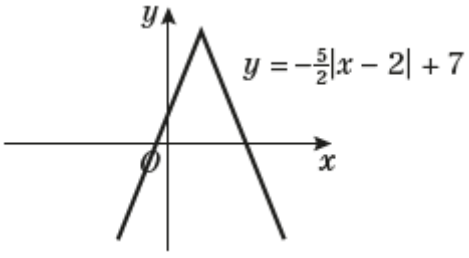
$y = |x - 2|$ is a horizontal translation of +2



$y = -\frac{5}{2}|x - 2|$ is a vertical stretch scale factor $-\frac{5}{2}$



$y = -\frac{5}{2}|x - 2| + 7$ is a vertical translation of +7



b The range is $g(x) \leq 7$

c At one point of intersection:

$$-\frac{5}{2}(x-2)+7 = x+1$$
$$-\frac{5}{2}x+12 = x+1$$
$$-5x+24 = 2x+2$$
$$22 = 7x$$
$$x = \frac{22}{7}$$

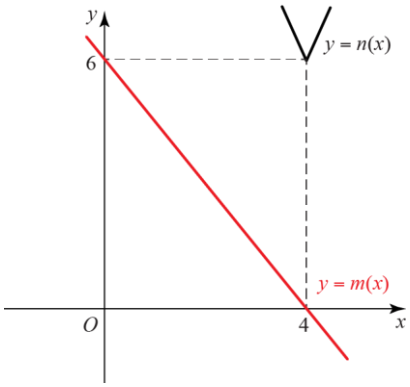
At other point of intersection:

$$\frac{5}{2}(x-2)+7 = x+1$$
$$\frac{5}{2}x+2 = x+1$$
$$5x+4 = 2x+2$$
$$3x = -2$$
$$x = -\frac{2}{3}$$

So the solutions are $x = -\frac{2}{3}$ and $x = \frac{22}{7}$

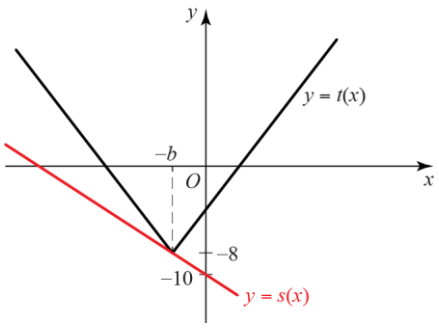
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6 For the equation m(x) = n(x) to have no real roots, it must be the case that y = m(x) and y = n(x) do not intersect.



The least value of
$y = \text{n}(x) = 3|x-4|+6$ is
$y = 6$ when $x = 4$
Hence, we need m(4) < 6 to avoid intersection
So $-2(4) + k < 6$
$-8 + k < 6$
$k < 14$

7 For the equation s(x) = t(x) to have exactly one real root, it must be the case that y = s(x) and y = t(x) intersect at the minimum point of t(x).



The least value of
$y = \text{t}(x) = 2|x+b|-8$ is
$y = -8$ when $x = -b$
Hence, we need s(–b) = –8 to ensure one intersection
$\Rightarrow -8 = -10 - (-b)$
$b = 2$

8 **a** The range is h(x) ≥ −7

b h(x) is many-to-one, therefore $\text{h}^{-1}(x)$ would be one-to-many, and so would not be a function.

c At one point of intersection:
$$-\frac{2}{3}(x-1)-7=-6$$
$$2x-2+21=18$$
$$2x=-1$$
$$x=-\frac{1}{2}$$
At other point of intersection:
$$\frac{2}{3}(x-1)-7=-6$$
$$2x-2-21=-18$$
$$2x=5$$
$$x=\frac{5}{2}$$
So the solutions are
$x = -\frac{1}{2}$ and $x = \frac{5}{2}$
h(x) < –6 between the two points of intersection, so the solution to the inequality h(x) < −6 is
$$-\frac{1}{2} < x < \frac{5}{2}$$

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8 d Since $\mathrm{h}(x) \geq -7$ and $\mathrm{h}(1) = -7$, then for the equation $\mathrm{h}(x) = \frac{2}{3}x + k$ to have no solutions, we require

$$\frac{2}{3}(1) + k < -7$$

$$\Rightarrow k < -\frac{23}{3}$$

9 a We can write h as

$$\mathrm{h}(x) = \begin{cases} a + 2(x+3), & x \leq -3 \\ a - 2(x+3), & x \geq -3 \end{cases}$$

