Integration 11I

1 a

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1	1.1260	1.2559	1.4142	1.6529

b i $\text{Area} \approx \frac{1}{2}h\left(y_0 + 2(y_1 + \ldots) + y_n\right)$

$= \frac{1}{2} \times \frac{\pi}{6}\left(1 + 1.6529 + 2 \times 1.2559\right) = 1.352$

ii $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{12}\left(1 + 1.6529 + 2(1.1260 + 1.2559 + 1.4142)\right) = 1.341$

2 a

θ	$-\frac{\pi}{5}$	$-\frac{\pi}{10}$	0	$\frac{\pi}{10}$	$\frac{\pi}{5}$
y	0	0.7071	1	0.7071	0

b $R = \frac{1}{2} \times \frac{\pi}{10}\left(0 + 0 + 2(0.7071 + 1 + 0.7071)\right) = 0.758$

c The shape of the graph is concave, so the trapezium lines will underestimate the area**.**

d $\int_{-\frac{\pi}{5}}^{\frac{\pi}{5}} \cos\frac{5\theta}{2}\,\mathrm{d}\theta = \left[\frac{2}{5}\sin\frac{5\theta}{2}\right]_{-\frac{\pi}{5}}^{\frac{\pi}{5}} = \frac{4}{5} = 0.8$

e $\text{Percentage error} = \frac{0.8 - 0.758}{0.8} \times 100\% = 5.25\%$

3 a

x	0	0.5	1	1.5	2
y	0.707	0.614	0.519	0.427	0.345

b Area using the trapezium rule:

$\approx \frac{1}{2}h\left(y_0 + 2(y_1 + \ldots) + y_n\right) = \frac{1}{4}\left(0.707 + 0.345 + 2(0.614 + 0.519 + 0.427)\right)$

= 1.04 to 2 decimal places

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4 a

x	1	1.5	2	2.5	3
y	1	0.7973	1	1.4581	2.0986

b i $\text{Area} \approx \frac{1}{2} \times 1\left(1 + 2.0986 + 2\right) = 2.549$

ii $\text{Area} \approx \frac{1}{2} \times \frac{1}{2}\left(1 + 2.0986 + 2(0.7973 + 1 + 1.4581)\right) = 2.402$

c Increasing the number of values decreases the interval. This leads to an approximation more closely following the curve.

d $\int_1^3 \left((x-2)\ln x + 1\right)\mathrm{d}x = \int_1^3 (x-2)\ln x \,\mathrm{d}x + \int_1^3 \mathrm{d}x$

Let $u = \ln x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$

$\frac{\mathrm{d}v}{\mathrm{d}x} = x - 2 \Rightarrow v = \frac{(x-2)^2}{2}$

$I = \left[\frac{(x-2)^2 \ln x}{2}\right]_1^3 - \int_1^3 \frac{(x-2)^2}{2x}\,\mathrm{d}x + 2$

$= \frac{1}{2}\ln 3 - \int_1^3 \frac{x^2 - 4x + 4}{2x}\,\mathrm{d}x + 2$

$= \frac{1}{2}\ln 3 - \int_1^3 \left(\frac{x}{2} - 2 + \frac{2}{x}\right)\mathrm{d}x + 2$

$= \frac{1}{2}\ln 3 - \left[\frac{x^2}{4} - 2x + 2\ln x\right]_1^3 + 2$

$= \frac{1}{2}\ln 3 + 2 - \left(\frac{9}{4} - 6 + 2\ln 3\right) + \left(\frac{1}{4} - 2\right) = -\frac{3}{2}\ln 3 + 4$

5 a

x	0	0.5	1	1.5	2
y	0	0.6124	1	1.0607	0

b $\text{Area} \approx \frac{1}{2} \times 0.5\left(0 + 0 + 2(0.6124 + 1 + 1.0607)\right) = 1.337$

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5 c $I = \int_0^2 x\sqrt{2-x}\ \mathrm{d}x$

Let $u = 2-x \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = -1$

$I = \int_2^0 -(2-u)u^{\frac{1}{2}}\ \mathrm{d}u = \int_2^0 u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\ \mathrm{d}u = \left[\frac{2}{5}u^{\frac{5}{2}} - \frac{4}{3}u^{\frac{3}{2}}\right]_2^0$

$= 0 - \left(\frac{2}{5}\sqrt{32} - \frac{4}{3}\sqrt{8}\right) = \frac{8}{3}\sqrt{2} - \frac{8}{5}\sqrt{2} = \frac{16}{15}\sqrt{2} = \frac{2^{\frac{9}{2}}}{15}$

