Functions and graphs 2C

1 a $\text{pq}(-8) = \text{p}\left(\frac{-8}{4}\right)$
$= \text{p}(-2)$
$= 1 - 3(-2)$
$= 7$

b $\text{qr}(5) = \text{q}[(5-2)^2]$
$= \text{q}(9)$
$= \frac{9}{4}$

c $\text{rq}(6) = \text{r}\left(\frac{6}{4}\right)$
$= \text{r}\left(\frac{3}{2}\right)$
$= \left(\frac{3}{2} - 2\right)^2$
$= \frac{1}{4}$

d $\text{p}^2(-5) = \text{p}(1 - 3(-5))$
$= \text{p}(16)$
$= 1 - 3(16)$
$= -47$

e $\text{pqr}(8) = \text{pq}[(8-2)^2]$
$= \text{pq}(36)$
$= \text{p}\left(\frac{36}{4}\right)$
$= \text{p}(9)$
$= 1 - 3(9)$
$= -26$

2 a $\text{fg}(x) = \text{f}(x^2 - 4)$
$= 4(x^2 - 4) + 1$
$= 4x^2 - 15$

b $\text{gf}(x) = \text{g}(4x+1)$
$= (4x+1)^2 - 4$
$= 16x^2 + 8x - 3$

c $\text{gh}(x) = \text{g}\left(\frac{1}{x}\right)$
$= \left(\frac{1}{x}\right)^2 - 4$
$= \frac{1}{x^2} - 4$

d $\text{fh}(x) = \text{f}\left(\frac{1}{x}\right)$
$= 4 \times \left(\frac{1}{x}\right) + 1$
$= \frac{4}{x} + 1$

e $\text{f}^2(x) = \text{ff}(x)$
$= \text{f}(4x+1)$
$= 4(4x+1) + 1$
$= 16x + 5$

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3 a $\text{fg}(x) = \text{f}(x^2)$
$= 3x^2 - 2$

b $\text{gf}(x) = \text{g}(3x-2)$
$= (3x-2)^2$

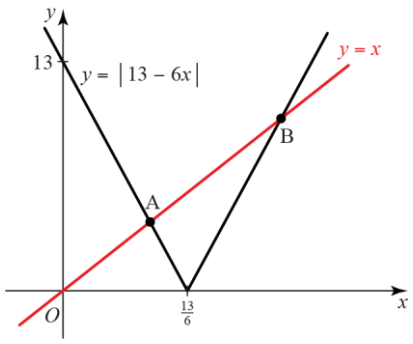
When $\text{fg}(x) = \text{gf}(x)$ then
$3x^2 - 2 = (3x-2)^2$
$3x^2 - 2 = 9x^2 - 12x + 4$
$0 = 6x^2 - 12x + 6$
$0 = x^2 - 2x + 1$
$0 = (x-1)^2$
Hence $x = 1$

4 a $\text{qp}(x) = \text{q}\left(\dfrac{1}{x-2}\right)$
$= 3 \times \left(\dfrac{1}{x-2}\right) + 4$
$= \dfrac{3}{x-2} + \dfrac{4(x-2)}{x-2}$
$= \dfrac{4x-5}{x-2}$

b If $\text{qp}(m) = 16$ then
$3\left(\dfrac{1}{m-2}\right) + 4 = 16$
$\dfrac{3}{m-2} = 12$
$3 = 12(m-2)$
$\dfrac{3}{12} = m - 2$
$\dfrac{1}{4} = m - 2$
$m = \dfrac{9}{4}$

5 a $\text{fg}(6) = \text{f}\left(\dfrac{3(6)-2}{2}\right)$
$= \text{f}(8)$
$= |9 - 4(8)|$
$= |-23|$
$= 23$

b $\text{fg}(x) = \text{f}\left(\dfrac{3x-2}{2}\right)$
$= \left|9 - 4\left(\dfrac{3x-2}{2}\right)\right|$
$= |9 - 6x + 4|$
$= |13 - 6x|$
Now $\text{fg}(x) = x$ when $|13 - 6x| = x$



At A: $13 - 6x = x$
$13 = 7x$
$x = \dfrac{13}{7}$

At B: $-(13 - 6x) = x$
$5x = 13$
$x = \dfrac{13}{5}$

The solutions are
$x = \dfrac{13}{7}$ and $x = \dfrac{13}{5}$

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6 a $f^2(x) = f\left(\frac{1}{x+1}\right)$

$$= \left(\frac{1}{\left(\frac{1}{x+1}\right)+1}\right)$$

$$= \left(\frac{1}{\left(\frac{1+x+1}{x+1}\right)}\right)$$

$$= \left(\frac{x+1}{x+2}\right)$$

b $f^3(x) = f\left(\frac{x+1}{x+2}\right)$

$$= \left(\frac{1}{\left(\frac{x+1}{x+2}\right)+1}\right)$$

$$= \left(\frac{1}{\left(\frac{x+1+x+2}{x+2}\right)}\right)$$

$$= \left(\frac{x+2}{2x+3}\right)$$

7 a $st(x) = s(x+3)$
$= 2^{x+3}$

b $ts(x) = t(2^x)$
$= 2^x + 3$

c $2^{x+3} = 2^x + 3$
$2^{x+3} - 2^x = 3$
$2^x \times 2^3 - 2^x = 3$
$2^x(8-1) = 3$
$2^x = \frac{3}{7}$
$x \ln 2 = \ln\left(\frac{3}{7}\right)$
$x = \frac{\ln\left(\frac{3}{7}\right)}{\ln 2}$

8 a $gf(x) = g(e^{5x})$
$= 4\ln(e^{5x})$
$= 4(5x)$
$= 20x$

b $fg(x) = f(4\ln x)$
$= e^{5(4\ln x)}$
$= e^{\ln x^{20}}$
$= x^{20}$

9 a $qp(x) = q(\ln(x+3))$
$= e^{3(\ln(x+3))} - 1$
$= e^{\ln(x+3)^3} - 1$
$= (x+3)^3 - 1$

Since $x > -3$, so $qp(x) > -1$

b $qp(7) = (7+3)^3 - 1$
$= 999$

c $qp(x) = (x+3)^3 - 1 = -126$
$(x+3)^3 = -125$
$x + 3 = -5$
$x = -8$

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10 $t^2(x) = t(5 - 2x)$
$= 5 - 2(5 - 2x)$
$= 5 - 10 + 4x$
$= -5 + 4x$

$t^2(x) - (t(x))^2 = 0$
$-5 + 4x - (5 - 2x)^2 = 0$
$-5 + 4x - 25 + 20x - 4x^2 = 0$
$-4x^2 + 24x - 30 = 0$
$2x^2 - 12x + 15 = 0$

Using the formula:

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4 \times 2 \times 15}}{2 \times 2}$$
$$= \frac{12 \pm \sqrt{24}}{4}$$
$$= \frac{12 \pm 2\sqrt{6}}{4}$$
$$= 3 \pm \frac{\sqrt{6}}{2}$$

11 a Range of g is $-8 \leq x \leq 12$

b From the graph,

$$g(x) = -\frac{1}{2}x + 12 \text{ for } 0 \leq x \leq 14$$
and $g(0) = 12$

So $gg(0) = g(12)$
$$= -\frac{1}{2}(12) + 12$$
$$= 6$$

c $$gh(7) = g\left(\frac{2(7) - 5}{10 - 7}\right)$$
$$= g(3)$$
$$= -\frac{1}{2}(3) + 12$$
$$= 10.5$$

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