The line which has gradient –2 and passes through (0, 4) is $y = -2x + 4$

So, for $x \geq -3$

$$-2(x + 3) + a = -2x + 4$$
$$-2x - 6 + a = -2x + 4$$
$$a = 10$$

b At P, $\mathrm{h}(x) = 10$ (from part **a**)

So $10 = 10 - 2(x + 3)$

$$-2x - 6 = 0$$
$$x = -3$$

At Q, $\mathrm{h}(x) = 0$

So $0 = 10 - 2(x + 3)$

$$4 - 2x = 0$$
$$x = 2$$

$P(-3, 10)$ and $Q(2, 0)$

c $\mathrm{h}(x) = \frac{1}{3}x + 6$

At one point of intersection:

$$10 - 2(x+3) = \frac{1}{3}x + 6$$
$$4 - 2x = \frac{1}{3}x + 6$$
$$12 - 6x = x + 18$$
$$7x = -6$$
$$x = -\frac{6}{7}$$

At other point of intersection:

$$10 + 2(x+3) = \frac{1}{3}x + 6$$
$$16 + 2x = \frac{1}{3}x + 6$$
$$48 + 6x = x + 18$$
$$5x = -30$$
$$x = -6$$

So the solutions are

$x = -6$ and $x = -\frac{6}{7}$

10 a The range of $\mathrm{m}(x)$ is $\mathrm{m}(x) \leq 7$

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10 b $m(x) = \frac{3}{5}x + 2$

At one point of intersection:

$$-4(x+3)+7 = \frac{3}{5}x+2$$
$$-4x-5 = \frac{3}{5}x+2$$
$$-20x-25 = 3x+10$$
$$-23x = 35$$
$$x = -\frac{35}{23}$$

At other point of intersection:

$$4(x+3)+7 = \frac{3}{5}x+2$$
$$4x+19 = \frac{3}{5}x+2$$
$$20x+95 = 3x+10$$
$$17x = -85$$
$$x = -5$$

So the solutions are $x = -5$ and $x = -\frac{35}{23}$

c For two distinct roots, there are two points of intersection, so $m(x) < 7$. Therefore, $k < 7$.

Challenge

1 a At A:

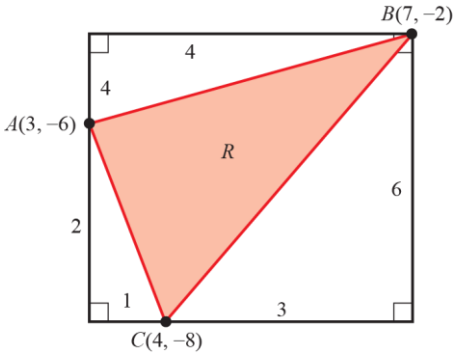
$$-2(x-4)-8 = x-9$$
$$-2x = x-9$$
$$-3x = -9$$
$$x = 3$$
$$y = 3 - 9 = -6$$

At B:

$$2(x-4)-8 = x-9$$
$$2x-16 = x-9$$
$$x = 7$$
$$y = 7 - 9 = -2$$

$A(3, -6)$ and $B(7, -2)$

b Taking the shaded triangle R and enclosing it in a rectangle looks like:



$$R = (4\times6) - \left(\frac{1}{2}\times4\times4\right) - \left(\frac{1}{2}\times6\times3\right) - \left(\frac{1}{2}\times2\times1\right)$$

$$R = 24 - 8 - 9 - 1$$
$R = 6$ units^2

2 At the first point of intersection:

$$x - 3 + 10 = -2(x-3) + 2$$
$$x + 7 = -2x + 8$$
$$3x = 1$$
$$x = \frac{1}{3}$$

At the other point of intersection:

$$-(x-3) + 10 = 2(x-3) + 2$$
$$-x + 13 = 2x - 4$$
$$-3x = -17$$
$$x = \frac{17}{3}$$

Maximum point of $f(x)$ is
$f(x) = 10$ when $x = 3$, so at (3, 10)
Minimum point of $g(x)$ is
$g(x) = 2$ when $x = 3$, so at (3, 2)

Area of a kite $= \frac{1}{2} \times$ width $\times$ height

$$= \frac{1}{2} \times \left(\frac{17}{3} - \frac{1}{3}\right) \times (10 - 2)$$
$$= \frac{1}{2} \times \frac{16}{3} \times 8$$
$$= \frac{64}{3} \text{ units}^2$$

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