$p = \frac{16}{15}, q = \frac{1}{2}$

d $\frac{16}{15}\sqrt{2} = 1.509$

Percentage error $= \dfrac{1.509 - 1.337}{1.509} \times 100\% = 11.4\%$

6 a $y = \dfrac{4x-5}{(x-3)(2x+1)}$

$y = 0 \Rightarrow 4x - 5 = 0 \Rightarrow x = \frac{5}{4}$

$A\left(\frac{5}{4}, 0\right)$

b

x	0	0.25	0.5	0.75	1	1.25
y	1.6667	0.9697	0.6	0.3556	0.1667	0

c Area $\approx \frac{1}{2} \times 0.25\left(1.6667 + 0 + 2(0.9697 + 0.6 + 0.3556 + 0.1667)\right) = 0.7313$

d $\dfrac{4x-5}{(x-3)(2x+1)} = \dfrac{A}{x-3} + \dfrac{B}{2x+1}$

$4x - 5 = A(2x+1) + B(x-3)$

Let $x = 3 : 7 = 7A \Rightarrow A = 1$

Let $x = -\frac{1}{2} : -7 = -\frac{7}{2}B \Rightarrow B = 2$

$y = \dfrac{1}{x-3} + \dfrac{2}{2x+1}$

$I = \int_0^{\frac{5}{4}} \left(\frac{1}{x-3} + \frac{2}{2x+1}\right)\ \mathrm{d}x = \left[\ln|x-3| + \ln|2x+1|\right]_0^{\frac{5}{4}}$

$= \ln\frac{7}{4} + \ln\frac{7}{2} - \ln 3 - \ln 1 = \ln 7 + \ln 7 - (\ln 4 + \ln 2 + \ln 3) = \ln\frac{49}{24}$

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6 e $\ln\dfrac{49}{24} = 0.7137$

$\text{Percentage error} = \dfrac{0.7137 - 0.7313}{0.7137} \times 100\% = 2.5\%$

7 a

x	0	0.5	1	1.5	2	2.5	3
y	2.7183	4.1133	5.6522	7.3891	9.3565	11.5824	14.0940

b $\text{Area} \approx \dfrac{1}{2} \times 0.5\left(2.7183 + 14.0940 + 2(4.1133 + 5.6522 + 7.3891 + 9.3565 + 11.5824)\right) = 23.25$

c $I = \displaystyle\int_0^3 \mathrm{e}^{\sqrt{2x+1}}\ \mathrm{d}x$

Let $t = \sqrt{2x+1} \Rightarrow \dfrac{\mathrm{d}t}{\mathrm{d}x} = \dfrac{1}{\sqrt{2x+1}} = \dfrac{1}{t}$

$I = \displaystyle\int_1^{\sqrt{7}} t\mathrm{e}^t\ \mathrm{d}t$

$a = 1, b = \sqrt{7}, k = 1$

d $I = \displaystyle\int_1^{\sqrt{7}} t\mathrm{e}^t\ \mathrm{d}t$

Let $u = t \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}t} = 1$

$\dfrac{\mathrm{d}v}{\mathrm{d}t} = \mathrm{e}^t \Rightarrow v = \mathrm{e}^t$

$I = \left[t\mathrm{e}^t\right]_1^{\sqrt{7}} - \displaystyle\int_1^{\sqrt{7}} \mathrm{e}^t\ \mathrm{d}t = \sqrt{7}\mathrm{e}^{\sqrt{7}} - \mathrm{e} - \mathrm{e}^{\sqrt{7}} + \mathrm{e} = \left(\sqrt{7} - 1\right)\mathrm{e}^{\sqrt{7}} = 23.20$

8 a

x	$\dfrac{\pi}{3}$	$\dfrac{5\pi}{12}$	$\dfrac{\pi}{2}$	$\dfrac{7\pi}{12}$	$\dfrac{2\pi}{3}$
y	1.15470	1.03528	1	1.03528	1.15470

b $\text{Area} \approx \dfrac{1}{2} \times \dfrac{\pi}{12}\left(1.15470 + 1.15470 + 2(1.03528 + 1 + 1.03528)\right) = 1.106$

c $I = \displaystyle\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \operatorname{cosec} x\ \mathrm{d}x = \left[-\ln\left|\operatorname{cosec} x + \cot x\right|\right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}}$

$= -\ln\left(\dfrac{2}{\sqrt{3}} - \dfrac{1}{\sqrt{3}}\right) - \left(-\ln\left(\dfrac{2}{\sqrt{3}} + \dfrac{1}{\sqrt{3}}\right)\right) = -\ln\dfrac{1}{\sqrt{3}} + \ln\dfrac{3}{\sqrt{3}}$

$= -\ln 1 + \ln\sqrt{3} + \ln 3 - \ln\sqrt{3} = \ln 3$

d $\text{Percentage error} = \dfrac{\ln 3 - 1.106}{\ln 3} \times 100\% = 0.67\%$

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